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## An Investigation In Journal Writing And Cooperative Learning With 8th Grade Geometry Students In The Construction Of Proof

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AN INVESTIGATION IN JOURNAL WRITING AND  
COOPERATIVE LEARNING WITH  
8<sup>TH</sup> GRADE GEOMETRY STUDENTS  
IN THE CONSTRUCTION OF PROOF

by

PAULWOJCIK

B.S. Indiana University of Pennsylvania, 1986

A thesis submitted in partial fulfillment of the requirement  
for the degree of Master of Education in K-8 Mathematics and Science  
in the Department of Teaching and Learning Principles  
in the College of Education  
at the University of Central Florida  
Orlando, Florida

Spring Term

2011

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## **ABSTRACT**

This action research study summarizes the investigation of journal writing and cooperative grouping with 8<sup>th</sup> grade geometry students in the construction of proof. Students written responses to prompts in journals were analyzed over the course of twelve weeks. Case studies of four students were developed from the researchers' three geometry classes. All four students in the study participated in an academic program called Pre International Baccalaureate Preparation.

Standardized test scores and an attitude scale categorized the four students. The ATMAT survey (Appendix A) measured each student's attitude toward mathematics. Writing prompts focused on the students' perceptions of the group process in constructing proofs and the development of geometric proof. The results suggested the students were engaged in learning within their cooperative groups but they also desired individual time before coming to a group setting. In addition, students' written responses to journal prompts may provide an informal assessment and help students convey their own understanding of proof before any formal assessments.

## ACKNOWLEDGMENTS

First, I would like to thank God for providing the time, patience, and flexibility in completing this process. Many people in my life have provided their time, a listening ear, and their willingness to help in many ways.

I would like to thank my chairperson, Dr. Erhan Hacıomeroglu, for helping me to complete the action research. The feedback you provided was invaluable. Your firm and sincere guidance made this work possible and I am eternally grateful. I would like to acknowledge the help and influence of Dr. Enrique Ortiz and Dr. Michelle Stephan. Over the course of the last three years of graduate study, being in or visiting your classrooms helped tremendously.

Finally, I would like to thank the many people I have met along this journey and those who have been part of my life for many years. My daughters Theresa, Molly, and Katrina provided me the inspiration to keep working. Their mom Mary supported me for many years and I am grateful. My Planetters whom I met because of this experience namely Maura, Megan, and Adam, your friendship and support was extremely helpful. I always looked forward to all the classes, projects, and our experiences together. And to all of my Lockheed Martin cohorts, I enjoyed all our time and mutual support.

I must recognize the many friends who listened to my concerns repeatedly: Michael, Ricardo, Brett, Tyler, Arthur, Jackie, and to our Higher Power. Many thanks to my geometry students this year who participated in the study and I am grateful to my parents Julia and John Wojcik and my sister Gigi for believing in me.

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## CHAPTER ONE: INTRODUCTION

### Introduction

Research has shown that academic achievement improves in cooperative group settings (Gilles & Boyle 2010). However, teachers struggle with implementing cooperative group activities in mathematics (Cohen, 1994, Gilles & Boyle, 2010). The concern may center on individual accountability. Are the student's involved in the group activities comprehending the standard or the task? Can a teacher be certain all students are equally participating? Questions such as these force me as a teacher to find a way to determine methods of accountability for the individual student. Written prompts from journal writings can inform the teacher about the individuals understanding (Aspinwall & Miller 2001).

In eighth grade geometry, academically talented students begin the process of formulating proof. Keeping a record of constructing proof, and how the group process affected the writing of proof appeared to be a reasonable task. My previous classroom experience in teaching geometry helped me to realize working cooperatively may provide assistance in developing proof. I planned to have student's record proof along with reflections after writing the proof so I could gauge their progress. My concerns were the use of cooperative groups, therefore the writing would provide me understanding on the mathematical development. I could not meet with every student each day, therefore the journal writing provided another tool to identify student progress. According to Kagan (1992), students can become a self-evaluator if given the opportunity on a regular basis.

Initially, I received data on my eighth grade geometry students by interpreting their previous years standardized test scores, the Florida Comprehensive Assessment Test (FCAT). It appeared that the students I taught were well equipped to work in cooperative settings, yet I wanted students engaged and I desired to know what was taking place during group work as I was facilitating the class as a whole.

Therefore, I felt students needed an opportunity to reflect on their proofs in the groups. I wanted to comprehend what each student was learning. According to Ramdass and Zimmerman (2008), writing provided opportunities for reflection. Therefore, I instituted a new practice in my classroom. I introduced journal writing at the beginning of an 8<sup>th</sup> grade geometry class. I wanted to determine if the journal writing along with cooperative learning provided insight into the learning of proof.

According to the Next Generation Sunshine State Standards (2009), students must meet certain language arts standards imbedded in the learning of their mathematics skills, especially at the Honors Geometry level. One of these skills is to pre-write using organizational strategies. Journaling can provide a venue for students to put any thoughts on the page. In geometry class students could draw the geometric figures, organize tables, charts, and theorems. This helped students organize for the writing of proofs in statement and reason form and paragraph form. Using journal writing as a tool may provide the students to become that individual who takes the initiative in their own educational experiences (Ramdass & Zimmerman 2008). In the past year, I had some success to get students to reflect on their problem solving experience. The writing process took time to initiate especially if I wanted the students to develop the amount of detail

that justified mathematical solutions to problems. Research has shown (Countryman 1992, Ramdass & Zimmerman 2008, Sanders 2009) students can reflect on their roles and become a self-advocate for constructing their own mathematical knowledge. As a byproduct of their own reflections, students may explain their own degree of understanding, and give me as their teacher an opportunity to direct my instruction.

### **Rationale**

The rationale behind the study is my desire to engage students in learning mathematics and to promote discussions within their groups. My vision of group work allowed individuals to argue their positions and learn from one another. As I instituted reform curriculum in previous years, I recognized student considered high achievers, were not producing work that I valued as proficient for the mathematical topic the student was studying. I had concerns about how I implemented reform curriculum. The National Council of Teachers of Mathematics (NCTM) indicated that communication needed to play a major role in the classroom. Students needed to talk to one another and place their mathematical ideas both in written and verbal form. In the previous two years, I was trained to use the Connected Math Program (CMP2). CMP2 (Lappan, Fey, Friel, & Phillips, 2006) provided tasks that required me to set up a problematic situation and then the students would explore within cooperative learning groups for the majority of the class period. My role was to facilitate each group by using a variety of questions. The last part of the class students shared their work and I synthesized the mathematics that was imbedded in the task. Here was where my level of concern grew. I felt within many of the tasks that we explored, the mathematics was not recorded and I did not get a good understanding of what the students were constructing. Again, the training promised that high achieving students would

expand their mathematical horizons. I understood that it was a different pedagogy and one in which I had not personally experienced with the exception of attending training sessions. I wanted more. I wanted to know what the students thought, and what they understood. Then, I would be in a position to help students achieve and grow. I had always valued students sharing their knowledge and presenting it to the class. I had been a proponent of students talking in front of their own classmates since I had started teaching over twenty- four years ago. Since I wanted to know what the students were thinking, I designed writing prompts to describe their thought process. I wanted the writing to describe the events within the groups. I wanted information that would help me make inferences about their understanding, improve my own teaching practices, and enhance the communication between students. Time did not permit me to ask every individual student, yet I hoped with selected journal prompts, opportunities for insight into what the student was thinking would be available as I read and commented on a student's entries. Students would have to organize and summarize a mathematical concept in their own words. Perhaps the writing could lead to students taking more initiative in their own work, and determine the usefulness of a collaborative group process.

In the following chapter, I examined the implementation of cooperative learning and the use of journal writing. Within the literature there appears to be a gap for the use of writing with geometry students. Studies on writing were completed for elementary and middle school math courses. Also, research has been done with journal writing with calculus students at the high school and in college (Aspinwall & Miller, 2001). Yet in a geometry class where writing was necessary due to the construction of mathematical proof, there was a gap in the literature. The most recent article on Writing and Geometry was in 2009. Sanders (2009) developed an article

for the Mathematics Teacher as a classroom teacher to give ideas on the ways to implement journal writing within the classroom. It appeared that all the information was original and based upon the authors experience. The following case-study hopes to add to the body of work on journal writing but incorporate geometric proof and the use of cooperative groups.

As a graduate student in the Lockheed Martin program at the University of Central Florida, I reflected on all aspects of my experience as a student. I wrote reflections on articles, websites, and classroom experiences. I felt that I became a student who could identify my own strengths and weaknesses. Therefore, I go back to a quote from Ramdass and Zimmerman (2008) “accurate self-reflection is important to student’s success in math” (p.21). Thus, the idea came about that I could give my students the same opportunity for self-reflection. The study in my classroom chose four students out of the fifty-three geometry students I taught. The journals track their progress over a period of twelve weeks. The experience in my geometry class produced one research question:

*Research question*

*How does journal writing reveal the nature of students’ cooperative learning in the construction of geometric proofs?*



## **CHAPTER TWO: LITERATURE REVIEW**

Taking into account my research question three areas were needed to review existing literature: proof in geometry, cooperative grouping, and journaling. Research was available on all three areas yet at this point, no intersection of these topics has occurred within the literature. I propose proof within geometry has been a challenge for students to master alone and the choice of the researcher to identify the affects of cooperative grouping and journaling may add to the existing literature.

Within my review of the literature on cooperative grouping and journaling, I continued to search for the connection to individual accountability. Themes that also appeared were the difficulties of teacher implementation with cooperative grouping, the structure of journaling and the bias in journaling. The following summary of the literature review note the influence on these themes.

### **Proof in Geometry**

Herbst (2002, p. 308) indicated, “proof is essential in mathematics education not only as a valuable process for students to engage in (such as their developing their capacity for mathematical reasoning) but, more importantly, as a necessary aspect of knowledge construction”. Herbst has documented the use of proof, specifically two-column proof since the turn of the century. Yet he believed proof by its own nature should reflect what a “community of knowers want to know more about “(p. 308). NCTM and Herbst encouraged teachers to look for other ways to help students in proving. The research Herbst completed since 1998 tends to focus on proof from the student’s responsibilities. He argued, “important questions that must be raised

concern what it takes to organize classrooms where students can be expected to produce arguments and proof” (2002, p. 284). Herbst vacillated between the idea of maintaining a classroom environment where two column proof is not the main focus in a high school geometry class. Current reforms in NCTM Reasoning and Proof Standard indicate students should be able to produce proof in different formats such as two column, paragraph or any other version (NCTM 2000, p. 345).

Although many teachers believe a student’s introduction to formal proof needs a structure such as two column or paragraph form, Herbst research indicates that for over a century school mathematics has utilized this form to assist teachers and students to see the logic of the argument. Teachers have the opportunity to make decisions based on the interactions of their students. Martin, Soucy McCrone, Wallace Bower, and Dindyal (2005) researched the interplay of teacher and student actions in the teaching and learning of proof. Martin and her associates agree that students need to create mathematical arguments and one of the first courses that allow students the structure to manifest an argument is a high school geometry class. Herbst and Martin et al. agree on the gap regarding the “pedagogical factors to the learning of proof” (Martin, Soucy McCrone, Wallace Bower, & Dindyal, 2005 p. 95). Martin et al. inspected a variety of methods a teacher used to assist students in constructing and developing proof. Among them were direct instruction, questioning, and cooperative learning. Herein was the opportunity for this researcher to combine the teacher’s choices of methodologies on student construction of proof takes place. Namely, utilize cooperative learning and journaling to assist students in documenting their findings within the proof construction. Furthermore, the cooperative learning allows students the opportunity to communicate ideas with one another and

either validate or question their own assumptions. Coupled with NCTM's Reasoning and Proof Standard (2000) and the Communication Standard students had the opportunity to organize their thinking and at the same time use the language of mathematics, geometric proof and the symbols associated with it, to express their ideas as individuals.

### **Implementation of Cooperative Learning**

Cooperative learning leads to communication, students taking on different social roles, establishing relationships, which may lead to the learning of mathematics (Brown, 2007). Traditional instruction solely focuses on the individual, while effective cooperative education involves group goals and individual accountability (Slavin, 1994). According to Brown members within a group constantly practice, renegotiate, and investigate within the cooperative learning community. The participation within a group helps students develop beliefs that mathematics can be constructed and was based on the level of work within their community. Even with this positive outlook on cooperative grouping, research has shown that teachers struggle with the implementation of cooperative group activities in mathematics (Cohen, 1994; Gillies & Boyle, 2008).

The concern is that students within the groups are “passive recipients” instead of “being active in creation.” “A reluctance to embrace cooperative learning may be partly due to the challenge it poses to teachers’ control of the channels of communication and the demands it places on curriculum organization” (Gillies & Boyle, 2010, p. 933-934). Gillies and Boyle state in their research that a teacher needs to provide situations where investigation and engagement occur between students. In other words, teachers must strive for a more student-centered

classroom to develop higher order thinking and problem solving skills. Within a geometry class, a student-centered classroom can be a place where students operate within an axiomatic system and begin to prove relationships among lines, triangles, and quadrilaterals. Students were given the opportunity to explore with simple tools such as a protractor and compass to determine relationships among lines, triangles, and quadrilaterals. Coupled with the teacher's options to utilize small groups for discussion, a student can begin to construct a proof.

Still with these struggles in communication and organization, the Principles and Standards for School Mathematics urge teachers to "build a sense of community" (National Council of Teachers of Mathematics [NCTM], 2000). According to NCTM, my role was to develop norms, worthwhile tasks, guide classroom discussions, and monitor learning.

Gillies (2003), Brown (2007), Cohen (1994), and Kagan (1992) all speak to group structure and group composition. Spencer Kagan's (1992) book on cooperative grouping set a standard for multiple structures within any curriculum. Kagan has coined over ninety structures of which I have used three to develop a sense of group community as well as individual accountability. The three structures used in this study were roundtable, team interview, and think pair share. Short amounts of time and practice within these group structure assisted students to make appropriate transition into groups and understand their individual role.

Research has indicated that student cooperation promotes significant results in achievement (Slavin, 1994). Von Glasersfeld (1993) states the opportunity for students to express themselves using their language and work at problems by explaining the process generates reflection. Student interaction contributes to individual understanding and knowledge.

This new found knowledge was constructed by the students. Students will reflect on their social interaction and change it “if a new result is desired” (Von Glasersfeld, 1993, p. 31). Meanwhile the teacher’s role is to monitor the group progress, intervene by identifying a student’s level of understanding and “challenge each student to progress” (p. 33).

Traditionally and historically, working together on making sense of problems is outside of the student’s experience (Wheatley, 1993). “The opportunities for students to discuss, reflect and negotiate meanings within tasks promote understanding since it is constructed from the student’s point of view” (p. 132). Written reflection is the next step. It “is a critical turning point in the entire culture development of the child” (Vygotsky, 1978, p. 106).

### **Journaling**

“One method of gaining insight into the social patterns of participation and the influences different individuals and groups achieve is the use of structured journal writing” (Brown, 2007, p. 119). Brown explained that students need to be given time to explain not only their social positions within a group, but also how it impacted their own train of thought. Students own identities will emerge as they communicate with others. Therefore, the work within cooperative groups and journal writing become part of the framework of the mathematical task. Actions such as solving the task and reflecting on it afterward become part of “interactive communication which constitutes the central activity of teaching” (Steffe, 1991, p.189).

Gathering reflections from student journal provide a teacher with a student centered analysis of one’s own comprehension (Brown, 2007). Borasi and Rose (1989) established a position in their study on journal writing and mathematics instruction that individual writing can

engage all students. A certain degree of individual instruction can occur due to entries that each student can reflect on situations of discovery or difficulty.

The keeping of a journal in mathematics allows for the student to “record events, feelings, and ideas of which the writer may not initially recognize the relevance or value but reporting them on paper creates a new awareness and may induce further reflection” (Borasi & Rose, 1989, p.353). Through this awareness a student manipulating symbols or memorizing procedures is not enough, the individual must “create their own meaning for symbols in order to express words on paper” (p. 355). Finally, this may lead an individual to “become more aware of how they do mathematics” (p. 356), and help foster further reflection or potentially a change in their thinking process.

Borasi and Rose (1989) suggest writing provides information to the teacher and the student. In their study, they indicated a new type of relationship may be formed. Both researchers commented that during class-time, an instructor may not be able to meet the needs of all students, yet with the use of journal entries, an informal conversation can take place. The written conversation may reveal strengths, weakness, and valuable information on a direction to take for the entire class or for that particular individual.

Aspinwall and Miller (2001) contend that the process of writing and self-reflection helps a student to improve mathematical communication. In their study with calculus students, written responses were constructed using words and complete sentences instead of the mathematical symbols to evaluate their own understanding. What Aspinwall and Miller discovered in their analysis of student writing was it helps to develop a shift from a “teacher-centered instruction to

a student-centered construction of knowledge” (p. 106). Writing “holds individuals accountable for the learning taking place in the classroom and provides students with opportunities to practice communicating their mathematical understanding” (p. 106).

The findings of Borasi and Rose (1989), Aspinwall and Miller (2001), and Brown (2007) point to the possibility of developing journal activities for the geometry student. As students develop proof, each individual can express concerns not only on the mathematics but also on the social positions within their group. Students may discover if cooperative group work benefits the construction of proof. As stated by Aspinwall and Miller, journaling provides each student further practice with their communication skill. As previously stated according to the Next Generation Sunshine State Standards (2009), students must meet certain language arts standards imbedded in the learning of their mathematics skills. Journaling can provide a venue for students to put any thoughts on the page, both proof related tasks and concerns about their understanding.

### **Structure of journal writing**

Journal writing can play a crucial role in learning (Borkowski, 2001; Huang & Normandia, 2007). Researchers such as Borasi and Rose (1989) contend that writing can be used to express feelings as well as academic content. Aspinwall and Miller (2001) focus on the academic content and ask students to utilize their language skills to explain difficulties and discoveries. Meanwhile, Stonewater (2002) focuses on a checklist of effective writing. Calculus essays were evaluated in Stonewater’s research, but the value in the research provided the instructor a framework to rank the individual’s work.

The focus of this research discovered how journal writing revealed the nature of student collaboration while creating geometric proof. The qualitative nature of this study allowed the researcher to take into account the affective and academic content of the writing.

Overall, there was a concern about potential bias in entries. The concern was that students will not record what they are learning but what they believe the teacher wants them to learn. Brown (2007) commented on this concern about bias. Brown stated, “factors such as establishing good relationships with the student and providing structured journal writing can add to the validity and reliability of student reflections” (p. 119). According to Brown, a sample of student writing over time will provide the teacher a view of the different social and academic positions that the individual establishes.

## **Conclusion**

The potential for observing individual reflection in journal writing combined with cooperative grouping has been discussed in the preceding review of the literature. The purpose of the literature review was to establish the reason for using journal writing in conjunction with cooperative grouping, not to prove or disprove the use of each well-researched and well-established practices. Creating an environment where the teacher and the student both have the opportunity to communicate their ideas was important. The following chapters indicate the methods and data this researcher utilized to determine what journal writing revealed within cooperative grouping while constructing proof.



## CHAPTER THREE: METHODOLOGY

### Introduction

My experiences within graduate school have caused me to become a reflective teacher. Reflection for me is to think about my thinking and to write about my thinking. According to Ramdass and Zimmerman (2008), “accurate self-reflection is important to a student’s success in math. (p. 21).”

I have taught at the middle school level for twenty-four years and the one area that I want to improve upon as a professional was the implementation of cooperative learning groups. In reading the research, I understand the usefulness, yet I also have experienced the pitfalls. I have had trouble where too few students were engaged and my frustration level became so great I desired to return to the well-ordered version of desks and rows, yet I was aware that constant use of traditional modes of instruction did not serve my students well. I wanted a tool that the students could use that would give me insight into their understanding and potentially give me a framework for future instruction.

Within the last several years of working with reform curriculum and gifted and talented students, I needed to develop a new tool for students working within cooperative groups. As students were given the opportunity to work collaboratively on mathematical tasks, I observed students negotiating their positions. At times students began to justify their reasoning, and the opportunities for engagement were high and students felt empowered. Especially in a geometry class, if a student was developing a proof there may be more than one way to construct an argument. I wanted to give the students a tool to use to reflect, self-evaluate, and provide me

with insights into their daily struggles and successes. Therefore, the purpose of this study focused on one question:

*How does journal writing reveal the nature of students' cooperative learning in the construction of geometric proofs?*

### **Design of the study**

Action research provided the venue for the teacher to become researcher and practitioner all in one. Nolen and Vander Putten (2007) identified the method as an opportunity for educators to use a method to “investigate their own teaching and their students’ learning in and outside the classroom” (p.401).

Journal writing in an 8<sup>th</sup> grade geometry class within cooperative group experiences provided a challenge. At first, I became immediately aware that I could not have random writing assignments. I searched for appropriate journal prompts within the literature and found a mix of academic and affective, but none that suited my needs.

I decided that the writing assignments needed connection to proof related tasks. The writing was a reflection of what took place during the tasks and any reflections that the individual student had on the experiences within their cooperative learning groups.

Initially the study surveyed fifty-three eighth grade geometry students. The students were grouped based on achievement using a Kagan (1992) approach by placing high achieving, two medium achieving, and one low achieving student within proximity of one another. According to Kagan, at first it may be more appropriate for the high achieving student to work with a medium achieving student, and a medium with a low achieving student. Yet, at the

beginning of the school year, the Kagan method would account for only the achievement piece. An instrument (Appendix A) entitled the Attitude Towards Mathematics and its Teaching(ATMAT) helped in the selection of students. Student Achievement and Attitude would be the two variable used to identify four students. Categorizing students using attitude and achievement provided rich data.

Along with the writing samples, teacher observation and short student interviews triangulated the data. The researcher chose four students from the original fifty-three geometry students that match the following characteristics: Diane, High Achievement and High Attitude (HH), Nancy, High Achievement and Low Attitude (HL), Dan, Low Achievement and Steven, High Attitude (LH), and Low Achievement and Low Attitude (LL).

The goal was to average one journal prompt per week over a course of twelve weeks. The first entry occurred after all parent permission and child assent forms were collected. Students received feedback from the teacher on their entries and a short grading rubric was established to give credit for their entries. The grading rubric (Braxton, 2010) was used as a guide to gauge the quality of responses from low, medium, high, to exceptional.

### **Assumptions and Limitations**

This study was conducted with the assumption that journal writing would cause students to be reflective about their role in cooperative groups and to justify their mathematical understanding. At first, the students were confused about writing in a mathematics class. To the students this was math class not language arts. Yet with the opening problem solving task

(Appendix B) the students started to realize that I wanted to document their thinking and their involvement in their groups by writing about it.

Another challenge was to determine how to make the choice of observing, and collecting data on four students. I was encouraged to do a case study to make the study manageable and worthwhile. A few good quality prompts could help me compare four students' actions and understanding in class.

Since this was an eighth grade geometry class, the overall achievement in the class was very high. Each of these eighth-grade students had been in a very advanced mathematics curriculum since elementary school. I decided to look at their achievement based on their seventh-grade Florida Comprehensive Assessment Test (FCAT). I recorded their overall score and their sub-scores within the five strands of the mathematics test. Again, since this was a geometry class and all of these students were taking a tenth grade class in eighth grade, the numerical gap in their (FCAT) scores was not very large. Yet, coupled with the ATMAT (Appendix A), an attitude survey, I could identify some intriguing pairings between FCAT achievement score and attitude scores from the four students I observed for this study.

## **Participants**

Students were selected out of the three honors geometry classes in the teacher's schedule. The researcher surveyed fifty-three geometry students and then chose four students for a case study. All fifty-three students received parental consent and child assent forms. The forms were made available on two occasions at a before school open house and in the opening weeks of the

school year. Copies were made available upon parental request. Assent and consent forms were kept in a locked cabinet to preserve anonymity.

Within the first two weeks, 41 students returned and agreed to be in the study. According to Frankel and Wallen (2009), the purposive sample was appropriate for my research needs. I was aware that geometry students would need to prepare written arguments or proof for their curriculum. Giving the student the opportunity to journal on a task, a proof, or a problem solving experience would be pertinent to the study.

**School setting**

The school district is an entire county within the southeastern United States. The middle school is in an urban setting of the county. The public school is a magnet school for the fine arts and communications and offers a self-select pre international baccalaureate preparation academic program (Pre IB) or a traditional academic program. At the time of the study, the total population was 1740 students.

Table 1

*School Setting*

	Total	Pre-IB gifted
Asian	3.80%	5%
Black	23.5%	17%
Hispanic	21.6%	10.9%
American Indian	0.3%	0.8%
Multi-Racial	7.3%	5.3%
White	43.5%	61%
Free/Reduced Lunch	56.20%	31.10%

. Table 1 describes the total population and the Pre IB students who are gifted. All of the participants in the research study are in the Pre IB program, and all geometry students are in the Pre-IB program. Out of the four students in the case study, one was classified as gifted and all four students were white.

### **Data Collection**

The researcher consulted with the school principal prior to the action research and written permission was obtained. Research proposals were documented at the county and at the University of Central Florida due to the researcher's connection to the institution. Parent and child assent forms were made available on at least two occasions with each participant detailing the scope of the research. After Institutional Review Board application at the university, the University of Central Florida reviewed the documentation of this study and determined that minimal risk was apparent and no permission was required from the university. However, the school district required parent consent and child assent forms before data collection began.

The maintenance of confidentiality will be achieved by assigning each case study participant a pseudonym which linked the data from journal writings, teacher notes, and short individual interviews with each participant. Data was tracked over a period of twelve weeks. The researcher gave students two weeks to return the appropriate documentation. Short writing assignments were given at the beginning of the first nine weeks to get students familiar with writing in a math class. Slowly the researcher implemented writing as a normal practice within

the classroom. By the beginning of the fourth week of school, students were ready to write on a large-scale problem-solving task that would help them develop paragraph proof in geometry.

An action research plan was followed due to the reflective nature of the study for the researcher and the student. Action research allows changes to occur within and throughout the study depending on the data. The purpose of action research allowed me the opportunity to discuss the theoretical aspects of the project, but be willing to be flexible based on, the actual events occurring in the classroom (Tanner & Jones, 1994).

Fifty-three eighth grade geometry students were sampled for this study. Within the curriculum of geometry, students must learn to construct arguments based on mathematical axioms and theorems. In fact, the geometry student's construct of proof was analogous to a scientist making a claim and using evidence to support that claim. Much in the same way, a geometry student must learn to support a hypothesis and bring it to its logical conclusion through a series of formal steps. From personal experience, formulating mathematical proof can be challenging and frustrating, even though it needs to be the backbone of the students' mathematical experiences (NCTM, 2001).

If students can be given opportunities for peer and self-assessment in their construction of mathematical arguments, the potential for success can be far greater (Tanner & Jones, 1994). Prior to writing proofs, students may need to reinforce previously learned algebra and problem solving skills that enhances their confidence (Tanner & Jones, 1994).

A case study involving the effects of journal writing on individual accountability within cooperative learning groups, can help promote "a problem solving" approach to the art of

teaching (Stevens, Harris, Aguirre-Munoz, Cobbs 2009). Tanner and Jones (1994) describe a study in Wales where the purpose was to develop a student's problem solving skills via peer and self-assessment. The research found that in order for students to be reflective, the assessment piece of their daily tasks needed to be structured and formative in nature. As students first began the process of solving problems their assessment of their peers and own self were weak in the beginning, yet as the class developed with guidance from the teacher, the individual student began to develop a more critical approach to their work. Thus given time and an adequate framework developed within the culture of the class students can become reflective on their own work and the framework of this study can be accomplished. Students can become individually accountable and use the process of reflection to get them there. With this in mind, the researcher of this study would like to implement cooperative learning groups as a vehicle for student discussion, writing, and eventually personal understanding of an eighth-grade geometry curriculum. It is within the writing that the researcher hopes to identify themes of how students become reflective and construct proof. Yet, the research indicates individual accountability stems from working with others (Tanner & Jones, 1994). Students will construct ideas, negotiate meaning, and eventually record their own thoughts in a process of self-reflection.

Writing samples were analyzed after the students returned parental consent and child assent forms. However, to institute a practice of writing within the curriculum, students at the beginning of the school year were given writing prompts to reflect on the day's learning objective. The opening of school required me to be flexible with planning and assignments since schedules can be rearranged, and a regular routine began during the second week.



Within the first three weeks of school, I used this time to establish cooperative learning groups. I used three structures: roundtable, team interview, and think-pair share. Each of the structures provided specific duties for the students. For example in roundtable if a student was writing a proof he would pass it to a student to his right and that student would add any missing steps to the proof or make a positive comment on his work. The process would continue until the proof returned to its owner. The purpose for each of these structures was reviewed before each use within the first three weeks to get students familiar with their social positions. Students would reflect in their own notebooks on what took place in their groups within the last five to seven minutes of class. Once or twice per week, the class had an opportunity to share their writing with the class.

By the fourth week of school, all paperwork had been collected and organized and the students had enough practice in their cooperative groups. The district provided an instructional plan and by this time of the school year, students were expected to begin developing reasoning and proof. The students were given a problem-solving task (Appendix B). I modified the task from NCTM (2002) to conform to the district plan on patterns and reasoning. The students used the problem solving task and their cooperative groups to determine a general pattern for the task. The task had a number of solutions, either using physical manipulatives, virtual manipulatives, or an algebraic representation. The students used three forty-five minute class periods to investigate and record their findings. The writing prompt contained a series of questions to help students construct a paragraph proof of their findings. The students recorded their data in a journal that was kept in the classroom. Each class had a color-coded journal, for example second

period was red, fourth period was yellow, and eighth period was purple. Color-coding made for ease of organization and identification since there were over fifty journals.

My experience with the first journal entry was positive, yet it was time consuming. I decided to keep subsequent journal entries shorter and focused on maintaining the use of cooperative groups with the district instructional plan. Future journal entries were made after students worked in groups and on average the students were given 5-7 minutes at the end of the class. If there was time, students shared their entries with the class. Writing prompts averaged one per week over the course of the study. I collected the journals at the end of each week and made comments to each student. The first problem solving task I used a rubric (Braxton 2010) to determine a class work grade for each student. Subsequent journal entries were used for students to provide personal feedback or information to me.

While students were involved in cooperative group tasks, I took field notes. I wrote down what students said in their groups. I wrote down what students wrote on their papers. I recorded questions the students asked one another or would ask me. At times the field notes led to short interviews with students within the study to clarify what they were doing within their groups or individually. The field notes and interviews were used in the results section of this research.

### **Selection of students**

Since the students within the study were academically talented students, choosing students on achievement level alone may not provide rich data. The students worked within

small groups and their attitude towards mathematics provided another insight as to how the students worked together in the construction of proof.

An instrument called Attitudes Towards Mathematics and its Teachings (ATMAT) was used. The ATMAT is located in Appendix A. From the ATMAT research, it was indicated that the higher the score on the survey, the more positive the attitude towards mathematics (Ludlow & Bell, 1996). The survey consisted of 29 items and a six-point Likert scale. The six-point scale gave the student the opportunity for three levels of agreement or disagreement and there was not a neutral option. "Items were worded in a positive and negative direction to minimize the threat of a response set on the part of the students"(p. 865). The ATMAT itself is a conglomeration of other attitude scale surveys. It was tested for content validity, and the initial Cronbach's alpha coefficient measured 0.95 (Ludlow & Bell, 1996). Ludlow and Bell (1996) describe a study on attitudes towards mathematics. The design of the study was with pre-service elementary teachers. The study focused on the teachers attitude towards teaching mathematics in the near future, yet an instrument was used which could be applicable to the study reported here. The instrument gave the researcher the opportunity to compare the student's attitude and achievement in mathematics. See Appendix A for a listing of questions within this survey. Permission from the publisher was sought to utilize a form of the ATMAT. The researcher used twenty of the twenty-nine original questions to determine attitudes toward mathematics within the group of fifty-three geometry students. The survey provided the researcher with data to make his choice on the case study of the four individuals.

Data was triangulated with the researcher's teacher notes, short student interviews and student writing samples. The first task will be of a problem solving nature and most likely viewed as a puzzle by the students. Two of these tasks are in Appendix B and Appendix C. The second task was used as a backup to the first if more data was required within the case study. Additional entries will be from the instructional plan provided by the school district for an Honors Geometry class. Students wrote and proved topics such as parallel lines, congruent triangles, and the properties of quadrilaterals.

All fifty-three geometry students took the ATMAT (Appendix A), but the data was used from the forty-one, which granted permission. I categorized the twenty questions into a chart and ranked each answer with a point value from +3 for very strongly agree to -3 for very strongly disagree. A six-point scale gave the student the opportunity for three levels of agreement or disagreement and there was not a neutral option. "Items were worded in a positive and negative direction. Out of the 20 survey questions, the author considered 11 labeled as positive and 9 labeled as negative. I totaled the student's responses to the positive questions and subtracted them from the student's responses to the negative questions. The higher the final total score, the higher the attitude toward mathematics.

I studied four student's entries over a period of twelve weeks. The students wrote nine entries within the twelve-week time-span. Seven of them related to a geometric proof. Two journal entries answered questions about the student's perceptions on the effectiveness of cooperative group work in constructing the proofs. I chose three entries that displayed the student's process of developing a proof. I chose one entry to describe the student's perceptions

on the effectiveness of working in groups. The entries were chosen at three to four week intervals to identify any change from one entry to the next.

I chose four students from the forty-one. All four geometry students were from one class. To preserve anonymity the names I used for the four students were Diane, Nancy, Dan, and Steven. I used achievement on standardized test scores (FCAT) and their ATMAT score to distinguish between the four students and to identify any similarities or differences in their journal writing.

The survey is located in Appendix A. Table 2 shows the results from the survey scores of the four students, Diane, Nancy, Dan, and Steven. Below each of their survey scores was their FCAT scores broken down into an overall score and the five categories that were administered in March of their seventh grade.

Each question in Table 2 was labeled as a positive question or a negative question and placed in a random order. According to Ludlow and Bell, this allowed the individual's response to be a more accurate representation of their attitude.

Diane had the highest ATMAT survey score, and I classified her as having the best attitude toward mathematics. Dan had the next highest attitude score. Steven and Nancy had the lowest attitude scores. Since all four of these students were advanced in their mathematics courses, their FCAT scores are all high. For comparison, the highest score possible for this grade level is a 2572. Diane and Nancy were in the highest developmental range, while Dan and Steven were just one level below. Yet, Dan and Steven's score still qualified them as more than proficient in their grade level standardized test scores.

Taking this data, I categorized Diane with high achievement and high attitude toward mathematics (HH). Nancy had high achievement and low attitude toward mathematics (HL). Dan had lower achievement and high attitude toward mathematics (LH). Steven had lower achievement and low attitude (LL). To make one item clear, Dan and Steven's classification of lower achievement was used in comparison to Diane and Nancy.

Table 2

*ATMAT Survey Results of Four Selected Students*

Question	Diane	Nancy	Dan	Steven
1+	3	1	2	1
2-	-3	-2	-2	-2
3-	-3	2	-3	-1
4-	-2	-1	-3	-3
5-	-3	-1	-2	-1
6+	3	-1	2	2
7-	-3	-2	-3	-2
8+	3	-1	2	1
9+	3	1	2	1
10-	-2	2	-1	-1
11-	-2	-2	-1	2
12+	3	1	1	-1
13-	-3	-2	-2	-1
14+	3	2	3	1
15-	-3	-3	-3	1
16+	1	1	1	-2
17+	3	2	2	1
18+	3	1	2	2
19+	3	-1	1	-1
20+	1	3	3	1
<b><i>ATMAT Score</i></b>	53	18	41	14
FCAT Overall	2136	2172	1995	1974
Number sense	100 %	100%	88.9%	77.8%
Measurement	88.9	88.9	88.9	100
Geo/Spatial	87.5	87.5	50	87.5
Algebra	88.9	100	77.8	66.7
Data/Prob	100	100	88.9	66.7

## **Data Analysis Plan**

The research question asked how journal writing revealed the nature of students' cooperative learning in an eighth grade geometry class while they constructed proof. At the beginning of the study, the researcher will administer the ATMAT survey (Appendix A), several problems solving writing tasks (Appendix B & C), proofs from the geometry curriculum on parallel lines, congruent triangles and quadrilaterals. Standardized test data was used to determine baseline achievement levels.

The first journal entry was the problem-solving task (Appendix B), and it was structured to allow students to write about their learning process. Future writing prompts were developed as the class studied proofs of parallel lines, congruent triangles, and properties of parallelograms. I used the curriculum, field notes and direct observation to construct future writing prompts. The prompts were geared to determine the nature of the cooperative groups in the construction of proof and the level of understanding the students were developing in geometric proof.

Journal writing, teacher notes, and short interviews were used to continue the study. The four data points will provide the researcher to identify themes within the individual writing and cooperative group conversations. The researcher described how journal writing revealed the events in cooperative groups while students constructed proof.

## **Summary**

Data from tasks outlined in Appendix B, writing samples and Appendix D through Appendix I will be opportunities for Geometry students in eighth grade to begin the process of formal proof. Within the experiences of a case study, my own reflection will permit me the



opportunity to grow as a teacher. My intention within this practice may influence the students to become reflective (Stevens, et al. 2009).

## **CHAPTER FOUR: RESULTS**

### **Introduction**

My experience as a middle school mathematics teacher and a graduate student caused me to reflect on identifying experiences to improve as a person and as a teacher. Personally, I had kept journals for many years and I hoped my students would benefit from the experience. Writing provided me a source of reflection, accountability and an opportunity to see growth. Student writing samples, my observations, and short interviews with the four students provided the format for this section.

### **Organization of results**

After much thought, I decided that an effective way to organize the results was to place four journal entries and describe the writings of the students. Each student had a distinct classification. Diane was characterized as high achievement and high attitude toward mathematics (HH). Nancy was characterized as high achievement and low attitude toward mathematics (HL). Dan was characterized as lower achievement and high attitude toward mathematics (LH). Stephen was characterized as lower achievement and low attitude toward mathematics (LL). The reader will be able to see what each student constructed. The focus will be on what the student wrote from each prompt.

### **Journal entry one**

The task in Figure 1 was presented to each student and they were asked to write and organize their findings based on the worksheet in Appendix B.

“There are seven stepping stones and six people. On the three left-hand stones, facing

the center, stand three of the people. The other three people stand on the three right-hand stones, also facing the center. The center stone is not occupied. Everyone must move so the people standing on the right hand stepping stones are on the left, and the people on the left move to the right hand stepping stones.”

*Figure 1.* Stepping-stones problem solving task.

The objective was to get all the people on the right to the left hand side and all the people on the left to the right hand side. The students had to follow rules within the task outlined in Appendix B. The first task allowed the class to begin the process of proof. Within the high school geometry curriculum the student was expected to provide proofs in either a statement and reason form or a paragraph form. The first problem-solving task in Figure 1 provided the students an opportunity to explore the problem in a variety of ways, and provide each student the opportunity to write about it. The point of this task was to determine how the student and their group could find the number of moves it took in the original task. As students worked with their groups, they were challenged to extend the pattern to “n” people on each side of the stepping-stones. Since the first problem-solving task became the first journal entry. I asked two questions to prompt the use of a hands-on model. I wanted students to describe or draw a hands-on model of this problem. The students were provided with materials to help them decide how to describe a model that worked for them. I provided small cubes, car mats, paper, web-sites for virtual manipulatives. Additionally I asked seven questions that helped students organize the task. I hoped students could develop a general pattern and I wanted the students to write at least one paragraph describing any patterns they found.

### **Diane HH entry one**

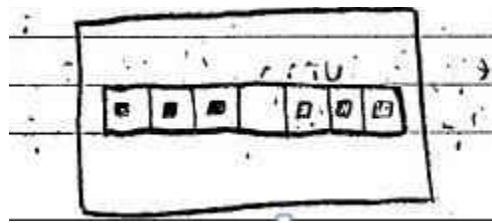
Diane was a high achieving student with a high attitude. Her journal entries were the most detailed out of all four students. She was very meticulous with her work. Diane wrote five pages of information to the problem solving text in Appendix B. Diane and her group worked for three forty-five minute class periods on this task. The time gave students the opportunity to explore the task physically by using other students in the class, develop and draw out a hands-on model, and use a virtual model found on the computer.

Diane started her entry by answering the questions she discussed with her group. She found that by solving a simpler problem a pattern developed. Diane worked with her partners to do this problem physically. Her group became involved with another group and physically walked through this problem. I provided the groups with cut up car mats to mimic the stepping-stones. Diane organized the group of people she worked with and had people move following the rules of the problem. She discovered that “there isn’t a pattern in the steps and jumps you make, but in the minimum number of moves you make.”

Diane returned to her group and wrote. Her journal entry contained two versions of her description of the events. One version contained a systematic process of what took place in the groups and then she constructed a paragraph version of the events. Diane’s writing was clear, answered the situation and she was willing to see beyond the task. She discovered that the stepping-stone problem was a model of a parabola, a second-degree polynomial equation. The following figures were Diane’s final page in her journal entry. She drew a picture, recorded a

table and summarized her findings with an equation at the very bottom of the entry, since Diane was asked to describe the pattern with  $N$  number of people on each side of the stepping-stones.

Diane's first paragraph detailed her vision of a hands-on model. In Figure 2 Diane showed the stepping-stones picture and she explained what she used to experiment with the model. She decided to use seven boxes with six little cubes to move on the paper. I observed that she followed the rules of the problem while she worked methodically on her own model.



*Figure 2.* Diane's diagram for the stepping-stones task.

Initially she organized six students from the class and used the car mats I provided to help in acting out this problem. Eventually she returned to her seat and used the diagram in Figure 2 to solve the problem and determine the pattern for  $N$  on both sides of the stepping-stones.

Diane's next paragraph explained that there was a pattern for two, four and six people. She indicated that she Diane described what she eventually used in her analysis of this problem, yet initially she organized six students from the class and used the car mats to solve this problem by acting it out. Eventually she could follow this pattern and extend it to eight or ten people. Diane was following the prompts at the bottom of Appendix B. She was systematic with her writing and each line logically progressed to the next. In Figure 3, Diane displayed the table that followed her description of the pattern.

	X	Y
total number of people	1	3
2	2	8
4	3	15
6	4	24
8	5	35
10		
	number of people on each side	number of moves

Handwritten annotations in the table include:
 

- Arrows pointing from the 'X' column to the 'Y' column with labels:  $\times 3$ ,  $\times 4$ ,  $\times 5$ ,  $\times 6$ ,  $\times 7$ .
- Curved arrows between 'Y' values:  $+5$ ,  $+7$ ,  $+9$ ,  $+11$ .
- Vertical arrows between 'Y' values:  $+2$ ,  $+2$ ,  $+2$ .
- The word "parabola" with a checkmark.

Figure 3. Diane’s table for stepping-stones task.

Diane identified what her variables represented and patterns created both vertically and horizontally within the problem. Her analysis continued as she explained what the table signified and extended her data from her group. She indicated it took “twenty four moves for eight people or four people on each side to exchange positions, thirty five moves for ten or five people on each side with eleven spaces.” Diane continued to explain that this pattern was a parabola. She showed by a series of finite difference that the pattern was a second-degree equation. It was evident from Diane’s analysis of her table that she extended the pattern to  $y=x(x+2)$  where  $x$  represented the number of people on each side and  $y$  represented the total number of moves on the stepping-stones.

The amount of detail was exceptional, and furthermore Diane reflected on not only on the problem presented, but she explored further. I observed that when she was given time to write in class she took approximately twenty minutes of class time to construct her final paragraph answer. As she was analyzing her table, she did ask for help from another student. She

requested that another member of her group count the number of moves as she checked her results within the table format.

I used a four-point grading rubric to provide feedback to Diane (Braxton 2010). She scored four out of a possible four points. Diane received an exceptional response, which was one that went beyond the requirements of the problem, showed original thought, gave strong supporting arguments, and displayed exceptional thinking and description. She recalled the finite difference analysis from algebra 1 to classify the pattern as a parabola. She was able to identify the connection between the  $x$  (the number of people on each side) and the  $y$  (the number of moves) to determine the equation. In her equation she calculated that the number of people on each side ( $x$ ) was multiplied by  $x+2$  to equal the minimum number of moves.

Diane's response showed me that she utilized the resources from her cooperative learning group by asking people for help. She provided feedback to others in her group and at the same time had to struggle with numerous trial and errors to establish the validity of the pattern. Her persistence was noteworthy, because in my experience Diane was the type of student who worked a problem repeatedly until she was satisfied with her results. Given the amount of detail in her entry, I was impressed that she kept a running log of her work over the period of three class periods and finally how she was able to bring all the ideas together.

### **Nancy (HL) entry one**

Nancy belonged to a different group than Diane at this time of the school year. She and her group produced different results. Figure 4 displayed how Nancy began her thought process.

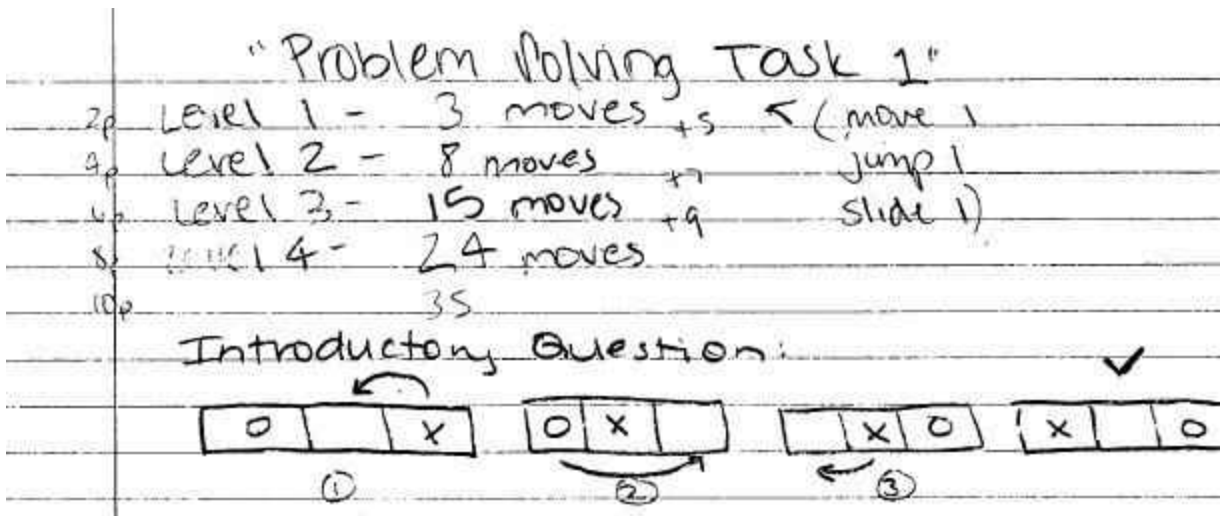


Figure 4. Nancy's introductory paragraph problem solving task 1.

Nancy did not use a table to organize her thoughts, yet she did draw some pictures to illustrate the rules in the task. She used logical reasoning to predict the outcome of the pattern, yet her writing was not able to extend to an expression or take it to a general case. The first nine lines of Nancy's writing described what she did with her group of people to determine what pattern she saw in the problem. She stated that her group used two people with three spaces and then four people with five spaces. Nancy indicated starting out with a simpler problem helped her develop a prediction. Nancy did not construct a table, but her description displayed a table in a sentence form. After each number of people and moves she wrote a number in parentheses with a plus sign, for example "...with 2 people it took 3 moves and with 4 people, 8 moves (+5)". Nancy continued to detail her work and she came to a prediction at the bottom of her writing. She wrote the numbers with an additive relationship between the numbers in parentheses. She recorded in her journal: 1 (+3), 4 (+5), 9 (+7), 16 (+9), 25 ...



Nancy and Diane, two students with high achievement, described the number of people on each side of the stepping-stones, and how many moves were needed. Nancy wrote “with 2 people it took 3 moves, 4 people 8 moves (+5), six people took 15 moves (+7), 8 people was 24 (+9) moves.” From this pattern, Nancy concluded it was a square pattern. When Nancy was asked to explain further, she stated that the “pattern reminded her of the same additive pattern between sequences of perfect squares”. Nancy predicted the correct number of moves but her writing indicated the the pattern connected back to perfect squares. The feedback I gave to Nancy was that she gave some strong supporting argument, and her response had a lot of detail. Yet, I felt she had minor errors in her conclusions.

Within Nancy’s writing she used the pronoun “we” five times, whereas Diane did not use the term. I observed Nancy relied on other people’s feedback within her cooperative group and she gave the entire group credit for the findings. Diane used the second person you in her writing. She used terms such as “you can figure out” or “you can create”. Nancy and Diane both described their work well, but their actions in their groups and the way each of them expressed their findings was different.

### **Dan LH entry one**

Dan’s entry for the first task is the shortest of the four. He described the amount of moves needed for 10 levels. Somehow, he wrote the general expression  $n(n+2)$  where  $n$  equals the number of people on each side. When I asked Dan how he arrived at the answer, he could not give me an adequate reason. He was in a different group than Diane, Nancy or Steven but other members of his group had come up with this same result. As I continued to inspect Dan’s

journal, I asked if he answered the introductory question to the task. The introductory question was to get the student to draw a model of the situation. Dan's response was that he did not need a picture, but I felt he evaded the question. The instructions were to create a picture of the problem. In addition to this, I was concerned that Dan had a correct answer but no supporting evidence. One of my concerns about group work was coming to the light. I wanted to see how the cooperative group work would help Dan construct the proof. I observed Dan participating in his group, and I wrote field notes on the questions he would ask to his group members. He was willing to ask a group member how they came to their answer, or show me what you mean. However, Dan's written entry was not descriptive enough to indicate if the work had been cooperative in nature. I wanted to know if Dan was also offering information to his group. My observations of Dan's work did not match his written entry. It appeared that Dan worked within a group, yet his group did most of the work. Figure 5 was Dan's written entry for the stepping-stone task. The class had spent three forty-five minute class periods investigating and writing about the task, and Dan's entry was not of the same quality of his three peers in this case study. I provided written feedback to Dan that his response did not have any errors but needed detail. I urged Dan to describe his thought process. I also wanted to positively reinforce his ideas were appropriate yet I wanted to understand his thought process.

Step, Jump, Jump, Jump, Jump, Step, Jump, Step	
Level 1 - 3 moves	Level 6 - 48 moves
Level 2 - 8 moves	Level 7 - 63 moves
Level 3 - 15 moves	Level 8 - 80 moves
Level 4 - 24 moves	Level 9 - 99 moves
Level 5 - 35 moves	Level 10 - 120 moves
Each step makes one more jump than the last.	
$n(n+2)$ $n = \#$ of people on each side	

Figure 5. Dan's analysis of stepping-stones task.

### Steven LL entry one

Steven's entry had a little more detail than Dan's, and his conclusion of  $n(n+2)$  was listed. Nancy and Steven shared the common theme of the pattern of odd numbers. Nancy stated how this was similar to perfect squares. Diane, Dan and Steven showed the correct answer to the problem, but Diane, HH, had the only complete justification. The error patterns for Nancy and Steven were similar. It appeared that Steven's results would match Nancy, but he stated the  $n(n+2)$  pattern. Steven, LL, listed his pattern in sentence form instead of using a table. Initially, Steven used the same method as Nancy, even though he was in a different group. Steven was able to discover that "four people took eight moves, six people took fifteen moves, eight people, took twenty-four moves, and for ten it took 35 moves". Steven stated the pattern

between the number of moves but there was no connection to the general expression. He indicated in his writing that “every time the people on both sides goes up the amount of moves goes up by an odd number...3,5,7,9,11”. His result seemed to match Nancy additive pattern, but somehow he jumped to  $n(n+2)$  without any further explanation.

He did attempt to make a diagram but it was incomplete and did not have any labels on it. In essence, Steven had the correct result, but he did not have the justification to reinforce it. Similar to Dan, I was confused how Steven arrived at the result. His writing, pictures and final analysis did not correspond to his conclusion. He made a leap from his initial data to the general expression. What I observed was typical of a geometry student construction of proofs. The initial details were correct yet there were gaps in reaching a conclusion. The feedback I provided for Steven was he displayed some understanding of the concepts, but in rereading his entry, more detail was needed.

### **Summary of Stepping-Stones Task**

My research question focused on the use of cooperative learning groups and journal writing to assist eighth grade geometry students in constructing proofs. The questions the students were given from Appendix B were intended to help students detail their construction of a paragraph proof. Diane and Nancy both of whom were high achievement detailed their thought process as they were solving the problem and then reconstructed their ideas in paragraphs and pictures. Meanwhile Dan and Stephen, both of whom were low achievement, showed little detail. Diane, Dan, and Stephen stated a correct result, yet Diane’s description justified why her answer made sense to her. Dan and Stephen did not elaborate on the result at all; it just

appeared. Three of the four wrote in first person terms “I did” or second person statements “you can solve”, only Nancy chose the collective “we” to describe her work. Nancy’s work relied on the group process and it was important to her formation of a solution. At this point in the research to make an inference on the connection between Nancy’s attitude toward mathematics and dependency on the group process was not possible.

### **Journal Entry two**

The second entry I chose to analyze was approximately four weeks from the first problem-solving task. At this time in the instructional plan, the students had been working on constructing proof on their own and in groups. All four students solved problems on the properties of parallel lines. The group work focused on not only proof, but also a review of algebra topics imbedded within geometry problems. By this time, the students worked in groups for six to seven weeks. The entry was designed to determine the student’s perception on the effectiveness of the group process. I attempted to identify at this point in the school year if students could explicate how cooperative groups affected their understanding. After working in groups with several problems, I asked each person two questions:

*How does working in a group help you determine your own solution?*

*Describe if it was helpful or harmful.*

The students had the last five to seven minutes to respond to each question. Each student’s entry was transcribed in the following section.

### **Diane HH entry two**

The prompt was given to elicit a response from the student about the group process. I tried to identify if Diane could self-evaluate while working in a group. The prompt provided her an opportunity to form her own solution or decide if there were aspects of the group that obstructed her progress. Her response was positive about her own experiences.

Diane's writing described that working in a group helped her since she could use "other's ideas and mistakes to form your own accurate solution." She stated it was helpful to work in groups. She continued, "we were able to help each other find patterns and we were able to learn for each other's mistakes." Diane's entry was succinct. This was the first time Diane used the collective "we" in describing the group process. Given her high achievement, this was a turning point for Diane in how she viewed cooperative learning groups. It seemed that she was not imposing her own work upon the group, but she became more willing to allow the group process affect her own work. It caused her to question her own conclusions and be willing to convince another person of her own solutions. The process became more reflective for Diane. Other individuals in the group provided her feedback and she provided others with feedback. I noticed she was willing to listen to others viewpoints even if they did not match her own. Diane would then ask questions in return and then record her solution to a problem.

### **Nancy HL entry two**

Nancy wrote that working in groups was helpful to her. She stated, "working in groups is really helpful for me, and I always tend to second guess/question myself. But I like hearing others opinions to help me decide." The interaction helped her decide if her own work was

valid. Overall, she believed that her role in the group and the overall group experience was positive. The opportunity to communicate ideas to others and receive feedback from others improved her confidence.

The writing continued to provide insight into Nancy's confidence level. Her achievement was categorized as high from her FCAT scores and class work. Yet Nancy's attitude towards her mathematical ability was different from her achievement level. Her score on the ATMAT survey indicated Nancy experienced doubt with her own capabilities. This feeling of doubt was a concern as Nancy wrote geometry proofs. In writing proofs she would need to become more confident. I became concerned that her lack of confidence may affect future achievement. She was a student who delivered correct solutions most of the time, but was unsure of her own work. The journal writing showed me that delivering a vote of confidence on my part or keeping her in a supportive cooperative group provided Nancy with opportunities for success. She felt good about herself when her mathematical work, especially writing proof, connected to interactions with others in a group setting.

### **Dan LH entry two**

Dan described in his second entry that working in a group helped him develop his own ideas. He commented, "working in a group helps me form my own decision because when you get an idea from someone, you can spread that idea and come up with another way to solve or say something." I noticed that Dan had a high level of interest in working together, but my sense was he would not move forward or commit to a solution until everyone agreed. Dan scored forty one out of a possible sixty on the ATMAT survey. His scores indicated that mathematics was

interesting to him and he believed he had a good mathematics background. I continued to get the sense from Dan's writing and group interactions that he appeared to know what he was doing. Yet in reality, I observed him wait on others to provide potential solutions and then he adopted them as his own. His comment from this writing "when you get an idea from someone" indicated that he was not contributing original thought to the proof solving process. It seemed that he believed he was offering a solution. I connected his words "spread that idea" to his final comments about the group process as "helpful because I can understand something better when my group helps me." Dan's process continued to show that group work may not be developing his own independent ideas, but he relied on others to form his own conclusions. I began to develop concerns about Dan's progress, because here was a student who seemed to believe he could manufacture his own results, yet the journal writing began to show that Dan had some gaps to fill mathematically. I noticed in Dan's work he wanted to make conclusions within a proof without the appropriate logical steps.

### **Steven LL entry two**

Steven stated very briefly, "it [group work] helps me to brainstorm ideas and then grow off those ideas." He indicated that being in a group was helpful because it gave him "an opportunity to voice ideas." My observation of Steven was that other members of his group were brainstorming, but Steven was not. When I asked Steven directly, he would provide his own insight. Steven's writing did not match his actions. He was passive in his group unless directly confronted by me or another member in his cooperative group.



At this point all four candidates are still in different groups. From Diane and Steven's writing, both indicated the group process helped them in constructing their proofs. Diane was classified as HH while Steven was LL, but their journal entries had similar themes. Group work was helpful, ideas can be exchanged, and work can be checked. Nancy HL and Dan LL expressed that being in a group validated their own ideas or sparked new ideas, but all of these actions were dependent on being in a group to generate the final product. This journal entry provided some insight in connecting the work of a proof to the use of cooperative groups and journal writing. Diane became more willing to utilize and trust others within her group. Nancy displayed her own work on proofs, yet she depended on the group interaction to believe her work was correct. Dan and Steven's actions were more on the passive side. They recorded that the group process was helpful, but their writing or their actions did not produce strong evidence either of them had the tools to develop a complete geometric proof on their own.

### **Journal Entry three**

Entry number three consisted of a two-day activity. It was approximately three weeks from entry number two. On the first day, the students were given four proofs on congruent triangles. The proofs were on four different sheets of paper and four different colors. The students received these proofs in a large envelope and they were separated into individual strips. The student's job was to reconstruct these proofs in the proper order. I chose to extend the problem and, I left out some of the steps to the proofs. Therefore, the students had to determine the gaps and record the missing steps in the proofs. To document their work the group had to record the proofs on large chart paper to display and report to the class.

The second day's activity consisted of one incomplete proof, which was similar to the previous four. Each student had to draw the diagram and complete the steps of the proof in statements and reasons form and paragraph form. After the students had individual work time to complete this task, they worked together to discuss their work. A journal prompt gave them an opportunity to reflect on their work after they finished.

At this point Diane (HH) and Steven (LL) were in the same group, while Nancy (HL) and Dan (LH) were in separate groups. Diane, who was HH, was a dominant leader. I wanted to observe if that would assist Steven (LL) or affect his attitude adversely. Meanwhile, Nancy (HL) voiced she had concerns in developing proof on her own, yet what she produced in class was meaningful quality work. Steven and Dan completed the work but were not as reflective in their journals about their justifications. Diane and Nancy took the time not only to complete the mathematical task, but commented on their process. Dan did not have an entry for this activity since he was absent from the class that day, yet he was able to do the follow-up activity.

Appendix D through Appendix I, were the proofs chosen for the class activity. I prepared packets for each group by cutting up the steps on four of the six proofs. The students were given the proofs. Each group had to decide how they were going to complete the task. They placed their results on chart paper to share with the class. I used proof one in appendix D as an assessment piece the next day, and I had one extra proof as a backup for a full class activity.

My interest was in the strategy the groups used. I observed groups in my classes give one proof to each person in the group, which gave everyone an equal share of the work. I noticed some groups worked on one proof at a time, while others paired up. Two people would work on

one proof. I felt the difficulty level of the proofs were manageable. In fact, some of my gifted students indicated that the proofs were easy, but convincing another group member of the appropriate order of the steps was more difficult than the actual proof.

Diane, Nancy, Dan, and Steven were given three writing prompts after this activity.

- 1) Describe what you did when you first started the activity.
- 2) How was the group process helpful in solving each proof?
- 3) How did you stay involved with your group? Explain any successes or difficulties.

### **Diane HH entry three**

Diane's entry indicated that her group subdivided the work. She explained that two group members worked on what they considered the hardest proof, while one person did an easier one, and one did the medium level of difficulty. No indication was given how this was decided, but there was mention that once a person finished their work they helped another group member.

In answering question's two and three from the prompts, Diane said the group learned from each other's mistakes and pieced ideas together to come to a common solution. Everyone had something to do since the checking process and recording the result on the chart paper took some negotiating.

Diane's entry again started to express a better sense of community within the cooperative learning groups. She stated, "we learned from each others' mistakes" and "we were able to stay

involved by each doing at least one proof.” From Diane’s record of the activity, I understood that Diane was not working alone. She found value in working together.

### **Nancy HH entry three**

Nancy and her group took a slightly different approach. She and a partner worked on one proof, while the other two members worked on another. She stated the “work got done faster, but we did spend a lot of time disagreeing” on ordering the proof. She felt successful since all the proofs were completed and that everyone had a job to do.

Nancy continued to write in her journal at how positive the group experience was in constructing the proofs. Her honesty and accountability towards painting a picture of what occurred both mathematically and socially helped in understanding her individual process. She stated that even though “work got done faster, there were many difficulties” such as “stuff out of order, messed up, etc.” Although, Nancy tried to keep a positive attitude, “at least we got it done.” Her next entry indicated that she was progressing at writing her own proof.

### **Steven LL third entry**

Steven commented that his group took another approach than the previous two. His group each worked individually. After I read Steven’s entry, his perception intrigued me. He was in the same group as Diane, yet his recollection of what took place was different.

After Steven completed his proof, I asked him what he could do next to help the group. He stated that he was waiting for everyone to finish. Behavior was not an issue, yet I noted a lack of engagement.

Steven's response to the final two prompts simply indicated the group process allowed the members to ask questions "is this right". He mentioned he did check other people's proofs eventually, but it got confusing and he was uncertain if any of the proofs were correct.

In contrast to Steven's (LL) work, Diane and Nancy, both high achieving students, used their group differently. Diane and Nancy used their group to make the activity meaningful for them. Each expressed how the opportunity to communicate within their groups was a key to constructing the mathematics and established validity to the proofs they created. As their teacher, I did not have to communicate whether they did it right or wrong. Diane and Nancy presented their findings to the class. Diane did this very well, but my perception was that the group discussion and the chance to write about it for Nancy helped her to present to the class.

I observed Steven's involvement in the group was minimal. His reflections were minimal. Ultimately, his understanding or perception of his understanding was weak. He expressed a degree of confusion that Diane and Nancy did not communicate. I observed that his behavior did not affect anyone negatively, yet his lack of involvement impeded his progress.

### **Journal Entry three part two**

In the follow-up activity (Figure 6) to the group proofs, Diane, Nancy, Dan, and Steven received a version of proof one from Appendix D.

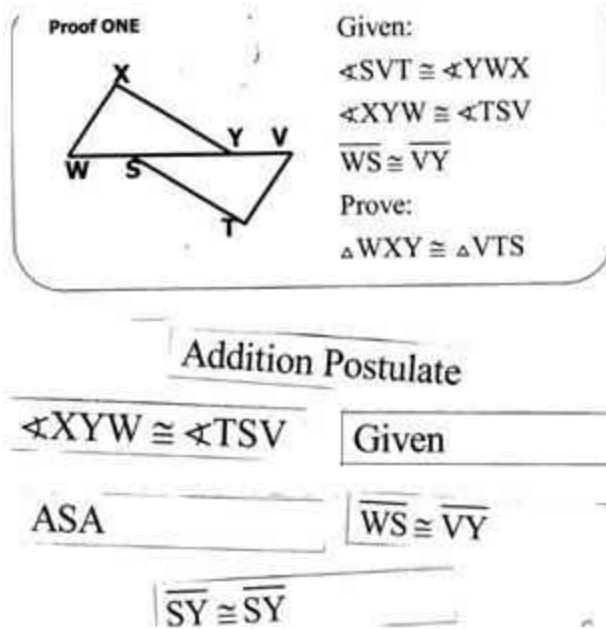


Figure 6. Individual proof after group proof activity.

I used Figure 6 as a self-assessment and a follow-up to the previous day's activity. The directions to this proof were:

Sally had some ideas to start the proof (Figure 6). Sam realized it was incomplete.

In your journal draw the diagram and complete the proof in statements and reasons form and in paragraph form.

In Figure 7 Diane recorded her proof. The original document is located in Appendix D.

Given:  $\angle SVT \cong \angle YWX$   
 $\angle XYW \cong \angle TSV$   
 $\overline{WS} \cong \overline{VY}$   
 Prove:  $\triangle WXY \cong \triangle VTS$

Statements	Reasons
1) $\triangle SVT \cong \triangle YWX$	1) Given
2) $\overline{WS} \cong \overline{VY}$	2) Given
3) $\overline{SY} \cong \overline{SY}$	3) Reflex. Prop. of $\cong$
4) $\overline{WS} + \overline{SY} = \overline{VY} + \overline{SY}$	4) Addition Postulate
5) $\overline{WY} = \overline{VS}$ or $\overline{WY} \cong \overline{VS}$	5) Segment Addition Postulate
6) $\triangle XYW \cong \triangle TSV$	6) Given
7) $\triangle WXY \cong \triangle VTS$	7) ASA

Proof:  $\triangle SVT \cong \triangle YWX$  and  $\overline{WS} \cong \overline{VY}$  because it is given.  $\overline{SY} \cong \overline{SY}$  because of the Reflexive Property of Congruence.  $\overline{WS} + \overline{SY} = \overline{VY} + \overline{SY}$  by the Addition Postulate, and  $\overline{WY} = \overline{VS}$  (or  $\overline{WY} \cong \overline{VS}$ ) by the Segment Addition Postulate.  $\triangle XYW \cong \triangle TSV$  because it is given. Therefore,  $\triangle WXY \cong \triangle VTS$  by ASA.

Figure 7. Diane's construction of proof one.

Diane showed work that was consistent with her attention to detail and accuracy. What intrigued me the most was Diane's reaction after given the opportunity to work in groups.

The journal prompt focused on the effectiveness of cooperative groups for the individual. I wanted to discover if the group process and journaling about their experience helped their own individual construction of proof. Diane answered, "I believe it is effective and ineffective to be in my group." She continued, "it was good to check my work, but I got my work done faster alone. It is best to work alone and then in a group." As I continued to read Diane's entry, she

explained that the value of group work for her was to provide a self check if “you don’t understand something.” It appeared that Diane had taken a similar stance back in her first entry. She expressed a desire to work independently but she could tolerate cooperating as long as she received some individual time to work. Diane made her point know not only in writing, but the class had time to share their journal entries with the class. Diane shared her point of view and others in her class agreed. Working cooperatively was beneficial, yet working individually first could provide a richer experience to the group process.

Nancy had work identical to Diane for her statements and reasons and her paragraph proof. Recall, that Nancy was in a different location of the room and had positive experiences with the previous day’s activity. Although both students are high achieving, Nancy’s reflection had a different tone than Diane’s entry. Even though her experiences the previous day gave her some feelings of success, Nancy doubted her work on the proof until she had the opportunity to check her work in the group. Her reflection was short but it gave me insight into an academically talented student who was not certain of her capabilities and attitudes toward mathematics. She explained that groups were effective for her because “I was able to double check my work.” “I always second guess myself.” In observing Nancy’s work, she utilized the tools we have put in place for constructing proof. She drew diagrams. Nancy created index cards of the theorems we studied in class and made comments on them for their use. While writing proof one (Appendix D), Nancy sifted through her index cards on triangles and line segments and developed the progression from the given statements to the end of the proof. Upon a closer inspection of Nancy’s work, she did not have any eraser marks within her journal. No changes were made from her original work. Still, her confidence level was lacking. We had



been writing for about nine weeks and this was a constant theme within Nancy's writing. Her achievement was fine. Nancy scored well not only on an FCAT test, but on in class assessments. Yet she continued to communicate her uncertainty.

Dan completed this task incorrectly (Figure 8). He had been absent for the group proof activity and his process was inaccurate and incomplete. He did not do a paragraph proof.

Statements	Reasons
$\angle X Y W \cong \angle T S V$	Given
$\angle S V T \cong \angle Y W X$	Given
$\overline{W S} \cong \overline{V Y}$	Addition Postulate
$\overline{S Y} \cong \overline{S Y}$	Reflexive Prop. of $\cong$
$\overline{W S} \cong \overline{V Y}$	ASA
$\angle X \cong \angle T$	SAS
$\triangle W X Y \cong \triangle V T S$	3rd Angle Theorem

Figure 8. Dan's construction of proof one.

He drew the diagram appropriately in his journal and it appeared he was accurate and took time on this part of the task, but completing the statements and reasons showed little logical progression. Dan's lack of experience with cooperative groups in the previous activity may have affected his own proof. Dan's previous journal entries indicated he relied on feedback from others just to get the process started. Unfortunately, an absence from class the previous day resulted in creating incorrect conclusions of SAS and third angle theorem.

Steven's proof was identical to Diane's entry. The result made sense since both were in the same group. The third entry was Steven best entry. He completed the statements and

reasons proof on his own and then he completed the paragraph proof while in groups. His reflection had some common themes to Diane even though the reflection was completed separately. For this entry, Diane's high achievement and high attitude helped Steven to raise his level of work output. Her dominance as a leader may have influenced Steven.

He stated an idea that he had expressed in a previous entry. Steven said, "working in groups was effective because you get ideas from other members." I struggled to interpret his second sentence. I believed Steven meant he liked working on his own at first. He indicated it was appropriate to share ideas after he had time to construct his own approach.

From previous observations and journal entries Steven had been more passive both in action and in his journal entries. Now he was stating that cooperative groups allowed him to get ideas from others, yet independent work was also productive. Steven and Diane were sharing a common outlook on their group work. Diane (HH) and Steven (LL) indicated individual work time helped make the cooperative group time more useful. In this entry, Steven showed signs of change within his writing and behavior in cooperative groups.

### **Analysis of third journal entry**

At this point, we were in the third month of school and had been journaling for approximately nine weeks. All of the students felt comfortable in the several group structures we used in geometry class. Diane and Steven's idea of wanting some independent time before working cooperatively appeared in other student journals. In fact, all three classes had approximately seventy-five percent of the students stating their perception of individual work was helpful before engaging cooperatively. The insight this gave to me was that student

appreciated individual time to write, organize for proofs, and then construct the proofs. Not only did I need to structure group activities but individual time as well. From this point, I gave student's individual time first. I set a timer for individual time and then reset the timer for cooperative group time. Student writings indicated that they used group time effectively as long as they had time to build their own understanding and then bring it to the group context. In the last entry, we were still working with proofs, yet now we worked with quadrilaterals. The last entry provided additional self-reflection on asking the student what they learned and how.

#### **Journal Entry four**

The last journal entry marked the twelfth week the students had been writing within geometry class. Before students got into groups, time was given to develop their own proof and the group members would have time to review, reconstruct, and argue over their process. All four members of this case study, Diane, Nancy, Dan, and Steven belonged to the same group. Figure 9 was Diane's fourth journal entry. Eventually it became a group activity on proving properties within quadrilaterals.

pg. 365 #32  
 Prove Thm. 6-4

Given:  $\square ABCD$

Prove:  $\angle A$  is supple. to  $\angle B$   
 $\angle A$  is supple. to  $\angle D$

Statements	Reasons
1. $\square ABCD$	1. Given
4. $m\angle A + m\angle B = 180$ $m\angle B + m\angle C = 180$ $m\angle C + m\angle D = 180$ $m\angle D + m\angle A = 180$	4. If a quadrilateral is a $\square$ , then its consecutive $\angle$ s are supplementary. (Theorem 6-4)
2. $AB \parallel CD$ & $BC \parallel DA$	2. Def. of a $\square$
3. $\angle A \cong \angle C$ & $\angle B \cong \angle D$	3. Thm. 6-5

Figure 9. Diane's proof of Theorem 6-4.

Again, Diane showed a great attention to detail. Yet after analyzing her response, I noticed Diane had made a mistake in her final line of her proof. I did not comment on the mistake until after I saw her journal. I had given the students two writing prompts after they constructed their proof. I had asked them to describe what they had learned and if they had any mathematical concerns over that day's topic or topics in the past. Diane's writing indicated that the proof in Figure 9 was connected to what "we" learned in the past from congruent triangles. She stated that if she could identify congruent triangles then she could use the corresponding parts to state what needed to be proven.

Within Diane's writing, she exhibited confidence in her own work. She believed she was correct in her proof, but in her writing, I discovered Diane and other students in the class had similar error patterns. Diane's next to last line of the proof used a theorem we had not proven.

In addition, the theorem that was used had to deal with opposite angles being congruent. She then proved the problem by using another theorem that the class had not proven. After I read more into Diane's argument, I discovered her conclusions were incorrect. This helped me plan for the next day's lesson since the same error was prevalent among the rest of the class.

Nancy, Dan, and Steven were in the same group as Diane. Each of these students used individual time and group time to solve this same proof. The interesting part was that all made similar errors but their reflections were different.

Nancy indicated in her reflection that she required assistance from all the members in her group to understand how to begin the proof. As she wrote about any concerns, she documented that proof writing troubled her, especially constructing an indirect proof. I wrote back and asked if it was how to use the theorems or not knowing where to start in a proof. Her response was "both". As I took field notes on the group's activity, I noticed Nancy utilized the group process to ask questions about where to start, how to use the note cards she had constructed on the theorems we were studying in class. In Figure 10, the uncertainty of Nancy's work can be seen, since she inserted step five between step one and step two.

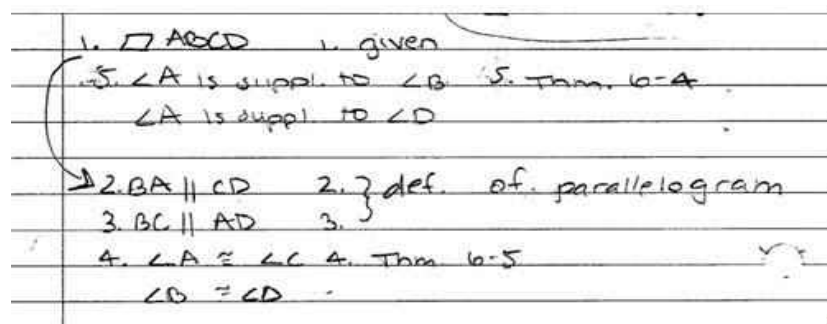


Figure 10. Nancy proof of Theorem 6-4.

She followed Diane’s lead to complete the proof, and in her reflection due to her reliance on Diane she stated she felt “more confident.” Once again, the group process allowed Nancy to receive feedback and ideas on how to construct the proof, even though it was incorrect.

Dan (LH) entry was the shortest of the four. He stated in his reflection that he did not have any concerns. Dan’s version of the proof (Figure 11) used three statements and his justification never lead to any logical argument. Dan’s error was he stated the theorem that he was trying to prove. The error was similar to Diane, Nancy and Steven’s, yet Dan did not use any intermediate steps from his given to the conclusion.

$\square ABCD$	Given
$\angle A$ is supplementary to $\angle B$	Theorem 6-4
$\angle A$ is supplementary to $\angle D$	Theorem 6-4

Figure 11. Dan’s proof of Theorem 6-4.

In Dan’s reflection he described, “Today I have learned how to make a proof with parallelograms. I learned it by working with my group.” However, his proof did not look like any of his teammates within his group. He expressed “I do not have any mathematical concerns.” Yet, the construction of Dan’s proof needed assistance. Dan held to his ideas in his proof even though his three other teammates had five steps in their proof Dan was content on using only two.

Steven (LL) wrote a proof that was identical to Diane’s proof. After discussing the proof in his group, he gave credit to Diane. I noticed that Steven appeared to be uninvolved in the

group process. I asked him a question. The question was, how did he decide on the steps in his proof? He stated that he learned how to do the proof by Diane talking about how to use the theorems that involved parallelograms.

### **Analysis of fourth journal entry**

The four incorrect results led me to believe that Diane was an influential leader in the group and Nancy, Dan, and Steven had come to rely on her authority. I noticed discussions taking place, yet it was obvious that no one questioned Diane. The incident led me to plan several lessons on not using the theorem one was trying to prove. It was similar to defining vocabulary words without using the original word. Diane, Nancy, Dan, and Steven along with the whole class struggled with this concept. My own personal experience with mathematical proof was challenged. I had to find methods and activities that would convince my geometry students how to get out of this circular argument.

Having the opportunity to view these tasks was like having one on one tutoring sessions with each of the four students. Diane was HH and her leadership role in her group also appeared in class. She was willing to ask questions to other students in the class, she was willing to challenge what was written in the text, and she was willing to challenge explanations or tasks I developed to practice geometric proofs. Diane's attention to detail in writing was part of her skills as a student. I learned that she was very confident in her work and was willing to work in a collaborative group, but she appreciated the opportunity to formulate her own ideas. Meanwhile, Nancy's scored well on in class assessments. Like Diane, Nancy was a high achieving student

but she doubted her abilities. Her writing indicated the group process helped her validate her findings and then move forward. She claimed to “second guess” her work in her journal entries.

Dan was a low achieving student with a high attitude. As he worked on his own, he believed he understood, but when he used the group process, he discovered new ideas in discussions. In reading some of Dan’s other entries, I came to understand that he felt like he was aware of the concepts and comprehended the proofs. Yet his work, did not always show this. Finally, Steven, who was LL, was not highly engaged. He was willing to let other individuals in the class like a Diane take the lead role in a group. He was fine being passive. Steven was satisfied with finishing the work, but not interested in truly understanding why.

According to the work of Aspinwall and Miller (2001) and Countryman (1992) the journal writing should provide me a picture of what my students understand. The four entries for Diane (HH), Nancy (HL), Dan (LH), and Steven (LL) provided information on their construction of proof, and information on the group process. The challenge was to determine if their attitude and achievement had any effect on their construction of proof and their perception of the group process. At this point, the one student who had a common theme throughout three of her four entries was Nancy. Nancy looked forward to using the group structures to complete her work. Meanwhile Diane and Steven concluded working individually first was more appropriate. Dan’s entries proved inconclusive.

Finally, my role became one of an observer and listener. I became more comfortable with posing questions instead of giving answers. The students in all the groups became aware of my practice of answering a question with a question. The students in all three of my geometry



classes became accustomed to me writing down ideas in my notebook as I listened to their conversations. Later in the class, these conversations became opportunities for more whole class conversations and students willing to defend or share their work. We were all constructing the mathematics, and then together we had the opportunity to reflect on our thinking by recording a journal entry. Over the course of twelve weeks in an eighth grade geometry class, writing had become more natural. I was noticing the types of language the students would use to help them in constructing the proof, and their reasoning behind their process. All the theorems, postulates, and proofs became part of the journal writing. Journal writing had become a regular part of the classroom.

In chapter five, I plan to indicate how the journal writing can have further classroom implications and continue to comment on my findings within this case study,

## **CHAPTER FIVE: CONCLUSIONS**

### **Introduction**

According to the NCTM's Principle and Standards (2000), communication is vital at all grade levels. From Pre K to Grade 12 NCTM's vision is for all students (p. 348):

- 1) Organize and consolidate their mathematical thinking through communication.
- 2) Communicate their thinking clearly to peers, teachers, and others.
- 3) Analyze and evaluate the mathematical strategies of others.
- 4) Use the language of mathematics to express ideas precisely.

The focus of this study was to determine how journal writing revealed the nature of cooperative learning in the construction of geometric proof for four eighth grade students. Journal prompts were analyzed over a period of twelve weeks of forty-one participating geometry students in eighth grade. Four students were selected based on their achievement and attitude toward mathematics. Writing became a part of the curriculum throughout the study. All students were expected to record information in their journals based on tasks assigned from the geometry curriculum and problem solving tasks designed by the teacher to elicit student response in a paragraph proof or two-column proof format. Students had opportunities to reflect on the group process, and its affect on the construction of proofs. Each student had opportunities to write about the task and their perception of their own understanding.

### **Overview of Results**

The use of journal writing and cooperative learning provided me with feedback on the student's progression of developing proof. Initially some of my main concerns were student

participation within cooperative groups may not yield enough independent work, and my hope was that the journal writing would provide an opportunity to determine what each student was thinking and doing. My observations, field notes, and short discussions with the four participants proved that three of the four students were actively involved throughout the research period. Diane (HH), Nancy (HL) and Dan (LH) were actively involved in each of their groups, while Steven (LL) was passive in two of the four entries. Diane, Nancy were willing to discuss the assigned tasks, work with other members of their group and record their work. Dan was willing to discuss and be actively involved in his group, yet he did not record what took place in his group or used it to construct a proof. Diane and Nancy recorded their work in detail, while Dan participated verbally while his writing was minimal. However, Steven's outcome was different. In two of the four cases, he needed to be prompted by either other members of the group or me to become engaged in the group activity. Similar to Dan, he did not record many details on the events of the group.

Student self-reporting was accurate and at times more demanding than my own assessment of their activities within the group structures, especially in Diane and Nancy's case. According to Countryman (1992), student journals were an opportunity for students to track their work and determine progress. Three out of my four students, within the case study, I believe experienced a roadmap of their work with proof. Overall, the journal writing provided insight into what the students were thinking. The writing became a "pathway into their cognition"(p. 27). Reading student journals provided information about their level of understanding. It also provided student perception of their activity and involvement within their cooperative learning groups. Correlated with my direct observation and field notes, the writing within a student's

journals provided an opportunity to gauge a student's understanding and their attitude. According to Borasi and Rose (1989), writing within the mathematics classroom offers the potential of working individually with students when time does not permit. Journal writing provided the virtual conference and insight into an eighth grader's academic and social concerns within the group construct. Recall that journal entry one was a problem solving activity (Appendix B) where the students had to prove a pattern generally, journal entry 2 was the student's perception on the effectiveness of the cooperative learning group process. Journal entry 3 was divided into two parts: part one was the strategy of the group proof, while part two was the individual proof the student created. Finally, journal entry four was a proof on quadrilaterals and a reflection on any mathematical concerns that the student had at the time.

### **Diane HH**

Diane's journal entries were detailed and organized. Recall her achievement level and attitude toward mathematics were very high, and she was explicit in what she described in each journal entry. Within each entry, Diane was able to clearly display her arguments and justify her position. In Diane's first entry she documented her results and went beyond the requirements of the writing prompts since she was not only able to identify a general pattern but she could classify the situation as a second-degree equation due to the use of finite differences. Diane continued to meet or surpass the requirements in the remaining three entries. I saw in Diane's writing and her participation in class geometric proof provided a challenge for her. Yet in three out of four entries, Diane was accurate with her proofs, and proficient in the process of writing a proof. She was willing to share her work within her cooperative group and with the class as a

whole. Diane was a student who wanted to display her thinking. This was evident in her writing, not only within the four samples in this study but in all entries.

I observed that Diane was one student who used all the writing time given. If I gave five to seven minutes for a short reflection, she used all the time. Her first entry she took twenty minutes to construct her final analysis and this was after four pages of previous documentation. The writing provided Diane with another tool to look at her own work and analyze it further. Her own writing brought her to the conclusion cooperative groups were appropriate for her learning process, yet she wanted time to think on her own prior to engaging in group-work.

Diane's writing style changed within the twelve weeks. Her first entry was written primarily in first person. As the study progressed, her second and third entry turned into a commentary on what her group did. Her final entry turned more back toward the first person, and this was about the time where her decision was made on wanting individual time before working in groups.

### **Nancy HL**

Nancy was a student who wrote two correct proofs out of four entries. Her first entry was partially correct and well detailed, but her conclusion was incorrect. Her second entry began to detail her dependence on the cooperative group process. Nancy achievement level was high but her attitude towards mathematics was classified as low. She detailed in three out of four entries that she perceived the groups were necessary for her to "double check" and validate her work, because in Nancy's own words she "second guessed herself".

Nancy's writing provided a voice for her concerns. She indicated how helpful the group process was for her. She diagnosed her own trouble spots. In entry four, Nancy noticed she was weak in indirect proof. From personal observation, Nancy was willing to put forth the effort, even if she was uncertain. The journal writing was a tool she utilized to enhance her efforts and reach out for the help she needed. The writing helped her become aware of her strengths and weakness, and at this point she realized that her active involvement within the group provided her information on her own progress. Nancy wrote in her journal using the words we did and we solved. Nancy's writing indicated her need for the cooperative group process. The writing coupled with the group work in Nancy's case provided a sense of security. Two out of four correct proofs was not a tremendous percentage, but from Nancy's writing her personal insight was a valuable learning tool.

### **Dan LH**

Dan's writing was interesting to read and he was an interesting case to observe. His attitude toward mathematics was classified high similar to Diane, yet the achievement level was very different. Out of four journal entries, Dan was able to record one correct proof, yet the one correct proof did not have much justification. There were little details to support his conclusion.

Further entries provided insight into how Dan operated within cooperative groups. His commentaries of "getting ideas from someone", forgetting to reflect on the third entry, and not having any mathematical concerns in the fourth entry showed that Dan was not able to construct a proof on his own. His fourth entry had no logical progression at all and did not even match any of the work within his group.

In my observations, Dan participated in his group. However, his participation and discussions were not detailed in his writing. I am uncertain if this was a product of Dan's unwillingness to write or a confirmation of his achievement level. Initially his achievement level was ranked third out of the four candidates, yet his geometry and algebra sub-scores within his FCAT testing were among the lowest of all four candidates.

### **Steven LL**

Steven managed to write two correct proofs out of four journal entries. Steven's progress was challenging to monitor since he wrote a minimal amount. His first entry had the correct result but his justification did not match his conclusion. His third entry, which was the group proof and a follow-up individual proof, was correct. Steven's third entry was his best out of the four. Along with a valid proof, I observed he was actively engaged in constructing the proof. The second and fourth entry were passive. In fact, Steven's lack of writing or a few choice words indicated his passivity. In entry two, he stated that group work "gave voice to his ideas", but his actions did not match his words. The journal writing can provide a documentation of a student's action but in this case, it can also inform of contradictory events. A student such as Steven can appear to be constructing proof in groups and individually, but the journal writing can prove otherwise.

Steven's initial classification as lower achieving and lower attitude, may have been an early indicator to keep his progress closely monitored. His score on the attitude scale was one of the lowest in his geometry class. One could conjecture the minimalist approach he took to the journal writing and cooperative group involvement may be directly connected to his achievement

and attitude. In direct contrast to Nancy whose attitude score was similar, yet her willingness to write and desire to participate in the cooperative learning groups was very different. The point of this research was to determine how journal writing revealed the nature of cooperative groups in the construction of proofs, yet Steven's lack of consistent involvement made determining results inconclusive. Direct observation and personal interviews led me to understand Steven chose to be uninvolved.

### **Summary**

The point of the research study was to determine how journal writing revealed the nature of cooperative learning in the construction of proof. The one main discovery many students in all three geometry classes confirmed was working in cooperative groups helped in the construction of proofs, yet the students appreciated having individual time to work on their own. The second journal prompt from this study: *How does working in a group help you determine your own solution?* was the point in the research where a majority of my students wrote and discussed the cooperative experiences were helpful. However many students want individual time before moving to groups. Without this journal prompt, I was not aware of this need within the class.

According to Borasi and Rose (1989), writing within the mathematics classroom offers the potential of working individually with students when time does not permit. Journal writing provided the virtual conference and insight into an eighth grader's academic and social concerns within the cooperative group setting. This can be especially helpful for a student who appears to be high achieving yet the attitude was low. Journal prompts can provide information as an informal assessment and help the student convey their understanding to the teacher before any



formal assessments. Also, Borasi and Rose (1989) commented journal writing can provide the teacher and the student to develop a relationship in writing and ask questions which otherwise may go unspoken. Steffe (1991, p 189) believes that this “interactive communication constitutes the central activity of teaching”

## **Recommendations**

While conducting my research on how journal writing and cooperative groups assist the construction of proofs, I reflected on my choice of journal prompts. For this research study, I used a series of nine questions from Appendix B, which was a problem solving task. My second journal prompt was focused on describing if working in a group was helpful in constructing a student’s own solution. The next two prompts came after the students wrote a proof either in groups or on their own. The main idea behind each of these prompts asked the students to describe how the group process helped them do the proof, and express any mathematical concerns. Now that my research period has ended, I would have liked to use a standard set of writing prompts or developed my prompts well in advance of the proofs that were taught. Generating the prompts well in advance may have provided richer data.

Another recommendation I have is to have the students read one another’s entries. In fact, using the group structure of roundtable could be an appropriate method to accomplish this. Students could read one another’s writing, comment in a constructive manner, and then add to their own writing after the roundtable. Using this technique could continue to provide more data for my question, since the writing and cooperative group structure can be used together.

Finally to get a broader lens on this research, I would like to conduct focus group discussions. Again it may provide more data to the question posed within this research. Students could be grouped by their achievement level or attitude level and questions be posed to augment the writing process. The questions would focus on the construction of proof and determine any other factors to assist in writing proof.

As a result of my research, I plan to use writing with both my algebra and geometry students. The information the journals provided for lesson planning was valuable. Also, some of my algebra students become my geometry students the following year and having students write over a period of two school years could provide data over a longer time period.

**APPENDIX A: ATTITUDES TOWARDS MATHEMATICS AND ITS TEACHING  
(ATMAT) SURVEY**

ID # \_\_\_\_\_

DIRECTIONS: Each of the statements on this oppinionaire expresses a feeling which a particular person has toward mathematics. You are to express, on a six-point scale, the extent of agreement between the feeling expressed in each statement and your own personal feeling. The six are Very Strongly Agree (VSA), Strongly Agree (SA), Agree (A), Disagree (D), Strongly Disagree (SD), and Very Strongly Disagree (VSD). You are to encircle the letter(s) which best indicates how closely you agree or disagree with the feeling expressed in each statement AS IT CONCERNS YOU TODAY. Please mark only one answer.

- |   |     |    |   |   |    |     |
|---|-----|----|---|---|----|-----|
| 1. Mathematics is very interesting to me, and I enjoy math courses.   | VSA | SA | A | D | SD | VSD |
| 2. My mind goes bank, and I am unable to think clearly when doing math.   | VSA | SA | A | D | SD | VSD |
| 3. I feel a sense of insecurity when doing math.  | VSA | SA | A | D | SD | VSD |
| 4. Mathematics makes me feel uncomfortable, restless, irritable, and impatient  | VSA | SA | A | D | SD | VSD |
| 5. I approach math with a feeling of hesitation resulting from a fear of not being able to do math.                         | VSA | SA | A | D | SD | VSD |
| 6. Mathematics is a course in school which I have always enjoyed studying.  | VSA | SA | A | D | SD | VSD |
| 7. It makes me nervous to even think about having to do a math problem  | VSA | SA | A | D | SD | VSD |
| 8. I feel a definite positive reaction to mathematics; it's enjoyable.  | VSA | SA | A | D | SD | VSD |
| 9. If I am confronted with a new mathematical situation, I can cope with it because I have a good background in mathematics | VSA | SA | A | D | SD | VSD |

10. I get flustered if I am confronted with a problem different from the problems worked in class.	VSA	SA	A	D	SD	VSD
11. I do not attempt to work a problem without Referring to the textbook or class notes.	VSA	SA	A	D	SD	VSD
12. I can draw upon a wide variety of mathematical techniques to solve a particular problem.	VSA	SA	A	D	SD	VSD
13. I do not feel that I have a good working knowledge of the mathematics courses I have taken so far.	VSA	SA	A	D	SD	VSD
14. I believe that if I work long enough on a mathematics problem, I will be able to solve it.	VSA	SA	A	D	SD	VSD
15. I have forgotten many of the mathematical concepts which I have learned.	VSA	SA	A	D	SD	VSD
16. I learn mathematics by understanding the underlying logical principles, not by memorizing rules.	VSA	SA	A	D	SD	VSD
17. If I cannot solve a mathematics problem, at least I know a general method of attacking it.	VSA	SA	A	D	SD	VSD
18. Problem solving fascinates me.	VSA	SA	A	D	SD	VSD
19. I have more confidence in my ability to deal with mathematics than in my ability to deal with other academic subjects.	VSA	SA	A	D	SD	VSD
20. Mathematics classes provide the opportunity to learn values which are useful in other parts of daily living.	VSA	SA	A	D	SD	VSD

**APPENDIX B: PROBLEM SOLVING TASK 1**

Objective: The students will be able to develop a pattern using manipulatives, tables, and develop a rule or an algorithm to generate the pattern at any level.

### **The Problem:**

There are seven stepping-stones and six people. On the three left-hand stones, facing the center, stand three of the people. The other three people stand on the three right-hand stones, also facing the center. The center stone is not occupied.(NCTM, 2002)

#### *Introductory Question:*

1. Can you describe a hands-on approach that depicts the problem statement?
2. Draw or model this description.

### **The Challenge: exchanging places**

Everyone must move so that the people originally standing on the right-hand stepping stones are on the left-hand stones and so that those originally standing on the left-hand stepping stones are on the right-hand stones, with the center stone again unoccupied.

### **The rules**

1. After each move, each person must be standing on a stepping stone.
2. If you start on the left, you may move only to the right. If you start on the right, you may only move to the left.
3. You may “jump” another person if the stone on the other side is empty. You may not “jump” more than one person.
4. Only one person may move at a time.

### **Discuss each of these questions in your group, and write at least one paragraph**

1. What if only two people and three spaces are involved, does this help in solving the original problem? Explain.
2. What if four people and five spaces are involved, does this help in solving the original problem? Explain.
3. How many moves are needed for four people to exchange positions?
4. What about six?
5. What about eight?
6. What about 10?
7. Is there a pattern occurring? Can you describe the pattern with  $N$  number of people?

**APPENDIX C: SUPPLEMENTARY PROBLEM SOLVING TASK 2**



The students will be told the mathematical tale behind the activity Tower of Hanoi and utilize physical and virtual manipulatives to generate a rule and a proof for the recursive function. The students will be asked to move all the discs from one tower to another in the least amount of moves.

**The rules:**

1. You can only move one disc at a time.
2. A smaller one can only be placed on top of any larger one.
3. The problem has been solved when all of the discs have been moved from one tower to another with the largest on the bottom to the smallest at the top with the minimum number of moves.

**The Procedure:**

1. Work as a pair or alone to move the discs according to the rules
  2. Use the web link as a help to repeat and process the solution:
- <http://www.mazeworks.com/hanoi/>
  - <http://www.cut-the-knot.org/recurrence/hanoi.shtml>

**Questions to discuss and write.**

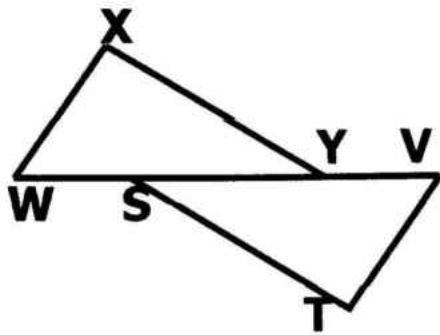
1. Can you identify your strategies of success or failure in solving this problem?
2. Explain why or why you were not successful in solving the problem.

**Complete the table below. Look for patterns**

Number of discs	Minimum # moves
1	
2	
3	
4	
N	

## **APPENDIX D: PROOF 1**

**Proof ONE**



Given:

$$\angle SVT \cong \angle YWX$$

$$\angle XYW \cong \angle TSV$$

$$\overline{WS} \cong \overline{VY}$$

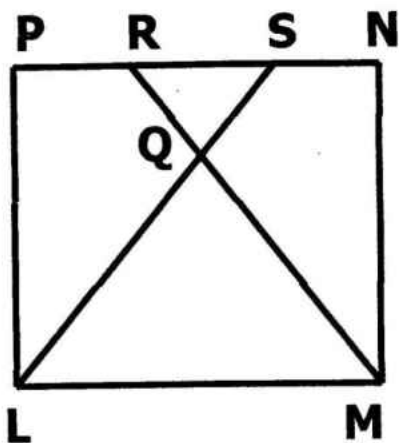
Prove:

$$\triangle WXY \cong \triangle VTS$$

$\angle SVT \cong \angle YWX$	Given
$\angle XYW \cong \angle TSV$	Given
$\overline{WS} \cong \overline{VY}$	Given
$\overline{SY} \cong \overline{SY}$	Reflexive Property
$\overline{WY} \cong \overline{VS}$	Addition Postulate
$\triangle WXY \cong \triangle VTS$	ASA

## **APPENDIX E: EXAMPLE OF GROUP PROOF**

**Proof TWO**



Given:

$$\overline{LP} \perp \overline{PN}$$

$$\overline{MN} \perp \overline{PN}$$

$$\overline{PR} \cong \overline{NS}$$

$$\overline{LP} \cong \overline{NM}$$

Prove:

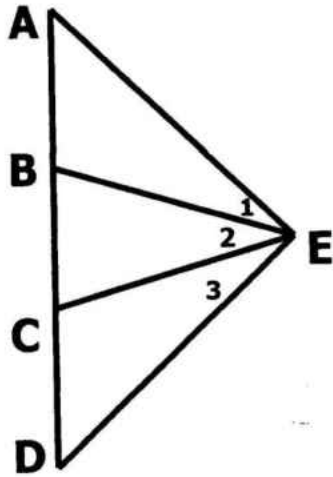
$$\triangle PLS \cong \triangle NMR$$

$\overline{LP} \cong \overline{MN}$	Given
$\overline{PR} \cong \overline{NS}$	Given
$\overline{RS} \cong \overline{RS}$	Reflexive Property
$\overline{PR} + \overline{RS} \cong \overline{NS} + \overline{SR}$	Addition Postulate
$\overline{PS} \cong \overline{NR}$	Substitution Postulate

$\overline{LP} \perp \overline{PN}$	Given
$\overline{MN} \perp \overline{PN}$	Given
$\sphericalangle LPS$ and $\sphericalangle MNP$ are right $\sphericalangle$ s	Definition of <i>perpendicular</i>
$\sphericalangle LPS \cong \sphericalangle MNP$	Right $\sphericalangle$ s are congruent
$\triangle PLS \cong \triangle NMR$	SAS

## **APPENDIX F: EXAMPLE OF GROUP PROOF**

**Proof THREE**



Given :

$$\sphericalangle 1 \cong \sphericalangle 2$$

$$\sphericalangle 2 \cong \sphericalangle 3$$

$$\overline{EB} \cong \overline{EC}$$

$$\overline{AE} \cong \overline{DE}$$

Prove:

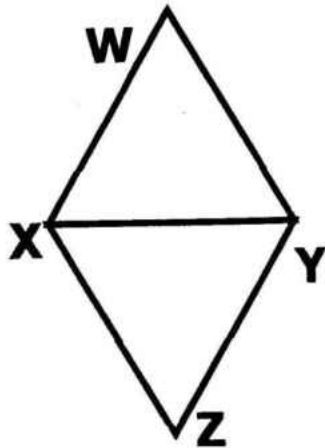
$$\triangle AEB \cong \triangle DEC$$

$\sphericalangle 1 \cong \sphericalangle 2$	Given
$\sphericalangle 2 \cong \sphericalangle 3$	Given
$\sphericalangle 1 \cong \sphericalangle 3$	Transitive Property
$\overline{EB} \cong \overline{EC}$	Given
$\overline{AE} \cong \overline{DE}$	Given
$\triangle AEB \cong \triangle DEC$	SAS



## **APPENDIX G: EXAMPLE OF GROUP PROOF**

**Proof FOUR**



Given:

$\overline{XY}$  bisects  $\sphericalangle WXZ$

$\overline{WX} \cong \overline{ZX}$

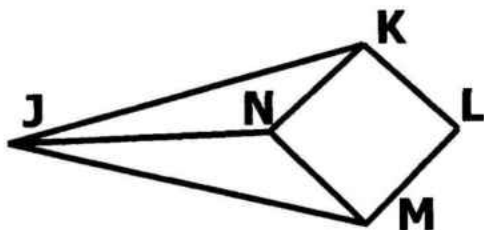
Prove:

$\triangle WXY \cong \triangle ZXY$

$\overline{XY}$ bisects $\sphericalangle WXZ$	Given
$\sphericalangle WXY \cong \sphericalangle ZXY$	Definition of $\sphericalangle$ bisector
$\overline{XY} \cong \overline{XY}$	Reflexive Property
$\overline{WX} \cong \overline{ZX}$	Given
$\triangle WXY \cong \triangle ZXY$	SAS

## **APPENDIX H: EXAMPLE OF GROUP PROOF**

**Proof FIVE**



Given:

$\square NKLM$  is a square

$$\overline{JK} \cong \overline{JM}$$

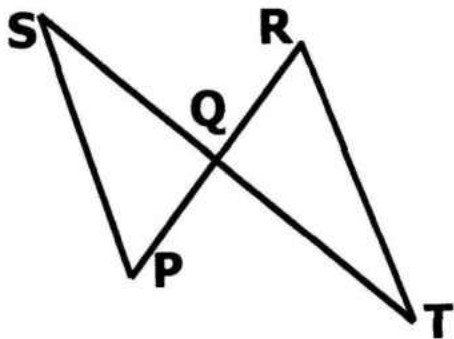
Prove:

$$\triangle JKN \cong \triangle JMN$$

$\square NKLM$ is a square	Given
$\overline{NM} \cong \overline{NK}$	All sides of a $\square$ are $\cong$
$\overline{JN} \cong \overline{JN}$	Reflexive Property
$\overline{JK} \cong \overline{JM}$	Given
$\triangle JKN \cong \triangle JMN$	SSS

## **APPENDIX I: EXAMPLE OF GROUP PROOF**

**Proof SIX**



Given:

Q is the midpoint of  $\overline{PR}$

$\sphericalangle P \cong \sphericalangle R$

Prove:

$\triangle SPQ \cong \triangle TRQ$

Q is the midpoint of $\overline{PR}$	Given
$\overline{PQ} \cong \overline{RQ}$	Definition of <i>midpoint</i>
$\sphericalangle SQP \cong \sphericalangle TQR$	Vertical $\sphericalangle$ s are $\cong$
$\sphericalangle P \cong \sphericalangle R$	Given
$\triangle SPQ \cong \triangle TRQ$	ASA

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