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Essays on Credit and Labour Market Frictions

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Doctor of Philosophy



The University of Edinburgh

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2016

Declaration of Own Work

I declare that this thesis was written and composed by myself and is the result of my own work unless clearly stated and referenced. This thesis has not been submitted for any other degrees or professional qualifications.

Yulia Moiseeva

*to my amazing husband Sergei whose massive support
cannot be described in words and my parents.*

Without you this thesis would not have been possible

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Abstract of Thesis

The financial crisis of 2008 was characterized by disruptions in credit markets and sharp rises in unemployment. This dissertation contributes to our understanding of the interaction of credit and labour markets. The first chapter studies the impact of credit frictions on labour demand given that the labour market is frictionless. The second chapter introduces search and matching to the labour market and studies the interaction between the two types of frictions. The third chapter investigates wages determined by surplus sharing between firms and workers in the environment with search and credit frictions.

In Chapter 1 I develop a partial equilibrium model where homogenous firms face credit frictions in the form of collateral constraints. As a result of these frictions firms' demand for capital depends on their net worth. Firms hire workers in the frictionless labour market with an upward-sloping labour supply curve. The model generates a large, although short-lived, response of capital demand to a negative productivity shock. Through complementarity of factors of production the decrease in capital affects employment and wages. As a result of a one standard deviation negative productivity shock employment falls by around 0.65% and wages fall by around 1.3% as opposed to 0.11% and 0.25%, respectively, in the first-best economy. I also find that changing capital and labour supply elasticities have different implications in the presence of credit frictions compared to the first-best economy.

Chapter 2 extends Chapter 1 by introducing search frictions to the labour side of the economy. On one hand, when buying capital firms have to deal with the credit frictions outlined above. On the other hand, when hiring workers they face standard search and matching frictions. I then study the interaction of the two frictions. Credit frictions affect labour demand through complementarity of capital and labour. Search frictions influence capital demand through wages: When wages are only partially flexible, the decline in firms' net worth is larger, and the resulting fall in capital is larger as well. I also find that the response of wages to wage flexibility is non-monotonic in the presence of credit frictions. This could potentially explain why we see wages fall little in data.

In Chapter 3 I use a model of search and credit frictions developed in Chapter 2 to investigate wages determined by surplus sharing in such environment. I find that credit frictions affect the surplus-sharing mechanism in such a way that they increase the worker's effective bargaining power. That is, the firm and the worker negotiate wages as if the worker had a higher bargaining power. This is due to the fact that under search and credit frictions the firm values workers more than under pure search frictions because output they produce increases the firm's net worth. However, the effective worker's bargaining power appears to be endogenous to the firm's capital holdings and the number of employees. The more capital the firm has, the less the firm is financially constrained, and the lower wages its workers are able to extract. Due to endogeneity of the worker's effective bargaining power, the effect of credit frictions on wages is ambiguous.

Lay Summary

The financial crisis of 2008 was characterized by disruptions in credit markets and sharp rises in unemployment. This dissertation contributes to our understanding of the interaction of credit and labour markets. It studies the impact of limited credit availability on firms' labour demand policies.

In Chapter 1 I develop a model where firms face credit frictions in the form of collateral constraints. So a firm cannot borrow more than the value of its capital. As a result of these frictions firms' demand for capital depends on their net worth. On the other hand, in order to produce output firms also need to hire workers. I assume that wages adjust to equate labour supply to labour demand. The model generates a large, although short-lived, response of capital demand to a negative productivity shock. Because firms need both capital and labour to produce output the decrease in capital affects employment and wages. As a result of a one standard deviation negative productivity shock employment falls by around 0.65% and wages fall by around 1.3% as opposed to 0.11% and 0.25%, respectively, in the first-best economy. I also find that changing capital and labour supply elasticities have different implications in the presence of credit frictions compared to the first-best economy.

Chapter 2 extends Chapter 1 by considering the fact that it takes time for firms and workers to meet and that posting vacancies is costly for firms. As a result of these labour market frictions, known as search and matching frictions, some work-

ers become unemployed. In the capital market firms have to deal with the credit frictions outlined above. I then study the interaction of the two frictions. Credit frictions affect labour demand because capital and labour complement each other in the production process. Search frictions influence capital demand through wages: When wages are only partially flexible, the decline in firms' net worth is larger, and the resulting fall in capital is larger as well. I also find that the response of wages to wage flexibility is non-monotonic in the presence of credit frictions. This could potentially explain why we see wages fall little in data.

In Chapter 3 I use a model of search and credit frictions developed in Chapter 2 to investigate wages determined by surplus sharing between firms and workers in such environment. I find that credit frictions affect the surplus-sharing mechanism in such a way that strengthens the worker's bargaining position. That is, the firm and the worker negotiate wages as if the worker had a higher bargaining power. This is due to the fact that under search and credit frictions the firm values workers more than under pure search frictions because output they produce increases the firm's net worth. However, the effective worker's bargaining power appears to depend on the firm's capital holdings and the number of employees. The more capital the firm has, the less the firm is financially constrained, and the lower wages its workers are able to extract. Due to this fact, the effect of credit frictions on wages is ambiguous.

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Chapter 1. Capital market frictions and labour market fluctuations

The paper studies the impact of credit frictions on labour demand given that labour market is frictionless. I develop a partial equilibrium model where homogeneous firms face credit frictions in the form of collateral constraints. As a result of these frictions firms' demand for capital depends on their net worth. Firms hire workers in the frictionless labour market with an upward-sloping labour supply curve. The model generates a large, although short-lived, response of capital demand to a negative productivity shock. Through complementarity of factors of production the decrease in capital affects employment and wages. As a result of a one standard deviation negative productivity shock employment falls by around 0.65% and wages fall by around 1.3% as opposed to 0.11% and 0.25%, respectively, in the first-best economy. I also find that changing capital and labour supply elasticities have very different implications in the presence of credit frictions compared to the first-best economy.

1.1 Introduction

The amplitude and propagation of unemployment fluctuations remains one of the key puzzles in the macroeconomics of labour markets. Shimer (2005) showed that the standard search and matching model is unable to generate the observed business-cycle fluctuations in unemployment and vacancies. Data from the financial crisis of 2008 shows an initial sharp increase in unemployment followed by a slow recovery process. Given that the availability of credit declined significantly during the Great recession it is natural to wonder if there is a link between the two markets and whether it could perhaps bring us closer to solving the puzzle.

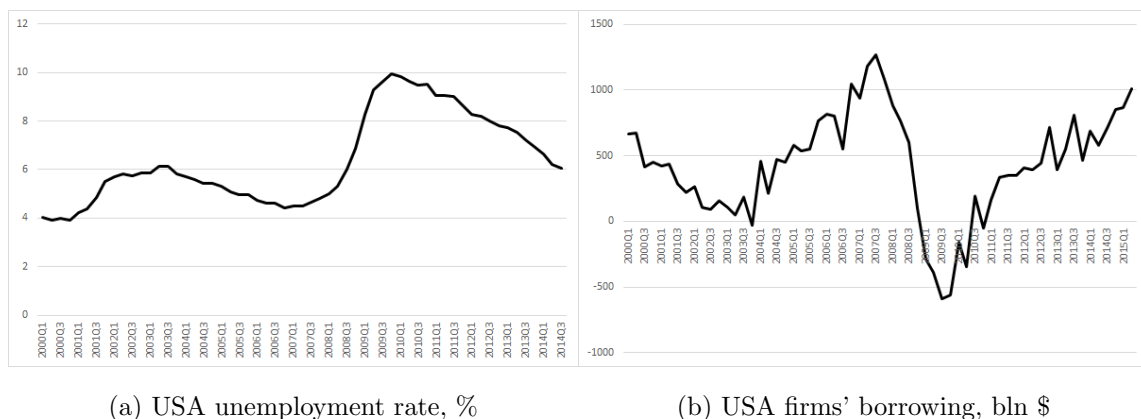


Figure 1.1.1: USA unemployment rate and firms' borrowing. Unemployment data is taken from OECD website. Borrowing data is taken from the Federal Reserve Z.1 Financial accounts of the United States table, flow series 'Nonfinancial business, debt securities and loans, liability'.

One channel through which credit availability may influence the labour market is the complementarity of labour and capital as factors of production. The idea is that as the credit market tightens, firms invest less, their capital holdings decline and so does their need for workers.

The ultimate goal of this research is to build a model with both credit and search frictions in which firms are heterogeneous in their credit limits and labour market decisions. This would provide a framework that can, in the future, address firm-level

data on capital and labour decisions. This paper is the first step towards reaching this goal.

I develop a partial equilibrium model where homogenous firms face credit frictions in the form of collateral constraints as in Kiyotaki and Moore (1997). In contrast to Kiyotaki and Moore (1997) firms also use labour as a factor of production. For now, I keep the labour market frictionless with an upward-sloping labour supply curve. Thus, there is no deep sense of unemployment in the model. I show that as a result of collateral constraints firms' capital demand depends on their net worth. The link between those creates a large, although short-lived, decline of capital demand to a negative productivity shock. Through complementarity of factors of production the decrease in capital affects employment and wages.

I find that as a result of one standard deviation negative productivity shock employment falls by around 0.65% and wages fall by around 1.3% as opposed to 0.11% and 0.25%, respectively, in the first-best economy. The responses raise hope that the model with included search frictions would have the potential to explain large unemployment fluctuations.

I proceed by discussing the implications of one of the assumptions of the model, the minimum dividend requirement. I conclude that the results are not greatly affected by this assumption.

I also explore how capital and labour supply elasticities affect impulse responses of the model variables. It turns out that the effects under credit frictions are quite different from the ones in the first-best case. In the first-best economy, the lower is the elasticity of capital supply, the more the price of capital falls as a result of a negative shock, and the less is the fall in the firm's capital. On the contrary, in the economy with credit frictions the lower is the elasticity of capital supply, the more the price of capital falls as a result of a negative shock, but the more the firm's

capital falls. The reason for this is that under credit frictions when the price of capital decreases, its net worth decreases as well, which leads to a fall in capital demand.

The effect of different labour supply elasticities in the economy with credit frictions is also different from the effects in the first-best economy. In the first-best economy, the higher is the labour supply elasticity, the less wages decrease as a result of a negative shock, and the more the firm's number of workers decreases. In the economy with credit frictions, the higher is the labour supply elasticity, the more the number of workers decreases, but the decrease in wages is larger. This is because when the labour supply is more elastic, employment decreases relatively more as a result of a shock, which leads to a greater decline in the net worth and capital demand. Thus, the marginal product of labour also declines by more, and so do wages.

The rest of the paper is organized as follows. Section 1.2 discusses the related literature. Theoretical model is explained in Section 1.3. Section 1.4 presents the calibration strategy and quantitative results. Section 1.5 concludes.

1.2 Literature review

Two major approaches to modeling credit frictions are presented in Bernanke et al. (1999) and Kiyotaki and Moore (1997). In Bernanke et al. (1999) credit frictions are based on the asymmetry of information between borrowers and lenders, and a related costly state verification problem. In their paper the realized return of a project is only known to borrowers, whereas lenders must pay an auditing cost to observe it. Kiyotaki and Moore (1997), on the other hand, choose to work with the limited enforcement problem where lenders cannot force borrowers to repay their

debt unless it is secured by a collateral.

Both papers arrive at the similar conclusion that a firm's demand for capital positively depends on its net worth. So when a negative shock hits the economy, the firm's net worth goes down and so does its capital demand. This is the first ingredient of the so-called "financial accelerator". The second ingredient is asset price variability. When the firm's demand for capital goes down the price of capital goes down as well. As a result, the firm's net worth and, consequently, its capital demand fall even more. Thus, the movement of the asset price contributes to the volatility of the firm's capital demand.

The main advantage of using Kiyotaki and Moore (1997) approach to model credit frictions is its tractability. The firm's problem and the collateral constraint lead to quite intuitive and tractable results. The key addition of this paper, and the rest of the thesis, to Kiyotaki and Moore (1997) is the exploration of the spillover effect of capital frictions to the labour market.

There are several papers that also investigate how credit frictions affect the labour market outcomes. Here I mention only the ones mostly related to this paper, with frictions in the capital market and frictionless labour market. In the following Chapters I discuss papers that model economies with frictions in both capital and labour markets.

Jermann and Quadrini (2012) consider a model with both debt and equity financing, as opposed to only-debt financing in my work, and study the impact of shocks that affect directly the financial sector of the economy. In their model a negative shock tightens the credit constraint, which is constructed in such a way that leaves two options for a firm, either to increase its equity, which is costly, or to lay off workers. They find that financial shocks contribute significantly to the observed dynamics of the labour market. The difference between their work and mine is I

concentrate on the link between the firm's capital and its net worth which Jermann and Quadrini (2012) do not consider. Also, the authors abstract from the powerful feedback from asset prices to credit constraints which is present in my model. Finally, I consider how productivity shocks affect the economy, whereas Jermann and Quadrini (2012) concentrate on financial shocks which affect the tightness of credit constraints.

There is also a paper by Buera et al. (2015) where heterogeneous firms face frictions in both credit and labour markets. But the labour market friction is different from the standard search and matching literature. In their model, unemployed workers can enter the centralized competitive hiring market only with a given probability. The authors model credit frictions by imposing a constraint on a firm's capital rental, so the firm cannot rent more capital than a certain fraction of its wealth. They find that a credit crunch causes a sharp decline in output and a protracted increase in unemployment. Unlike in my model, there is no asset prices feedback to credit constraints in Buera et al. (2015). The authors also simulate the credit crunch as a sudden tightening of credit constraints, as opposed to a decline in productivity in this paper.

In the following section I describe my model in which firms face collateral constraints in the spirit of Kiyotaki and Moore (1997) but they also need to hire workers in order to produce output. I start with a general setup and then move on to analyzing each of the three markets of the economy: credit, capital and labour.

1.3 Model

The model is based on the basic model in Kiyotaki and Moore (1997). Consider a discrete-time infinite-horizon economy populated by a mass of firms, normalized

to 1, a representative household and capital suppliers. The household consists of a continuum of workers that supply their labour to firms, which are owned by the household.

A typical firm produces output y , taken as the numeraire, using capital k_{-1} , installed last period, and workers l :¹

$$y = AF(k_{-1}, l), F_k > 0, F_{kk} < 0, F_l > 0, F_{ll} < 0, F_{kl} > 0. \quad (1.1)$$

A represents the aggregate productivity level, the evolution of which is known by firms and households in present and future periods. There are no idiosyncratic shocks so all firms are identical in their decisions.

There are markets for capital and labour where firms buy capital at price q and hire workers at wage rate w . Each period capital depreciates at rate δ .²

There is also a credit market in the economy which operates in the following way. Each period a firm may sign a one-period debt contract with the household which allows it to borrow b units of output at gross interest rate R . Next period it will need to repay Rb . Following Kiyotaki and Moore (1997) I assume the following contracting problem. When the debt contract is signed in the current period the firm cannot pre-commit to produce its output in the following period. Also, its production requires the firm's specific technology, meaning that once capital has been installed only this firm can convert it into output. Hence, the firm may use the possibility of withdrawing itself from production as a credible threat to negotiate a smaller repayment of its loans. Creditors protect themselves by requiring the firm

¹Hereafter, I denote last period values with a subscript, $_{-1}$, future values with a prime, $'$, derivatives with respect to a particular variable or variables with their corresponding subscripts.

²I assume for convenience that the capital depreciates after the next period's production is complete ($y = AF(k_{-1}, l)$) rather than at the beginning of next period ($y = AF((1 - \delta)k_{-1}, l)$).

to use its capital as borrowing collateral:

$$Rb \leq q'(1 - \delta)k. \quad (1.2)$$

The credit constraint (1.2) says that the firm's debt repayment must not exceed next period's market value of depreciated capital. So, in the event of default, creditors have the opportunity to sell the defaulted borrower's capital and cover the debt repayment. Note that in the representative agent framework default does not occur in equilibrium.

The firm also faces a profit constraint. The profit constraint is needed to restrict the firm to borrow from the household only via debt contracts described above. The profit constraint is crucial for the financial accelerator effect because it links the firm's capital demand to its net worth. The credit constraint, on the other hand, brings the asset prices feedback into the model. This will be discussed in more detail further.

The profit constraint works in the following way. Each period after producing output and paying the wage bill the firm has to pay a fixed share \bar{c} of its gross profit to the household as minimum dividend:

$$\bar{c}(AF(k_{-1}, l) - wl). \quad (1.3)$$

Here I appeal to "seniority" rules on how the firm's revenue is distributed. In general, firms pay their workers before their shareholders, and capital investment takes places after the dividend has been paid.³

³Loosely speaking, the "seniority" rule in accounting works in the following way. Net income is equal to revenue minus costs (including wages), interest payments, taxes etc. Net income is then split between dividend payments and retained earnings (earnings that the firm keeps to itself), which could be spent on the firm's expansion, investment, paying debts etc.

The minimum dividend requirement ensures that in steady-state equilibrium the firm is financially constrained which greatly simplifies solving the model. I investigate how the value of \bar{c} changes the model results in Section 1.4.3.

The profit constraint dictates that at the end of each day the firm's flow profit must be either positive or zero:

$$(1 - \bar{c})(AF(k_{-1}, l) - wl) + b - Rb_{-1} - q(k - (1 - \delta)k_{-1}) \geq 0. \quad (1.4)$$

In the former case the firm distributes the remaining profit as extra dividends to the household. The profit constraint, thus, ensures that debt contracts are the firm's only source of borrowing. In other words, the firm cannot borrow from the household as its shareholder.

The timing of the model is as follows. At the beginning of a period the firm hires workers and produces output, then it pays wages and distributes the minimum dividend to the household. After that, the firm's capital depreciates, the firm repays its debt held from the last period, borrows from the household and invests into capital. At the end of the period, if it has some output left, it distributes it as extra dividends to the household.

The objective of each firm is to maximize its respective discounted value of life-time profits by choosing capital, debt and the number of workers:

$$\Pi(k_{-1}, b_{-1}, A) = \max_{k, b, l} \left\{ AF(k_{-1}, l) - wl + b - Rb_{-1} - q(k - (1 - \delta)k_{-1}) + \beta \Pi(k, b, l, A') \right\} \quad (1.5)$$

subject to the credit constraint

$$Rb \leq q'(1 - \delta)k \quad (1.6)$$

and the profit constraint

$$(1 - \bar{c})(AF(k_{-1}, l) - wl) + b - Rb_{-1} - q(k - (1 - \delta)k_{-1}) \geq 0. \quad (1.7)$$

In the following sections I discuss supply and demand sides of the three markets: credit market, capital market and labour market, where the demand sides follow from the firm's problem (1.5)-(1.7).

1.3.1 Credit market

In credit market equilibrium, the interest rate R^* is determined by supply and demand for debt. Consider an equilibrium in which firms borrow.

Consider first the supply of debt. I assume that the household's discount factor is equal to β^h , so they are willing to lend as long as the interest rate is greater or equal to the inverse of their discount factor $\frac{1}{\beta^h}$. I do not explicitly model the behaviour of the household. Instead, I make this fairly standard assumption about it. This comes at the expense of not having a general equilibrium model. But it does come with the benefit of analytical tractability.

Therefore, the supply curve of debt is a horizontal line which intersects the R -axis at $\frac{1}{\beta^h}$.

I make the following assumption about the values of firms' discount factor β and β^h .

Assumption 1. $0 < \beta < \beta^h < 1$.

Assumption 1 states that the household is more patient than firms. This ensures that in and around steady-state equilibrium firms borrow.

Consider now the demand for debt. The first-order condition of the problem

(1.5)-(1.7) with respect to b states:

$$1 - \beta R - \lambda R + \mu - \mu' R = 0, \quad (1.8)$$

where λ denotes the Lagrange multiplier on the credit constraint and μ denotes the Lagrange multiplier on the profit constraint. Solving for λ in steady state gives:

$$\lambda^{SS} = \frac{1 - \beta R}{R}(1 + \mu). \quad (1.9)$$

Provided that $\beta R < 1$ the representative firm always borrows up to the maximum in steady state. Thus, the aggregate demand curve is a horizontal line which intersects the R -axis at $\frac{1}{\beta}$ but is bounded by the credit constraint (1.2) on the B -axis. Credit market equilibrium is depicted in Figure 1.3.1.

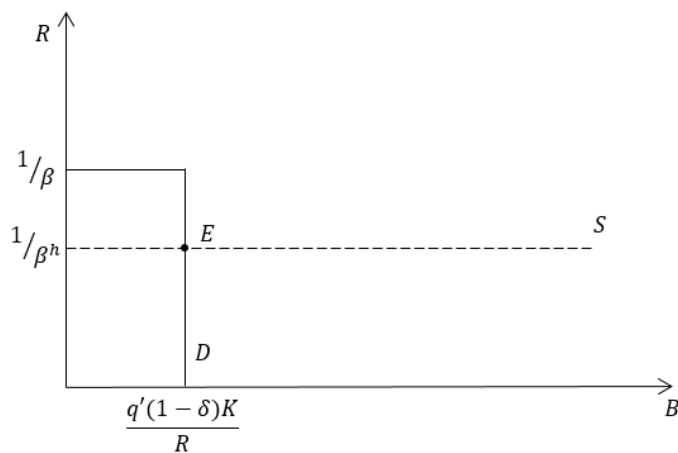


Figure 1.3.1: Credit market equilibrium.

Implicit in Figure 1.3.1 is an assumption that the size of operations in the credit market is limited by firms' credit constraint rather than by the household's assets. This ensures that the credit constraint binds in equilibrium. In other words, the household is always willing to lend at least $\frac{q'(1-\delta)k}{R}$ as long as $R \geq \frac{1}{\beta^h}$. Hence, the

equilibrium interest rate is equal to:

$$R^* = \frac{1}{\beta^h}. \quad (1.10)$$

The condition $\beta R^* < 1$ is therefore satisfied, and the firm does borrow up to the credit limit in steady-state equilibrium.

Consider now the behaviour of firms out of steady state. The Lagrange multiplier on the credit constraint becomes:

$$\lambda = \frac{1 - \beta R}{R} + \frac{\mu - \beta R \mu'}{R}. \quad (1.11)$$

Thus, the firm also borrows up to the maximum given that μ' is not too much greater than μ .⁴ The intuition for this result is as follows. Imagine the firm experiences an unexpected negative productivity shock today that is known to revert back up in the future. Then, the firm's profit constraint binds today more than it will bind in the future, so $\mu > \mu'$. Hence, the firm would prefer to borrow up to the maximum today and repay later.

Now imagine that the firm experiences an unexpected positive shock today that is known to revert back down in the future. In this case the firm will not necessarily want to borrow up to the maximum today because in the future, when it will have to repay the debt, its productivity will be lower and the profit constraint will bind more, $\mu' > \mu$. Depending on how much more the profit constraint will bind in the future compared to today the firm will make its decision whether to borrow up to the maximum today or not.

If the firm is going to be significantly more profit-constrained in the future, it

⁴ $\lambda > 0$ as long as $1 - \beta R > -\mu \left(1 - \beta R \frac{\mu'}{\mu}\right)$, which means that the credit constraint binds if μ and μ' are sufficiently close. I check that this holds in simulations.

may prefer to borrow less than the credit limit today because it does not want to repay a large debt in the future. In fact, it may well choose to become a creditor and lend some output to get an extra return in the future. If, on the other hand, the firm is going to be just slightly more profit-constrained in the future, then it may still decide to borrow up to the maximum because of the attractive interest rate.

In the simulations that follow in Section 1.4, I check that that the value of λ is positive, and thus the credit constraint binds.

Aggregating over firms and assuming that they do always borrow up to the maximum gives the following expression for the equilibrium firms' debt:

$$B = \frac{q'(1 - \delta)K}{R}. \quad (1.12)$$

As it will be shown later, the economy with the credit constraint only is not much different from the first-best economy. However, when combined with the profit constraint, the two induce a substantive financial accelerator effect whereby the responses of the economy to shocks become substantially different from those of the first-best one. I now turn to discussion of the profit constraint and the capital market.

1.3.2 Capital market

Capital suppliers face an increasing marginal cost $S(K)$, where K denotes their aggregate capital holdings. As in House and Shapiro (2008), I assume an isoelastic form:

$$S(K) = K^{\frac{1}{\nu}}, \quad \nu > 0. \quad (1.13)$$

On the other hand, suppliers' marginal benefit from selling a unit of capital is the difference in its price today and its price tomorrow corrected for depreciation. Hence,

when optimizing capital suppliers follow the rule of the marginal cost being equal to the marginal benefit:

$$S(K) = q - (1 - \delta) \frac{q'}{R}. \quad (1.14)$$

The parameter ν thus reflects the steady-state elasticity of capital supply. The chosen functional form of $S(k)$ allows me to pick the value for ν directly from House and Shapiro (2008) estimation results.

From the optimization rule (1.14) it follows that the price of capital varies not only with today's values of capital but also with the future values as well.⁵ As a result, two multiplier effects emerge, so-called static and dynamic multiplier effects, explained in more detail further, which allow for large fluctuations in capital given a relatively small productivity shock.

Consider now the demand for capital. The first-order condition of the firm's problem (1.5)-(1.7) with respect to capital reads:

$$\begin{aligned} -q + \lambda(1 - \delta)q' - \mu q + \beta A' F_k(k, l') + \beta(1 - \delta)q' + \beta(1 + \mu'(1 - \bar{c}))A' F_k(k, l') + \\ + \beta(1 + \mu')(1 - \delta)q' = 0. \end{aligned} \quad (1.15)$$

Using equation (1.11) to substitute in for λ gives:

$$(1 + \mu'(1 - \bar{c}))\beta A' F_k(k, l') = (1 + \mu) \left[q - \frac{(1 - \delta)q'}{R} \right]. \quad (1.16)$$

Expression (1.16) is different from the no-frictions rule, $\beta A' F_k(k, l') = q - \beta(1 - \delta)q'$, in three ways. First, the marginal benefit of having an extra unit of capital is larger than in the frictionless case, $(1 + \mu'(1 - \bar{c}))\beta A' F_k(k, l') \geq \beta A' F_k(k, l')$. Buying more

⁵Assume $\lim_{s \rightarrow \infty} \left(\frac{1-\delta}{R}\right)^s q_s = 0$ to rule out bubbles. Then, it follows from (1.14) that $q_t = \sum_{s=0}^{\infty} \left(\frac{1-\delta}{R}\right)^s S(K_{t+s})$. So the price of capital takes into account all, today and future, values of capital.

capital today increases future production which affects the firm's profit directly but also relaxes tomorrow's profit constraint. Second, the marginal cost of buying an extra unit of capital is higher because buying another unit tightens today's profit constraint, $(1 + \mu)uc \geq uc$, where uc stands for the user cost of capital. Finally, the user cost $(q - \frac{(1-\delta)q'}{R})$ reflects that capital is not only used in the production process but also as collateral. Against a unit of capital the firm can borrow $\frac{(1-\delta)q'}{R}$ units of output. Therefore, the firm demands more capital compared to the no-frictions case ($q - \frac{(1-\delta)q'}{R} < q - \beta(1 - \delta)q'$ by Assumption 1).

Since this optimality condition holds for the representative firm, aggregate demand for capital is:

$$(1 + \mu'(1 - \bar{c}))\beta A' F_K(K, L') = (1 + \mu) \left[q - \frac{(1 - \delta)q'}{R} \right]. \quad (1.17)$$

In equilibrium, the price of capital ensures that the capital demand (1.17) is equal to the capital supply (1.14).

Two special cases shed light on the importance of the profit constraint and the minimum dividend requirement. The first is when the profit constraint is always-binding and the second is when the profit constraint is never-binding.

Imagine that the value of \bar{c} is high enough so that the profit constraint binds for all sequences of productivity shocks:

$$(1 - \bar{c})(AF(k_{-1}, l) - wl) + b - Rb_{-1} - q(k - (1 - \delta)k_{-1}) = 0 \quad \forall A. \quad (1.18)$$

Using the credit constraint (1.2) to substitute in for b and solving for capital gives the relationship between the firm's capital demand, its net worth and the user cost

of capital:

$$k = \frac{1}{q - \frac{q'(1-\delta)}{R}} [(1 - \bar{c})(AF(k_{-1}, l) - wl) + q(1 - \delta)k_{-1} - Rb_{-1}]. \quad (1.19)$$

The firm's demand for capital depends negatively on the user cost of capital and positively on its net worth, which comprises the gross profit after the minimum dividend payment and the market value of installed capital less the loan repayment.

In what follows, I assume that the production function exhibits constant returns to scale. The assumption of constant returns may be relaxed, but it simplifies aggregation. Thus, if the profit constraint of the representative firm always binds, aggregate demand for capital becomes:

$$K = \frac{1}{q - \frac{q'(1-\delta)}{R}} [(1 - \bar{c})(AF(K_{-1}, L) - wL) + q(1 - \delta)K_{-1} - RB_{-1}]. \quad (1.20)$$

This case is the one considered by Kiyotaki and Moore (1997). In a model that abstracts from labour demand they show that if the economy is hit by a negative productivity shock, then the decrease of capital would be substantial due to the presence of static and dynamic multiplier effects, to which I now turn.

The static multiplier can be described in the following way. Holding the future constant, a decrease in productivity lowers the capital demand by decreasing firms' marginal product of capital. As K goes down the price of capital goes down as well, since its user cost decreases to clear the market: $K^{\frac{1}{\nu}} = q - (1 - \delta)\frac{q'}{R}$. The decrease in q results in a further decrease in the net worth and, therefore, in an even deeper decline in capital.

The dynamic multiplier effect occurs because less capital today means lower net worth tomorrow. This decreases tomorrow's capital demand and its price q' . Hence, the amount of credit that firms can get today against a unit of capital goes down,

firms can borrow less, and so they can invest less. Hence, the decline in today's capital becomes larger.

Note that the dynamic multiplier effect takes place because of the nature of the credit constraint. The fact that firms borrow against the future market value of capital allows for the feedback effect from future to present. Without the credit constraint, capital demand would not include any future values:

$$K = \frac{1}{q}[(1 - \bar{c})(AF(K_{-1}, L) - wl) + q(1 - \delta)K_{-1} + B - RB_{-1}]. \quad (1.21)$$

So the profit constraint brings the capital-net worth relationship into the model, whereas the considered form of the credit constraint creates the future-to-present feedback in asset prices.

Imagine now that the profit constraint never binds, $\mu = 0$ for all periods. The smaller is the minimum dividend, the more likely this is. If there is no minimum dividend requirement, $\bar{c} = 0$, in steady state the profit constraint would not bind, since if it did it would mean that the firm is constantly experiencing net losses and would be better off not operating at all. Aggregate capital demand is then

$$\beta A' F_K(K, L') = q - \frac{(1 - \delta)q'}{R}, \quad (1.22)$$

which is only different from the frictionless outcome due to the adjusted user cost implied by the credit constraint. In some sense, this is the "first-best" capital allocation in this economy. Firms are credit-constrained in terms of borrowing at rate R but they can borrow unlimited amount from the shareholders at a higher rate of $\frac{1}{\beta}$. The impact of a shock here is minimal because it only affects the marginal product of capital. I compare this "first-best" case with the case where the profit constraint binds in my simulations.

I now turn to describing the labour market.

1.3.3 Labour market

Consider the supply of labour. I assume that the representative household consists of a continuum of members. The members differ according to their disutilities of work, u_i , the distribution of which is known and is described by the cumulative distribution function $F(u)$.

Each of the members supplies her unit of labour inelastically as long as the wage payment is greater than u_i . Thus, aggregating over the workers I obtain the following labour supply curve:

$$L^S = F(w). \tag{1.23}$$

I assume that $F(w)$ is isoelastic and is equal to $F(w) = \bar{L}w^\epsilon$, where \bar{L} is a constant and ϵ is the Frisch elasticity of the labour supply (the elasticity of the quantity supplied to wage keeping the marginal utility of wealth constant). One could think of \bar{L} as of the size of the labour force; when ϵ is equal to zero $L^S = \bar{L}$.

Substituting in for $F(w)$ gives:

$$L^S = \bar{L}w^\epsilon. \tag{1.24}$$

The isoelastic form allows to calibrate the value of the elasticity by taking it directly from the literature.

Labour demand of an individual firm is given by the first-order condition of the firm's problem (1.5)-(1.7) with respect to labour:

$$(1 + \mu(1 - \bar{c}))(AF_l(k_{-1}, l) - w) = 0. \tag{1.25}$$

Since Lagrange multipliers can only either be positive or zero, the first term in brackets is always positive. This means that the profit constraint has no *direct* effect on the labour demand. The firm chooses its number of workers according to the standard rule that the marginal product of labour should be equal to the marginal cost:

$$AF_l(k_{-1}, l) = w. \quad (1.26)$$

Aggregating over the firms gives:

$$AF_L(K_{-1}, L) = w. \quad (1.27)$$

Instead, this simple rule implies that credit frictions affect the labour demand *indirectly* through a complementarity channel. Since capital and labour are complements in the production process, a negative productivity shock that lowers the firm's capital demand implies a decrease in the marginal product of labour. Provided that $\epsilon > 0$ this results in decreases in both employment and wages as they fall to equate supply to demand.

This concludes the description of the model. I now turn to its quantitative applications.

1.3.4 Steady-state equilibrium

In the simulations that follow, for a range of positive \bar{c} , I have been able to find only one steady-state equilibrium in which both the credit constraint and the profit constraint bind. The equilibrium is described by six equations which jointly determine steady-state values of capital, employment, debt, the price of capital, wages and the interest rate, $\{K^*, L^*, B^*, q^*, w^*, R^*\}$.

1. Equilibrium interest rate is determined by the household's discount factor:

$$R^* = \frac{1}{\beta^h}. \quad (1.28)$$

2. Firms borrow up to their credit limit:

$$B^* = \frac{q^*(1-\delta)K^*}{R^*}. \quad (1.29)$$

3. Firms invest as much as possible into capital. As a result, firms' capital demand is determined by their net worth:

$$K^* = \frac{(1-\bar{c})(AF(K^*, L^*) - w^*L^*)}{q^* \left(1 - \frac{1-\delta}{R^*}\right)}. \quad (1.30)$$

Note that firms' net worth in steady state is their revenue after wage and minimum dividend payments. If compared to the dynamic capital demand equation (1.21), the terms $q^*(1-\delta)K^*$ and R^*B^* are missing in (1.30) because in steady state the value of land is exactly offset by required debt repayment.

4. Labour demand is derived by equating the marginal product of labour to wage:

$$AF_L(K^*, L^*) = w^*. \quad (1.31)$$

5. Capital supply, which in equilibrium should be equal to capital demand:

$$K^{*\frac{1}{\nu}} = q^* \left(1 - \frac{1-\delta}{R^*}\right). \quad (1.32)$$

6. Labour supply, which in equilibrium should be equal to labour demand:

$$L^* = \bar{L}w^{*\epsilon}. \quad (1.33)$$

1.4 Quantitative applications

1.4.1 Calibration

I calibrate the model using one week as time period. The productivity shock evolves according to the following standard autoregressive process in logarithms:

$$\ln A_t = \rho \ln A_{t-1} + \xi_t, \quad \xi \sim N(0, \sigma^2). \quad (1.34)$$

Following King and Rebelo (1999) I assume quarterly values of ρ and σ to be 0.979 and 0.0072, respectively, which implies a weekly autoregressive parameter of 0.998 and a weekly standard deviation of 0.002.⁶

When calibrating the discount factors I follow Iacoviello (2005) and set $\beta^h = 0.999$ and $\beta = 0.998$ which corresponds to his quarterly values of 0.99 and 0.98, respectively. Hence, the internal rate of return for a firm $\left(\frac{1}{\beta} - 1\right)$ is twice as large as the rate of return faced by the household.

I assume a conventional capital share of output equal to 1/3. Capital depreciates at a standard rate of 10% a year, that is 0.19% a week.

I experiment with several values of \bar{c} , the minimum dividend fraction of gross profit. For the baseline calibration I choose a value which is supported by dividend

⁶Given the shock process, $E[\ln A_{t+m} | \ln A_t] = \rho^m E[\ln A_t]$. Thus, $\rho_{week} = \rho_{quarter}^{1/13}$. Also, since $Var(\ln A_t) = \frac{\sigma^2}{1-\rho^2}$ keeping variances of quarterly and weekly processes constant requires $\sigma_{week}^2 = \sigma_{quarter}^2 \frac{1-\rho_{week}^2}{1-\rho_{quarter}^2}$.

data. According to the Factset report,⁷ more than 350 companies included into S&P500 index have been paying dividends every year for the past 15 years. At the same time, dividend payments averaged from 25% of net income in 2011 to almost 55% in 2008. I set the value of \bar{c} to 30% which is a median dividend payout ratio (a ratio of dividend payment to net income) across S&P500 firms over the last 10 years.

House and Shapiro (2008) obtain estimates of investment supply elasticities in the interval between 6 and 14. Note that there is no difference between investment and capital supply elasticities in steady state. Thus, I set ν to be equal to 10, which is a midpoint of their estimates.

In the baseline calibration I normalize the size of the labour force \bar{L} to be equal to one.⁸ Chetty et al. (2011) report the intensive margin Frisch supply elasticity to be around to 0.54 and the extensive margin Frisch supply elasticity to be around 0.28. I set ϵ to be equal to 0.5, but I also experiment with its value and report the findings further in the text.

Table 3.6.1 provides a summary of the parameter values discussed above. In what follows, I discuss impulse responses of the main variables to a productivity shock.

1.4.2 Impulse responses: baseline vs "first-best"

In this section I analyze the responses of main model variables to an unexpected persistent negative productivity shock. Firms do not expect the shock to hit the economy but once it does they know that it is going to last. I take the size of the shock to be equal to one standard deviation. Figure 1.4.1 presents the shock.

Figure 1.4.2 presents the responses of variables to the shock and compares them

⁷http://www.factset.com/websitefiles/PDFs/dividend/dividend_9.28.15

⁸ I check how \bar{L} affects simulations, and confirm that, while it does change steady-state values, it has no effect on impulse responses.

Table 1.4.1: Calibrated model parameters

Description	Parameter	Value
Firm's discount factor	β	0.998
Households' discount factor	β^h	0.999
Capital share	α	1/3
Capital depreciation	δ	0.0019
Minimum dividend	\bar{c}	0.3
Capital supply elasticity	ν	10
Labour force	\bar{L}	1
Labour supply elasticity	ϵ	0.5
Autoregressive parameter	ρ	0.998
Volatility	σ	0.002

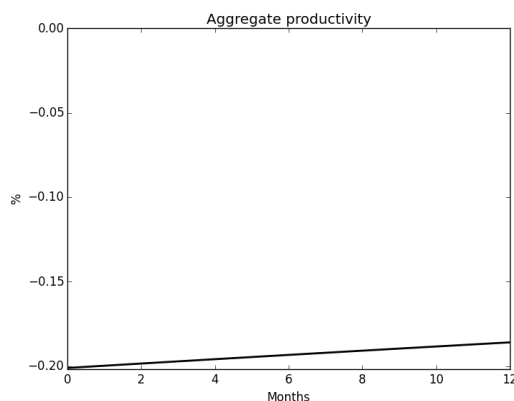


Figure 1.4.1: One standard deviation negative productivity shock.

to those under "first-best" economy.

When firms do not face the profit constraint, so the variables reach their "first-best" values, variables respond to the shock by declining by roughly the same order of magnitude as the shock. Capital, labour, capital price and wage all decline by 0.2%-0.5% from their steady-state values when productivity declines by 0.2%. Note that employment and wages decrease more one period after the shock rather than directly on impact. This is due to the nature of the production function, the fact that capital takes one period to install in particular. On impact, the marginal product of labour falls because of the decline in aggregate productivity. One period after, it

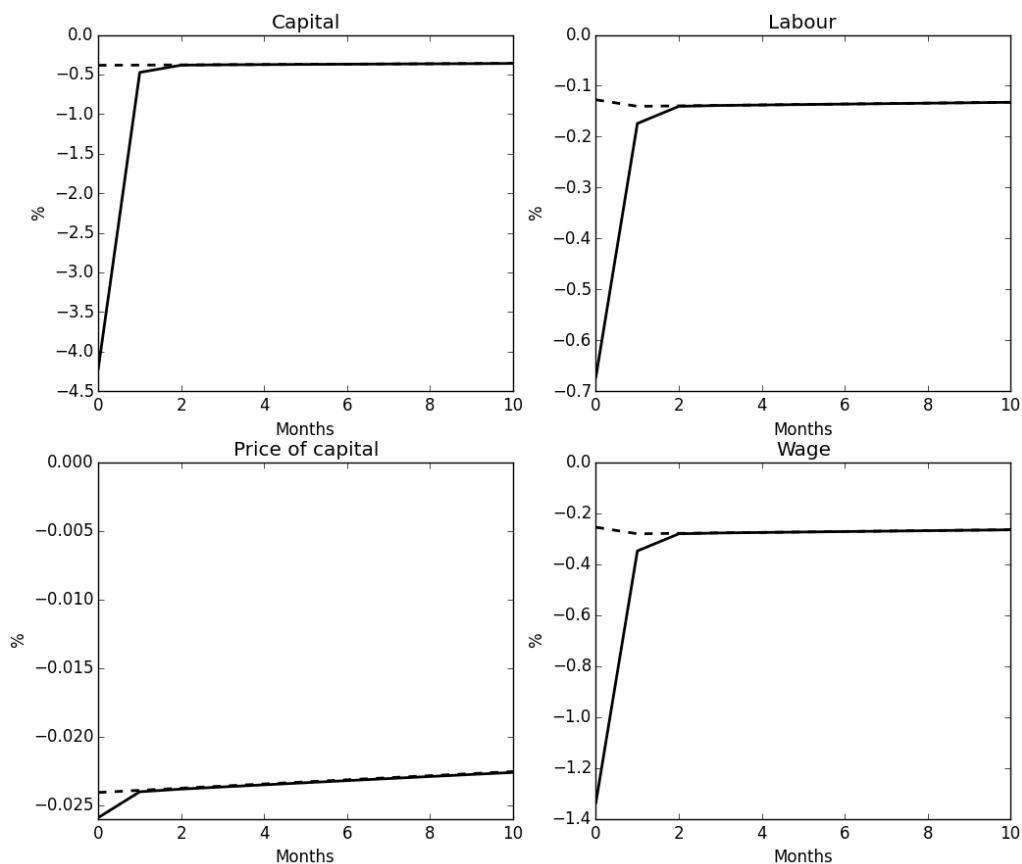


Figure 1.4.2: Model impulse responses to productivity shock in deviations from steady state. Dashed line - "first-best", solid line - credit frictions.

further declines because of the capital's response last period.

The situation is very different in the presence of credit frictions (solid lines). Due to a 0.2% productivity decline firms' capital falls by around 4%. The large decline in capital is explained by a large decrease in firms' net worth. When productivity declines not only do firms produce less, but they also have to repay a relatively large debt, inherited from yesterday. In addition to that, the price of capital declines, and this reduces their net worth even more. Perhaps surprisingly, the price of capital does not decrease by much more than it does in the "first-best" scenario. This is a result of capital supply elasticity being quite high. In the next section I experiment with different values of ν .

Wages and employment also display far larger decreases compared to the "first-best" economy. Employment falls by around 0.7%, and wages by around 1.3%. This is evidence that financial frictions have a substantial effect on the marginal product of labour. Also, this raises hope that once search and matching frictions are introduced a small negative productivity shock will lead to a significant unemployment increase.

Overall, the model generates promising impulse responses in terms of amplification. Unfortunately, however, the model generates little propagation. The effects of the shock are short-lived, after the first two months differences between the economy with credit frictions and the "first-best" analogue disappear.

One way of improving this would be adding a delay in the investment decisions. For example, Kiyotaki and Moore (1997) consider a situation when in a given period a firm may have an opportunity to invest only with a given probability. A drawback of this approach, however, is that this only influences responses to a positive, rather than a negative, productivity shock. When the economy experiences an increase in productivity, firms want to invest but, in case of Kiyotaki and Moore (1997), not all of them can. As a result there is a delay in capital's response. But if there is a negative shock, then it does not matter that they can't invest, they would not want to even if they could. So the amplification remains large but there is almost no propagation.

Bernanke et al. (1999) use a one-quarter investment delay, which improves propagation. More importantly, they also assume Calvo pricing in the goods market, which means that prices could be adjusted only with a given probability. This generates hump-shaped responses of the variables to a shock even without the financial accelerator. The responses are amplified when it is present.

Numerical simulations show that as a result of a 0.2% positive productivity shock the credit constraint stops binding. Intuitively, a positive shock increases firms' net

worth. Because they still want to invest as much as possible, they buy a lot of capital. In principle, they should also borrow a lot according to the credit constraint. However, firms know that in the next period the productivity is going to be lower, because it converges to the steady state, and they will have to repay a relatively large debt from today. Hence, borrowing up to the limit is not as attractive anymore, and firms prefer to borrow less. In this model this happens already for very small positive productivity shocks.

Now I turn to exploring what happens to impulse responses if I change the values of some of the parameters. I start with the minimum dividend share, then capital supply elasticity, and finally labour supply elasticity.

1.4.3 Minimum dividend requirement

Consider what happens if there is no minimum dividend requirement, or $\bar{c} = 0$.

Consider the implications for the model's steady state. If $\bar{c} = 0$, then the profit constraint requires the firm's flow profit to be greater or equal to 0. If this is not true in steady state, that is the firm makes losses every period, the firm would be better off not operating at all. Therefore, in steady state the profit constraint does not bind.

Now consider what happens if the economy is hit by a negative productivity shock while $\bar{c} = 0$. There are two possible scenarios. The first is that the profit constraint still does not bind, firms have enough net worth to afford the "first-best" level of capital. In this case impulse responses would look like the "first-best" ones in Figure 1.4.2. The second possible scenario is that the shock is big enough for firms not to be able to afford the "first-best" level of capital. In this case the profit constraint binds, the firm invests all of its net worth into capital.

It appears that when $\bar{c} = 0$ the profit constraint does bind for an unexpected

0.2% negative shock considered previously. The impulse responses are presented in Figure 1.4.3, along with the impulse responses under the baseline calibration.

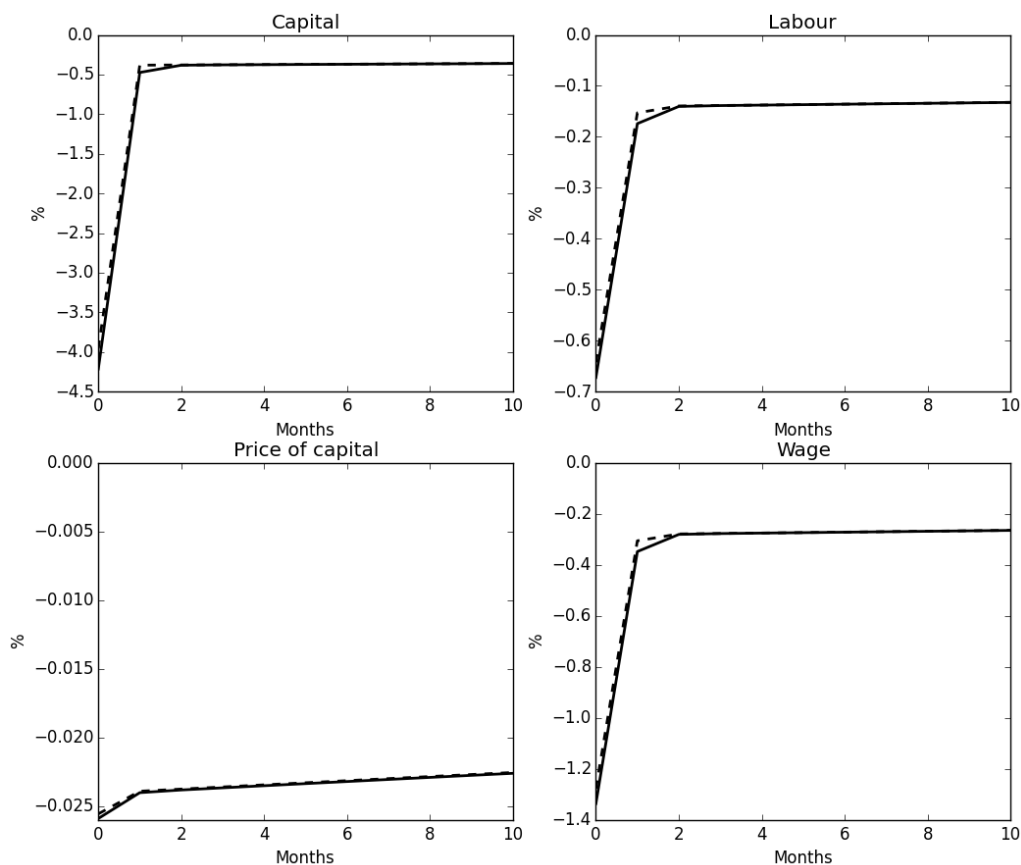


Figure 1.4.3: Model impulse responses to productivity shock in deviations from steady state. Dashed line - $\bar{c} = 0$, solid line - baseline, $\bar{c} = 0.3$.

There is almost no difference between the two cases. This is because firms' net worth is affected in a similar way in both cases. When the shock hits, in both cases firms have to repay a relatively large debt inherited from yesterday. This makes the profit constraint bind when $\bar{c} = 0$. In case of $\bar{c} = 0.3$ this makes the already-binding profit constraint bind even more. Hence, the net worths decline by similar orders of magnitude when $\bar{c} = 0$ and when $\bar{c} = 0.3$, and this leads to large decreases in capital, and other variables.

In the period following the shock there is a slight difference depending on the

value of \bar{c} . The economy with $\bar{c} = 0$ reverts to the "first-best" economy faster than the one with $\bar{c} = 0.3$. When the firm that has no minimum dividend requirement, it can afford the "first-best" level of capital sooner.

Therefore, this is evidence that the assumption about minimum dividend requirement is not as strong as one might think. One great advantage of the requirement is that the profit constraint becomes almost always-binding, at least in steady state and around it. And this considerably simplifies solving the model in the presence of aggregate uncertainty, which I consider in the following chapters.

1.4.4 Capital supply elasticity

In this section I analyze how the model reacts to changes in capital supply elasticity. Consider first the "first-best" economy. If the capital supply elasticity is high, this means that the supply curve is quite flat, and the quantity supplied responds a lot to a change in capital price. Then, when a negative shock occurs, demand for capital decreases, and this results in a large decrease in capital and a fairly small decrease in its price. This is what one could observe from Figure 1.4.4.

In the economy with higher ν (dotted line) capital responds more to the shock than in the economy with lower ν (dotted line). I choose values for high and low ν based on upper and lower bounds of the estimates in House and Shapiro (2008).

Such behaviour has some interesting implications for the economy with credit frictions. The impulse responses are presented in Figure 1.4.5.

On impact, capital falls more when its supply elasticity is low compared to when it is high, and that is the opposite to what we observe in the "first-best" economy. After the first couple of months it reverts back to the "first-best" level as it does in the baseline case.

The difference on impact could be explained by looking at what happens to firms'

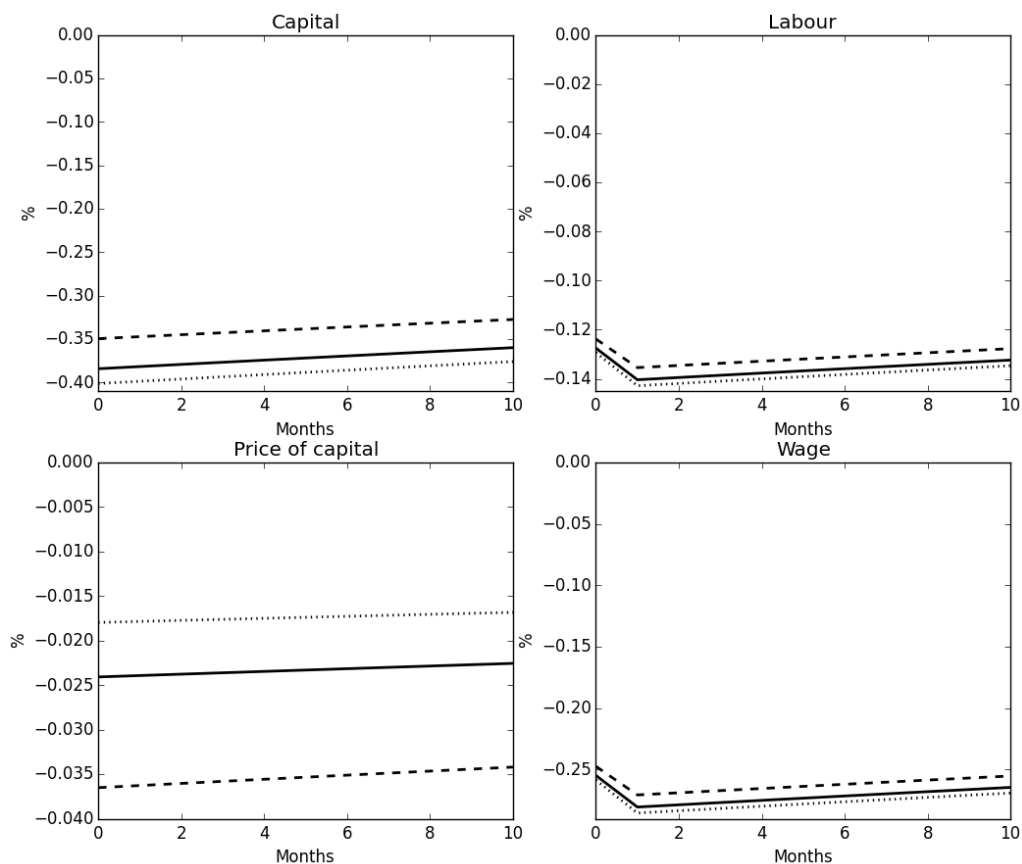


Figure 1.4.4: "First-best" economy impulse responses to productivity shock in deviations from steady state. Dashed line - $\nu = 6$, dotted line - $\nu = 14$, solid line - baseline, $\nu = 10$.

net worth. On impact, the net worth falls significantly because firms have to repay a large steady-state debt given today's lower productivity. As a result capital falls as well, and so does its price, and precisely because of the lower supply elasticity the price falls more when $\nu = 6$ compared to when $\nu = 14$. Moreover, static and dynamic multipliers also contribute more to the price decrease when $\nu = 6$ rather than when $\nu = 14$. Once firms start anticipating productivity shocks correctly, this effect disappears.

Thus, despite the fact that usually, if supply elasticity is low, prices respond to shocks more than they do if supply elasticity is high and quantities respond less, in the presence of credit frictions on impact the opposite happens. It is because of the

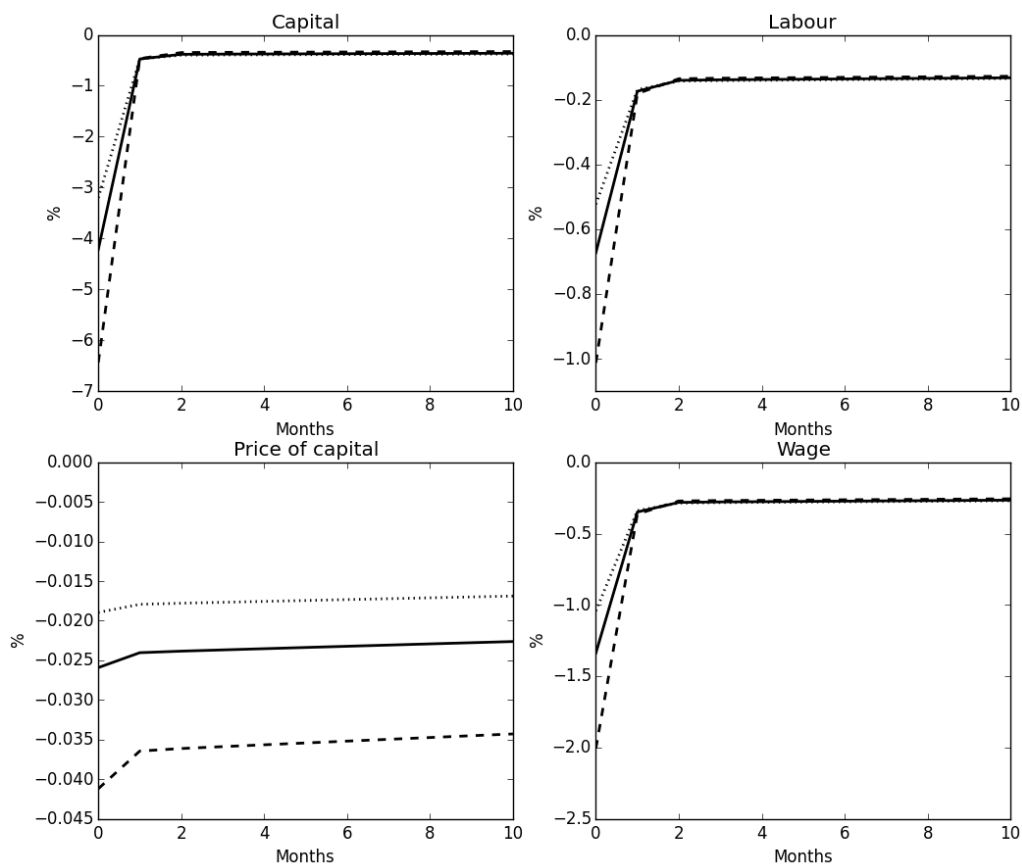


Figure 1.4.5: Model impulse responses to productivity shock in deviations from steady state. Dashed line - $\nu = 6$, dotted line - $\nu = 14$, solid line - baseline, $\nu = 10$.

fact that capital price declines more when capital supply elasticity is lower, firms' net worth goes down more leading to a greater decline in capital.

Employment and wages follow capital's pattern because of the complementarity of factors of production. They also go down more when capital supply elasticity is lower.

1.4.5 Labour supply elasticity

In the previous section I discussed the consequences of changing capital supply elasticity in the model. In this section I turn to changing the labour supply elasticity. It turns out that results are again different in the model with credit frictions compared

to the "first-best" economy model.

Consider first how the "first-best" economy reacts to changing the elasticity. In Figure 1.4.6 dashed lines are impulse responses if $\epsilon = 0$, so the labour supply is perfectly inelastic, dotted lines present impulse responses if $\epsilon = 1$, so the labour supply is a 45 degree line.

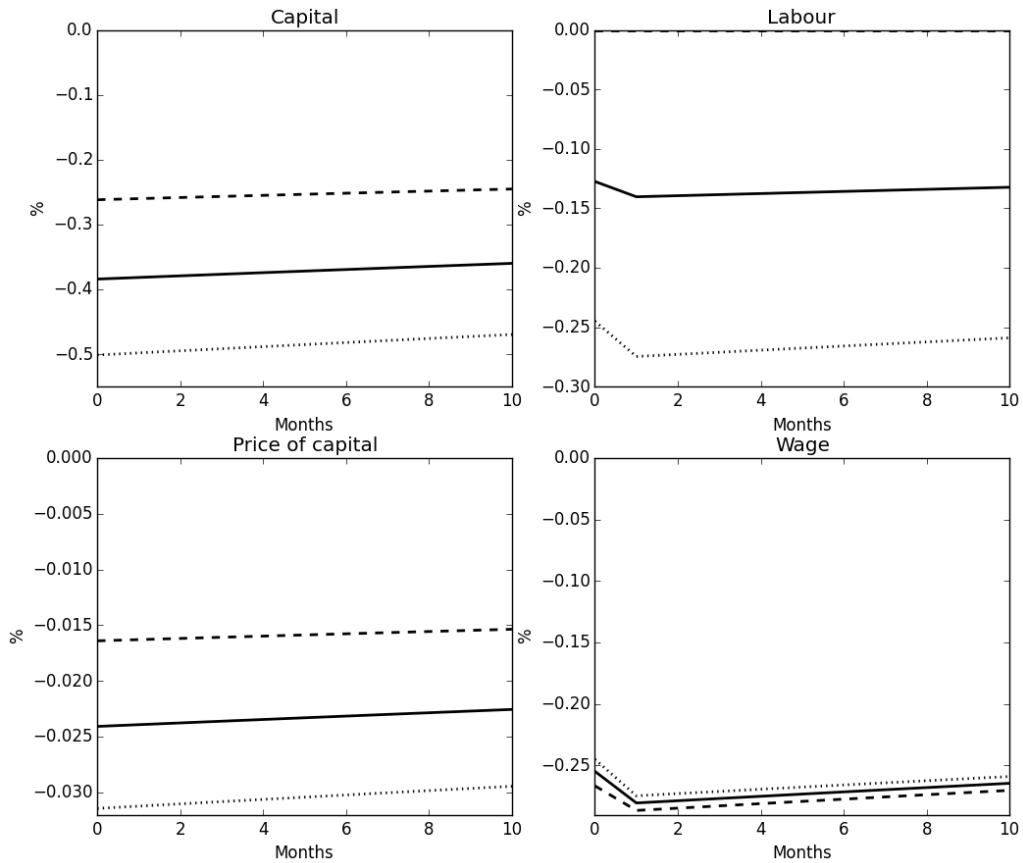


Figure 1.4.6: "First-best" economy impulse responses to productivity shock in deviations from steady state. Dashed line - $\epsilon = 0$, dotted line - $\epsilon = 1$, solid line - baseline, $\epsilon = 0.5$.

In case of perfectly inelastic labour supply, employment stays at its steady-state value. Wages decline by around 0.27% on impact and then slightly more because of the nature of the production technology. As one would expect, when labour supply elasticity is higher, $\epsilon = 1$, in equilibrium employment falls from its steady-state value (by almost 0.25%), and wages fall as well but by less compared to when $\epsilon = 0$.

As in case with capital supply elasticities discussed in the previous section, the implications of different labour supply elasticities are different under credit frictions. In Figure 1.4.7 wages and employment both decline more when when labour supply elasticity is higher.

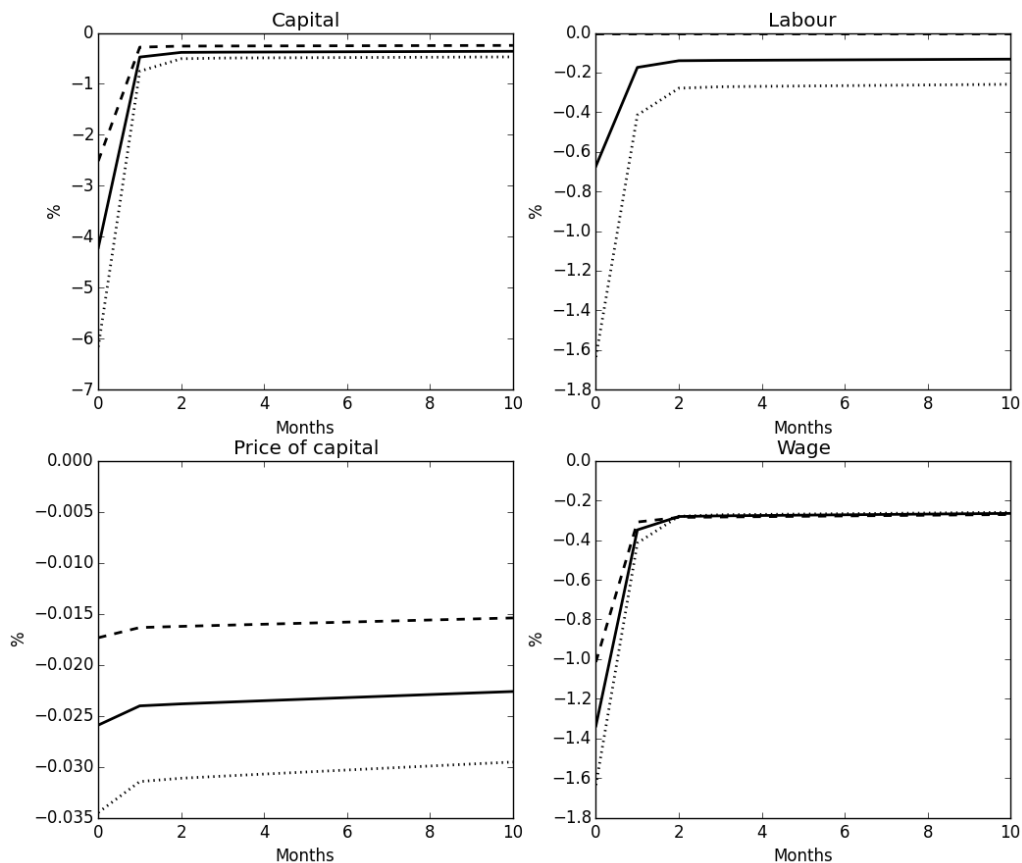


Figure 1.4.7: Model impulse responses to productivity shock in deviations from steady state. Dashed line - $\epsilon = 0$, dotted line - $\epsilon = 1$, solid line - baseline, $\epsilon = 0.5$.

An explanation for this result lies in the behaviour of the marginal product of labour, specifically in how it is affected by capital. Suppose the labour supply elasticity is high. Then, as a result of an unexpected productivity shock employment falls by relatively more compared to the situation with a low labour supply elasticity. Now notice that as employment falls, so does firms' net worth, even despite

the fact that wages fall as well.⁹ As the net worth declines, so does capital. Due to the financial accelerator the fall in capital is substantial, more importantly it is far greater than the decrease in labour. Therefore, the marginal product of labour declines as a result of a negative shock. Thus, the more elastic labour supply is, the more employment falls and the more capital falls. In turn, the more capital falls, the more marginal product of labour falls and, hence, wages.

Again, credit frictions change how wages behave straight after a productivity shock depending on how elastic labour supply is.

1.5 Conclusion

In this paper, I have developed a partial equilibrium model with frictionless labour market where firms face credit frictions in the form of collateral constraints as in Kiyotaki and Moore (1997). I show that the collateral constraint together with the profit constraint, which prevents a firm from borrowing from its shareholders, creates a link between the firm's capital demand and its net worth. This link leads to a large capital decline in response to a negative productivity shock. Through complementarity of the factors of production the decrease in capital affects employment and wages. As a result of one standard deviation negative productivity shock employment falls by around 0.65% and wages fall by around 1.3% as opposed to 0.11% and 0.25%, respectively, in the first-best economy.

The results from the model indicate that an introduction of search and matching frictions into it could bring some interesting results. First of all, amplification of a shock could be significantly increased, which contributes to the solution of Shimer (2005) puzzle. Second, it would be interesting to study the interaction of the two

⁹ When wages are equal to the marginal product of labour, the gross profit is $Ak_1^\alpha l^{1-\alpha} - wl = \alpha Ak_1^\alpha l^{1-\alpha}$, which is increasing in l .

frictions, particularly because of the potential spillover effect from search frictions to credit ones. The effect is likely to happen through the wage bill, and that will surely influence the firm's net worth. Finally, having heterogenous firms would allow to take the model to firm-level data and confront its predictions.

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Appendix

A Simulations

Basic model

The model is described by the following system of equations:

- credit constraint

$$B = \frac{q'(1-\delta)K}{R} \quad (1.35)$$

- capital demand

$$K = \frac{1}{q - \frac{q'(1-\delta)}{R}} [(1-\bar{c})(AF(K_{-1}, L) - wL) + q(1-\delta)K_{-1} - RB_{-1}] \quad (1.36)$$

- capital supply

$$K^{\frac{1}{\nu}} = q - \frac{q'(1-\delta)}{R} \quad (1.37)$$

- labour demand

$$AF_L(K_{-1}, L) = w \quad (1.38)$$

- labour supply

$$L = \bar{L}w^\epsilon. \quad (1.39)$$

In order to obtain the impulse response functions in the main text I proceed as follows.

1. Solve for the model's steady state and obtain the steady-state values of capital K^* , labour L^* , debt B^* and capital price q^* .
2. Set $K_{-1} = K^*$ and $B_{-1} = B^*$.

3. Guess q_0 which denotes the price of capital in the period when the shock occurs.
4. Use the guessed value of q_0 , q_0^{guess} , together with the following equations to solve for K_0 , L_0 and w_0 .

$$K_0 = \frac{1}{K_0^{\frac{1}{\nu}}} [(1 - \bar{c})(AF(K^*, L_0) - w_0 L_0) + q_0^{guess}(1 - \delta)K^* - RB^*] \quad (1.40)$$

$$AF_L(K^*, L_0) = w_0 \quad (1.41)$$

$$L_0 = \bar{L}w_0^\epsilon \quad (1.42)$$

5. Use the following equations to solve for K_i , L_i and w_i where $i \in [1, T]$.

$$K_i = \frac{1}{K_i^{\frac{1}{\nu}}} [(1 - \bar{c})(AF(K_{i-1}, L_i) - w_i L_i)] \quad (1.43)$$

$$AF_L(K_{i-1}, L_i) = w_i \quad (1.44)$$

$$L_i = \bar{L}w_i^\epsilon \quad (1.45)$$

Note that $q_i(1 - \delta)K_{i-1} = RB_{i-1}$ for $i \in [1, T]$ because the path of all shocks (except for the initial one) is known to all agents in the economy.

6. Solve for the price of capital backwards. In the aftermath of the shock the economy eventually reverts back to the steady state.¹⁰ By imposing $q_T = q^*$ one could use the following expression to solve for q_i for $i \in [0, T - 1]$.

$$q_i = K_i^{\frac{1}{\nu}} + \frac{(1 - \delta)q_{i+1}}{R} \quad (1.46)$$

7. Use the solution from step 6 to update q_0^{guess} .

¹⁰In my simulation I use $T = 3000$. I also check that the shock goes back to its steady-state value of 1 in that time frame.

8. Repeat steps 4-7 until convergence, that is until $|q_0^{guess} - q_0^{fromstep6}| < \zeta$.

First-best economy

The 'first-best' economy differs from the economy with credit constraints because of the way how capital demand is determined. In the first-best case the capital demand equation is

$$\beta AF_K(K, L') = q - \frac{q'(1 - \delta)}{R}. \quad (1.47)$$

In this case I solve for K , L and w by using equations (1.35), (1.37)-(1.39) and (1.47). The solution for the price of capital q is then obtained as described in step 6 in the previous section.

Economy without minimum dividend requirement (Section 1.4.3)

To solve for the impulse responses of the economy where firms do not face the minimum dividend requirement, that is $\bar{c} = 0$, I follow these steps:

1. Solve numerically for K , L , w and q as if the profit constraint does not bind or, in other words, obtain their first-best solutions.
2. Check the value of the flow profit in each period given solutions from step 1.
3. In periods when the flow profit is negative impose the binding profit constraint and resolve the model using the strategy described above.
4. Check that the solution is correct by verifying that in periods where the basic model solution is adopted the Lagrange multipliers associated with the profit constraint μ are positive, whereas in periods when the first-best solution is adopted the flow profit is not negative. If any of the checks fail, apply the other solution (basic model or first-best) and recheck.

Chapter 2. Interaction between credit and search frictions

I introduce a model in which a representative firm faces two types of frictions. On one hand, when hiring workers it faces standard search and matching frictions. On the other hand, it faces credit frictions in the form of collateral constraints which determine its ability to borrow and invest. I study the interaction of the two frictions. Credit frictions affect labour demand through complementarity of capital and labour as factors of production: due to the financial accelerator, a negative shock induces a much greater rise in unemployment than in the model with search frictions only. Search frictions influence capital demand through wages: When wages are only partially flexible, the decline in firms' net worth is larger, and the resulting fall in capital is larger as well. I also find that the response of wages to wage flexibility is non-monotonic in the presence of credit frictions. There exists a scenario where more flexible wages fall less as a result of a shock compared to more rigid ones. This is in contrast to the model with search frictions only where more flexible wages always fall more in response to a shock. This could potentially explain why we see wages fall little in data.

2.1 Introduction

The financial crises of 2008 was characterized by disruptions in credit markets and sharp and prolonged rises in unemployment. It is natural to postulate a link between the two markets which may potentially shed light on yet unexplained macroeconomic fluctuations in labour markets. It was shown in Chapter 1 that credit frictions, by affecting firms' capital demand, also have an influence on firms' employment decisions. That is because capital and labour are complementary in the production process. This Chapter continues investigating the link between the two markets, but now I consider not only the effect of credit frictions on labour market outcomes, but also a potential effect of search frictions on capital market outcomes.

A worsened financial position of a firm in the presence of credit frictions is characterized by a reduction in its net worth. The depressed net worth reduces the firm's ability to invest, and the firm's capital demand decreases. Note that the wage bill has a direct effect on the firm's net worth. With search and matching frictions in the labour market, wages are no longer determined by the labour market equilibrium, and therefore, play an important role in forming the firm's net worth.

It is natural to imagine that the size of the search frictions effect on capital demand depends on wage rigidities. If wages are rather rigid, they should fall less as a result of a negative technology shock leading to a larger decrease in capital demand which can then further depress labour demand through complementarity of factors of productions.

To address the question of interaction between frictional capital and labour markets I consider a model in which a representative firm faces two types of frictions. Firstly, it faces credit frictions in the form of collateral constraints, as in Kiyotaki and Moore (1997). Secondly, it faces standard Mortensen and Pissarides (1994) search and matching frictions in the labour market. I consider how the model compares to

one with credit frictions only, and also to one with search frictions only. I find that under both frictions responses of variables, including capital and employment, to a productivity shock are much larger than the ones under only one friction. This indicates that there indeed are effects going from credit frictions to the labour market and from search frictions to the capital market.

I then proceed by examining how wage flexibility affects the model outcomes. I find that more flexible wages dampen the responses of variables to the shock, as expected. The more flexible wages are, the less firms' net worth falls as a result of a negative shock. Thus, capital demand also declines less when wages are flexible. Through complementarity of capital and labour, firm's labour demand falls less as well.

Interestingly, I find that the response of wages to different wage flexibilities is non-monotonic. This is in contrast to the model with search frictions only where more flexible wages always fall more as a result of a negative shock leading to a smaller decline in employment. This is not always true under search and credit frictions where more flexible wages sometimes fall less as a result of the shock. The reason is that under search and credit frictions, as net worth falls considerably less under flexible wages, capital and employment also decline significantly less, and so does the marginal product of labour. Because it is natural for more flexible wages to depend more on the marginal product of labour they also decline less. This result points to a possible explanation of why in times of financial distress wages are observed to fall little in data.

The rest of the paper is organized as follows. Section 2.2 describes the related literature. The model is described in Section 2.3. In Section 2.4 I discuss the calibration strategy and quantitative applications of the model. Section 2.5 concludes.

2.2 Related literature

There is a vast literature that considers how credit frictions affect macroeconomic fluctuations, including fluctuations in the labour market. Some of it I have already discussed in Chapter 1. Here, I would like to concentrate on papers which consider frictional labour markets. This research could be divided into two strands which differ according to their strategies of modelling capital markets. In the first strand, which this paper is a part of, capital markets are modeled explicitly. Firms use both capital and labour to produce output. They face credit frictions when they invest, and they face search frictions when they hire workers. The second strand of literature assumes that firms produce output using only labour, and they need financing from frictional credit markets in order to post vacancies.

Garín (2015), Zanetti (2015) and Mumtaz and Zanetti (2016) belongs to the first strand of literature. Garín (2015) expands Jermann and Quadrini (2012) model with debt and equity financing. He includes asset prices feedback in credit constraints and search and matching frictions in the labour market. In his model, as in Jermann and Quadrini (2012), in the event of a negative shock firms decide whether to issue equity, which is costly, or to fire workers. One of the difference between my research and his is that in my model firms are not allowed to issue equity. This results in a link between capital demand and net worth which results in large responses of capital to small productivity shocks. Another difference is how wages are determined. In Garín (2015) wages are determined by Nash bargaining but when firms make capital and labour decisions they take wages as given. In other words, firms abstract from the fact that wages of existing workers change when an additional worker is hired due to the diminishing marginal product of labour. This effect is present in the model that follows.

Zanetti (2015) also extends the model in Jermann and Quadrini (2012) by including search and matching frictions. However, as opposed to this work, he models frictions in the labour market by following the approach of Blanchard and Galí (2010) in which vacancies get filled immediately by paying a hiring cost. Thus, the surplus of a match to a firm is just equal to the hiring cost. Wages are then determined by Nash bargaining. The author does not consider a link between net worth and capital demand, and thus does not study the influence of the wage bill on net worth, which is one of the goals of this paper.

Mumtaz and Zanetti (2016) combine approaches in Bernanke et al. (1999) and Blanchard and Galí (2010) to develop a model of credit and labour market frictions. On one hand, as in Bernanke et al. (1999), they use the asymmetry of information between creditors and borrowers, and a related costly state verification problem, to model credit frictions. On the other hand, they follow the approach in Blanchard and Galí (2010) to model labour market frictions. As in this paper, the authors do consider the effect of labour market frictions on capital demand. However, they find that labour market frictions affect capital demand through depressed hiring only, whereas in this paper both depressed hiring and changes in the wage bill influence capital demand. Moreover, as indicated by experiments with wage flexibility presented in this Chapter, the importance of the latter channel cannot be ignored.

The second strand of literature, in which capital markets are not explicitly modeled, includes papers by Monacelli et al. (2011), Petrosky-Nadeau and Wasmer (2013) and Petrosky-Nadeau (2014).

Monacelli et al. (2011) consider a standard search and matching model in which firms issue debt under limited enforcement. Lenders are assumed to be able to recover only a part of the debt in the event of default. When a firm is created it chooses debt this period but produces and bargains wages only next period. Under this

assumption, the authors argue that the firm strategically chooses to borrow up to the limit in order to bargain lower wages with its worker. Therefore, in times of financial distress, modeled as negative credit shocks rather than negative productivity shocks which I consider in this paper, lower debt puts firms in less favourable bargaining position in wage negotiations, and this results in depressed hiring.

The main difference between Monacelli et al. (2011) and my work is that I consider a different transmission channel of credit frictions into the labour market, which is through complementarity of capital and labour, and I also explore how the wage bill affects firms' ability to invest.

Petrosky-Nadeau and Wasmer (2013) and Petrosky-Nadeau (2014) both consider setups where firms need to borrow from frictional credit markets in order to post vacancies. Petrosky-Nadeau and Wasmer (2013) present a model of an economy with double search frictions. In their model, projects get matched to bankers in order to form a firm, then firms are matched to workers in order to produce output. The authors find that as a result of credit frictions firms face increased costs of hiring which increase the elasticity of labour market tightness to productivity shocks. Petrosky-Nadeau (2014) considers an economy in which firms need external financing to post vacancies but lending relationships are subject to a problem of costly state verification. Moreover, the author considers the terms of financial contracts which are time-varying and counter-cyclical. He finds that credit frictions increase firms' hirings costs, and more so in a recession. This improves the responses of labour market tightness to productivity shocks compared to the standard search and matching model.

My work is different from both of the papers described above because modelling capital demand explicitly allows to study the interaction between search and credit frictions, the effect of search frictions on capital demand through wages in particular.

I now turn to describing the model with search friction in the labour market and credit frictions in the capital market.

2.3 Model

The model in this section extends that in Chapter 1 as follows. First, workers face search and matching frictions in the labour market, and second, firms face uncertainty over future aggregate productivity shocks.¹ Capital side of the economy mimics Chapter 1. For completeness, I present it as before but draw attention to differences.

Consider a discrete-time infinite horizon economy which is populated by a mass of firms, normalized to 1, a representative household, and capital suppliers. Firms are owned by the household which consists of a mass of workers equal to the labour force, \bar{L} .

A typical firm produces output y , taken as the numeraire, using capital k_{-1} , installed last period, and workers l :²

$$y = AF(k_{-1}, l), F_k > 0, F_{kk} < 0, F_l > 0, F_{ll} < 0, F_{kl} > 0. \quad (2.1)$$

In equation (2.1) A represents a stochastic aggregate productivity level, the current value of which is revealed to firms and the household at the beginning of a period.

There are no idiosyncratic shocks so all firms are identical in their decisions.

The firm buys capital on a competitive capital market at price q . Capital is

¹In Chapter 1 I abstracted from aggregate uncertainty because I wanted to investigate how minimum dividend requirement affects the model results. As it was discussed in Chapter 1, without this assumption the profit constraint does not bind in steady state but may bind around it. This introduces occasionally binding constraints into the mode which are much easier to deal with in deterministic environment.

²Hereafter, I denote last period values with a subscript, $_{-1}$, future values with a prime, $'$, derivatives with respect to a particular variable or variables with their corresponding subscripts.

assumed to depreciate every period at rate δ_k .³

In contrast to Chapter 1, in order to hire workers the firm must post vacancies v at cost c per vacancy. I assume that the labour market is subject to Mortensen and Pissarides (1994) search and matching frictions. Consequently, it takes time for a firm to fill a vacancy, and similarly, it takes time for an unemployed worker to find a job. If, in total, there are U unemployed workers in the market and V vacancies are posted, then the number of job-worker matches is given by a matching function $M = M(U, V)$. As is conventional, the matching function is assumed to exhibit constant returns to scale. An unemployed worker thus faces a job-finding probability $f(\theta) = M/U = M(1, V/U)$, and the firm faces a vacancy-filling probability $\varphi(\theta) = M/V = M(U/V, 1)$, where $\theta = \frac{V}{U}$ is a ratio of vacancies to unemployment, also known as labour market tightness.

The firm pays wages $w(k_{-1}, l, A)$ to its workers; these will be discussed in detail below. Workers separate from jobs at exogenous rate δ_l at the end of each period.

As in Chapter 1, credit frictions affect the firm via two constraints, à la Kiyotaki and Moore (1997). Firstly, the firm faces a credit constraint. Each period the firm may sign a one-period debt contract with the household which allows the firm to borrow b units of output at a gross interest rate R . Because ex post the firm may withdraw itself from production, and use this as a credible threat to renegotiate a smaller repayment of the loan with its creditors, ex ante creditors impose the following constraint:

$$Rb \leq Eq'(1 - \delta_k)k. \quad (2.2)$$

The constraint says that the debt repayment value should be smaller or equal to expected market value of the firm's capital corrected for depreciation. As in Iacoviello

³As the reader will see further, I assume for convenience that the capital depreciates after the next period's production is complete ($y = AF(k_{-1}, l)$) rather than at the beginning of next period ($y = AF((1 - \delta)k_{-1}, l)$).

(2005), the constraint (2.2) generalizes the credit constraint in Kiyotaki and Moore (1997) to a stochastic setup.

Secondly, the firm faces a profit constraint. Consider the firm's gross profit:

$$AF(k_{-1}, l) - w(k_{-1}, l, A)l - cv. \quad (2.3)$$

In contrast to Chapter 1, it now also takes into account non-zero vacancy costs. The firm has to pay a fixed share \bar{c} of its gross profit to the household as a minimum dividend. Thus, because the firm's only way of borrowing is by signing debt contracts, described above, it faces the following constraint:

$$(1 - \bar{c})(AF(k_{-1}, l) - w(k_{-1}, l, A)l - cv) + b - Rb_{-1} - q(k - (1 - \delta_k)k_{-1}) \geq 0. \quad (2.4)$$

The constraint says that at the end of each period the firm's flow profit net of minimum dividends must be either positive or zero. In other words, the constraint (2.4) restricts how exactly the firm may borrow from the household, and that is only through a debt contract. If the flow profit turns out to be positive, the firm distributes it as extra dividends.

The two constraints (2.2) and (2.4) are crucial for the financial accelerator effect to take place. The credit constraint restricts how much the firm can borrow from the household through debt contracts. Additionally, it introduces a powerful asset price effect because the amount that the firm can borrow depends on the expected capital price. So, if the expected price of capital decreases, the firm is able to borrow less. In turn, the profit constraint ties capital demand to the firm's net worth. If net worth declines, then capital demand declines as well. Moreover, with capital price variability, the price of capital also decreases, which leads to an even deeper decrease in the firm's net worth, and its capital demand. Therefore, a small

productivity shock is able to generate a substantial response of capital demand.

The firm's objective is to maximize the expected present discounted value of its profits by choosing capital, debt and the number of vacancies to post:

$$\begin{aligned} \Pi(k_{-1}, b_{-1}, l_{-1}, A) = \max_{k, b, v} \left\{ AF(k_{-1}, l) - w(k_{-1}, l, A)l - cv + b - Rb_{-1} - q(k - (1 - \delta_k)k_{-1}) + \right. \\ \left. + \beta E\Pi(k, b, l, A' | A) \right\}, \quad (2.5) \end{aligned}$$

subject to the credit constraint

$$Rb \leq Eq'(1 - \delta_k)k, \quad (2.6)$$

the profit constraint

$$(1 - \bar{c})(AF(k_{-1}, l) - w(k_{-1}, l, A)l - cv) + b - Rb_{-1} - q(k - (1 - \delta_k)k_{-1}) \geq 0, \quad (2.7)$$

and also the fact that, by the law of large numbers, the number of hires should be equal to the expected number of filled vacancies:

$$l - (1 - \delta_l)l_{-1} = \varphi(\theta)v. \quad (2.8)$$

Using equation (2.8) to substitute in for v gives the firm's value function, which I use in the remainder of the paper:

$$\begin{aligned} \Pi(k_{-1}, b_{-1}, l_{-1}, A) = \max_{k, b, l} \left\{ AF(k_{-1}, l) - w(k_{-1}, l, A)l - \frac{c}{\varphi(\theta)}(l - (1 - \delta_l)l_{-1}) + b - Rb_{-1} - \right. \\ \left. - q(k - (1 - \delta_k)k_{-1}) + \beta E\Pi(k, b, l, A' | A) \right\}. \quad (2.9) \end{aligned}$$

In the next section I turn to discussing the timing of the model in more detail,

as well as the laws of motion of aggregate employment and unemployment.

2.3.1 Timing

The firm inherits from the previous period capital k_{-1} and debt Rb_{-1} , which has to be paid back to the household this period, from the previous period. Workers arrive to the period in one of the two states: employed or unemployed. Denote by U the pool of unemployed workers at the beginning of the period.

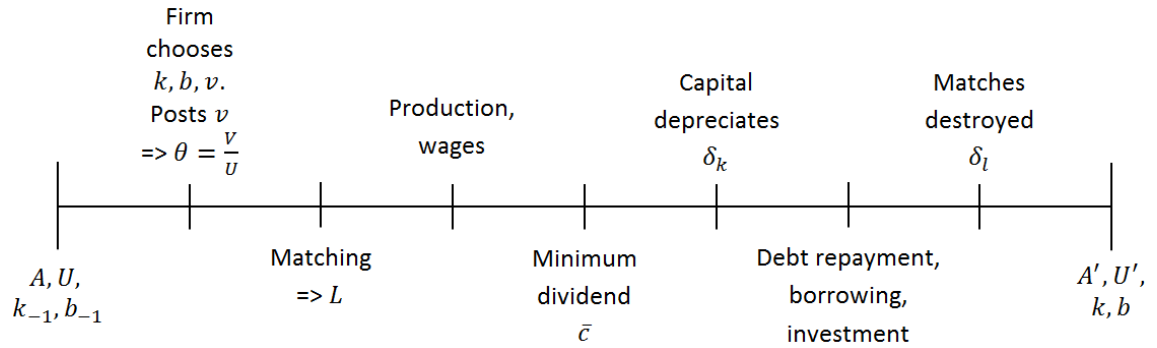


Figure 2.3.1: Timing.

At the beginning of the period the firm observes a realization of the productivity shock. Based on the realization, it makes its decision on how much capital k to buy, how much funds b to borrow, and how many vacancies v to post. The firm then posts vacancies according to chosen v which implies aggregate vacancies which, together with aggregate unemployment U , determine labour market tightness, θ , and hence the job-finding, $f(\theta)$, and vacancy-filling, $\varphi(\theta)$, probabilities. Workers are then matched to firms, and that determines the aggregate employment level in the economy L .

When the matching process is complete, the firm produces, pays wages and pays the minimum dividend to the household. Capital k_{-1} then depreciates, the firm repays its debt, borrows again by signing a new debt contract with the household

and invests into capital. It also pays extra dividends to the household if it has any funds left. At the end of the period, exogenous separations occur, and the economy moves into the next period.

The timing implies that the stock of unemployed workers observed at the beginning of each period evolves according to the following rule:

$$\Delta U' = \delta_l L - f(\theta)U. \quad (2.10)$$

So the change in unemployment depends positively on the number of workers exogenously separated from their employers, and negatively on the number of unemployed workers who have found jobs. Also, given the adding-up constraint $U' + (1 - \delta_l)L = \bar{L}$, the law of motion for aggregate employment is characterized by:

$$\Delta L = f(\theta)U - \delta_l L_{-1}. \quad (2.11)$$

The next two sections describe credit and capital markets and the firm's optimal levels of debt and capital. The markets function very similarly to the ones in Chapter 1, so sections below are summaries of what was described in more detail in Chapter 1.

2.3.2 Credit market

Consider a credit market equilibrium in which firms borrow. The equilibrium interest rate R^* is determined by intersection of supply and demand curves.

The supply of debt is provided by the household. The household has a discount factor β^h . Thus, it is willing to lend as long as the interest rate is greater or equal to the inverse of β^h . I assume that the household is more patient than firms.

Assumption 1. $\beta < \beta^h$.

This ensures that in and around steady-state equilibrium firms borrow and the household lends.

The firm's demand for debt is dictated by the first-order condition of the problem (2.9) with respect to b :

$$1 - \beta R - \lambda R + \mu - E\mu'R = 0, \quad (2.12)$$

where λ denotes the Lagrange multiplier on the credit constraint and μ denotes the Lagrange multiplier on the profit constraint. Rewriting equation (2.12) in terms of λ gives:

$$\lambda = \frac{1 - \beta R}{R} + \frac{\mu - \beta RE\mu'}{R}. \quad (2.13)$$

λ is greater than 0, and the credit constraint binds, meaning that the firm borrows up to the maximum, if two conditions are satisfied.

The first one is $\beta R < 1$. Imagine that the firm does borrow up to the maximum, then the competitive credit market ensures that the representative household makes zero profit from loans. It is implicitly assumed that the household does have at least $\frac{Eq'(1-\delta_k)}{R}$ of free funds that it is willing to lend. This ensures that the credit constraint binds in equilibrium. Thus, the equilibrium price of debt is

$$R^* = \frac{1}{\beta^h}. \quad (2.14)$$

Therefore, by assumption 1, $\beta R^* < 1$, and the first condition is satisfied.

The second condition for the credit constraint to bind is that $E\mu'$ should not be much greater than μ .⁴ The intuition for this is as follows. If the firm expects its

⁴ $\lambda > 0$ as long as $1 - \beta R > -\mu \left(1 - \beta R \frac{E\mu'}{\mu}\right)$.

future net profit to be greater than the current one, $E\mu' < \mu$, then it would borrow up to the limit, first, because the interest rate is attractive, and, second, because it will be able to repay the debt in the future. The same intuition applies when $E\mu' = \mu$ or $E\mu'$ is even slightly smaller than μ (see the footnote). On the other hand, if the firm expects its future net profit to be significantly lower than the current one, it may not be optimal for it to borrow up to the limit today because it will have to then repay a relatively large debt in the future. In this case, it does not borrow up to the maximum today. In fact, it may actually decide to become a creditor and lend some its profit to the household. In this case, it will have an extra return in the future when its profit is small.

Thus, the second condition for the credit constraint to bind need not always hold. However, in simulations reported in Section 2.4 I check that the credit constraint does bind.

Aggregating over all firms, and assuming that they do always borrow up to the maximum, gives the following expression for firms' equilibrium debt:

$$B = \frac{Eq'(1 - \delta_k)K}{R}. \quad (2.15)$$

I now turn to a brief discussion of the capital market.

2.3.3 Capital market

Capital suppliers face increasing marginal costs $S(K)$, where K denotes their aggregate capital holdings. As in House and Shapiro (2008), I assume an isoelastic form:

$$S(K) = K^{\frac{1}{\nu}}, \quad \nu > 0. \quad (2.16)$$

On the other hand, capital suppliers' marginal benefit from selling a unit of capital is the difference between its price today and its price tomorrow, corrected for depreciation. When optimizing, capital suppliers use the rule of the marginal cost being equal to the marginal benefit:

$$S(K) = q - \frac{Eq'(1 - \delta_k)}{R}. \quad (2.17)$$

The parameter ν thus reflects the steady-state elasticity of capital supply.

Now consider the firm's capital demand. The first-order condition of the firm's problem (2.9) with respect to capital is:

$$\begin{aligned} - (1 + \mu)q + \lambda(1 - \delta_k)Eq' + \beta E(1 + \mu'(1 - \bar{c}))[A'F_k(k, l') - w_k(k, l', A')l'] + \\ + \beta(1 - \delta_k)E(1 + \mu')q' = 0. \end{aligned} \quad (2.18)$$

Rearranging and using equation (2.13) to substitute in for λ gives:

$$\begin{aligned} \beta E(1 + \mu'(1 - \bar{c}))[A'F_k(k, l') - w_k(k, l', A')l'] = (1 + \mu) \left[q - \frac{(1 - \delta_k)Eq'}{R} \right] - \\ - \beta(1 - \delta_k)cov(\mu', q'). \end{aligned} \quad (2.19)$$

Equation (2.19) equates the marginal benefit of buying an extra unit of capital to the marginal cost. It is different from the one in Chapter 1, first, because it includes the covariance term due the uncertainty over productivity shocks, and second, because there is a potential effect of having more capital on wages. The covariance term is subtracted from the marginal cost of buying capital today because, if it is positive, buying more capital today makes the firm's net worth higher tomorrow and, at the same time, it relaxes tomorrow's profit constraint. So the marginal cost of

buying capital is lower. As for the wage effect, this will be discussed in more detail below, but the basic idea is that it is natural for wages to depend on the marginal product of labour, which in turn depends on how much capital the firm has. So $[A'F_k(k, l') - w_k(k, l', A)l']$ is essentially a share of the firm's marginal product of capital.

As in Chapter 1, equation (2.19) is different from the no-credit-frictions capital demand rule, $\beta[A'F_k(k, l') - w_k(k, l', A)l'] = q - \beta(1 - \delta_k)Eq'$, in three ways. First, the marginal benefit of having an extra unit of capital is larger with credit frictions because buying more capital today relaxes tomorrow's profit constraint. Second, the marginal cost of buying an extra unit of capital is higher because buying another unit tightens today's profit constraint. Finally, the user cost $(q - \frac{(1-\delta_k)Eq'}{R})$ reflects that capital is not only used in the production process, but also as collateral. Thus, when facing credit frictions the firm demands more capital $(q - \frac{(1-\delta_k)Eq'}{R} < q - \beta(1 - \delta_k)Eq'$ by Assumption 1).

Consider a scenario when the profit constraint binds for all sequences of productivity shocks, $\mu > 0$ for any A and $E\mu' > 0$. Then,

$$(1-\bar{c}) \left[AF(k_{-1}, l) - w(k_{-1}, l, A)l - \frac{c}{\varphi(\theta)}(l - (1-\delta_l)l_{-1}) \right] + b - Rb_{-1} - q(k - (1-\delta_k)k_{-1}) = 0 \quad \forall A. \quad (2.20)$$

Using the credit constraint to substitute in for b gives a familiar expression, which links the firm's capital demand to its net worth and the user cost of capital:

$$k = \frac{1}{q - \frac{Eq'(1-\delta_k)}{R}} \left[(1-\bar{c}) \left[AF(k_{-1}, l) - w(k_{-1}, l, A)l - \frac{c}{\varphi(\theta)}(l - (1-\delta_l)l_{-1}) \right] + q(1-\delta_k)k_{-1} - Rb_{-1} \right]. \quad (2.21)$$

The firm's net worth includes its gross profit and the value of its capital holdings less the debt repayment.

In contrast to Chapter 1, the firm's net worth takes into account vacancy costs and wages which are no longer determined by the labour market equilibrium. The latter is an important channel of transmission of search frictions to capital demand. Wages influence the financial accelerator effects on capital demand by affecting both the static and dynamic multiplier effects introduced by Kiyotaki and Moore (1997). Consider how each of those are affected in turn.

The static multiplier is a current period multiplier that occurs holding the future constant. As today's decrease in productivity lowers today's capital demand, the price of capital declines as well. This is because the user cost has to decrease to clear the market. The fall in q lowers the firm's net worth, which decreases its capital demand even more. The static multiplier effect, however, is different from the one in Chapter 1 because of wages. Notice that when today's productivity decreases, wages decrease as well. So the overall decline in $AF(k_{-1}, l) - w(k_{-1}, l, A)l$ depends on how flexible wages are. If wages are more flexible, they will fall more and the resulting decline of the net worth and capital will be relatively small. If they are more rigid, the decline will be relatively large.

Note that wage rigidity also plays a role in the first-best allocation of capital, $\beta E[A'F_k(k, l') - w_k(k, l', A')l'] = q - \beta(1 - \delta_k)Eq'$. However, since capital demand in this case does not depend on net worth, the financial accelerator effects do not take place and the effects of a decrease in productivity are much smaller. Therefore, the effects of wage rigidity are a lot smaller too.

The dynamic multiplier is a multiplier that links the future to the present via the credit constraint. As the firm's capital today goes down because of lower productivity, so does its net worth tomorrow. Again, the more rigid wages are, the more

the net worth falls, and the more tomorrow's capital demand falls. The decrease in tomorrow's capital demand lowers its expected price. This tightens the credit constraint *today*, and the firm is forced to borrow less. Thus, it is able to invest less, and its capital demand decreases further.

In this scenario, to obtain aggregate capital demand I assume that the production function exhibits constant returns to scale. This assumption may be relaxed, but it simplifies aggregation. Thus, adding up individual capital demands, given by equation (3.6), aggregate capital demand takes the following form:

$$K = \frac{1}{q - \frac{Eq'(1-\delta_k)}{R}} \left[(1 - \bar{c}) \left[AF(K_{-1}, L) - w(K_{-1}, L, A)L - \frac{c}{\varphi(\theta)}(L - (1 - \delta_l)L_{-1}) \right] + q(1 - \delta_k)K_{-1} - RB_{-1} \right], \quad (2.22)$$

which exhibits all of the features of individual capital demands described above.

In Chapter 1 I also describe what happens when the profit constraint never binds. The consequences of that in this model are very similar.

When simulating the model I consider cases when the profit constraint does always bind. I check that this holds in the simulations that I report in Section 2.4. This, of course, may not hold for every sequence of productivity shocks. If the value of A is high enough, the firm may not be profit constrained anymore. Although assuming an always-binding constraint is not entirely accurate in terms of the resulting numerical solution, it has a great advantage of simplifying the solution of the model.

In the next section I discuss consequences of having search frictions in the labour market and wages.

2.3.4 Labour market

Labour demand

As already mentioned, labour supply in the economy is fixed and equal to the size of the labour force, \bar{L} . Consider labour demand. The first order condition of the firm's problem (2.9) with respect to l gives:

$$(1 + \mu(1 - \bar{c}))AF_l(k_{-1}, l) = (1 + \mu(1 - \bar{c}))\left[w(k_{-1}, l, A) + w_l(k_{-1}, l, A)l + \frac{c}{\varphi(\theta)}\right] - \beta E(1 + \mu'(1 - \bar{c}))\frac{c}{\varphi(\theta')}(1 - \delta_l). \quad (2.23)$$

Imagine for a moment that the profit constraint does not bind for any values of A , so $\mu = \mu' = 0$. Then, equation (2.23) simply states that the marginal product of labour should be equal to the marginal cost of hiring an extra worker. The latter comprises the following costs. First, it includes the worker's wage. Second, it takes into account current hiring costs net of expected future hiring costs, corrected for separations. Expected future hiring costs influences today's labour demand because hiring an extra worker today saves the cost of hiring her tomorrow. Finally, the marginal cost also includes the effect of hiring an extra worker on wages of existing workers. When an additional worker is hired the marginal product of labour decreases. To the extent that wages are linked to the marginal product of labour, the wage of each worker falls.

Now consider the effect of the profit constraint on labour demand. First, note that in (deterministic) steady state it does not have any effect on the labour demand. Since in steady state the Lagrange multipliers satisfy $\mu = \mu' \geq 0$, this implies that $1 + \mu(1 - \bar{c}) > 0$. Therefore, dividing both sides of equation (2.23) by $1 + \mu(1 - \bar{c})$

gives a labour demand equation as if $\mu = \mu' = 0$:

$$AF_l(k, l) = w(k, l, A) + w_l(k, l, A)l + \frac{c}{\varphi(\theta)} - \beta \frac{c}{\varphi(\theta)}(1 - \delta_l). \quad (2.24)$$

Here, credit frictions affect the labour demand only through the complementarity of capital and labour, as in Chapter 1. When the firm's capital declines the marginal product of labour falls as well. Therefore, the firm's labour demand decreases, and it hires less workers.

Out of steady state, however, labour demand is affected by the relative sizes of μ and $E\mu'$. In other words, it depends on how much more or less the profit constraint binds today compared to how much it is expected to bind tomorrow. Consider the right-hand side of equation (2.23). It can be split into two parts, today's marginal cost of hiring a worker, $(1 + \mu(1 - \bar{c})) \left[w(k_{-1}, l, A) + w_l(k_{-1}, l, A)l + \frac{c}{\varphi(\theta)} \right]$, and the savings of future marginal hiring costs $\beta E(1 + \mu'(1 - \bar{c})) \frac{c}{\varphi(\theta)}(1 - \delta_l)$. Whereas the first part is affected by *today's* profit constraint, the second part is affected by *tomorrow's*. Imagine that, in expectation, the profit constraint binds tomorrow relatively more than today, $E\mu' > \mu$. Then, in expectation, it is very costly for the firm to hire tomorrow, and thus the firm is better off hiring more workers today and hence saving on vacancy costs tomorrow. So relative to the steady state the firm overhires. In the opposite situation, when the constraint binds relatively less tomorrow than it does today, $E\mu' < \mu$, expected savings on future vacancy costs are small. The firm knows that it is going to be relatively cheap to hire tomorrow, so it underhires today relative to the steady state.

While $E\mu' > \mu$ is likely during the aftermath of a positive shock, $E\mu' < \mu$ is likely during recovery after a negative shock. The latter contributes to the propagation of the recovery process when firms effectively delay hiring until it is cheaper in terms

of the impact on net worth.

Wages

The main purpose of this paper is to explore the feedback effect from labour frictions back to capital demand. As already discussed, this feedback effect exists because the wage bill influences the firm's net worth, and therefore, its capital demand. A very important question, then, is how wages are determined.

Search and matching frictions in the labour market imply that each employment relationship creates a rent to be split between the firm and the worker. As pointed out by Hall (2005), these rents could be split in many different ways which means that there exists a range of feasible wages in each job-worker match.

In Hall (2005) model firms produce output according the production function $y = Al$ which omits capital and exhibits constants returns to scale. In this setting he considers the following wage schedule:

$$w = \bar{w}A^\tau, \quad 0 \leq \tau < 1, \quad (2.25)$$

where \bar{w} is a constant and τ is the elasticity of the wage with respect to productivity. The chosen wages are partially rigid with respect to labour productivity.

Michaillat (2012) then appeals to Hall (2005) in a setting with large firms. Given diminishing returns to labour in production function $y = Al^\alpha$, which also omits capital, he considers wages given by (2.25). In his setting, this form of wages means that they are fully rigid with respect to the number of workers and partially rigid with respect to aggregate productivity.

However, this form of rigid wages is critiqued in Brügemann (2014). He shows that they are privately inefficient off the equilibrium path, and therefore cannot

be a result of any theory of wage determination that yields private efficiency. The intuition for this result is as follows. Over the course of one period the firm makes two decisions about the number of workers to hire. First, it decides how many vacancies to post (which relates to how many workers it wants to hire through the law of large numbers). Second, after the recruiting costs are sunk and the matching is complete, it decides how many workers to actually employ among those who got matched to the firm.

Now if the firm's profit, excluding vacancy costs, is a (weakly) increasing function in l , then private efficiency would require that the number of workers resulting from the first decision should be equal to the number of workers resulting from the second decision. To see this, consider a firm's problem adapted from Brügemann (2014):

$$\max_l \left\{ \pi(l, A) - \frac{c}{\varphi(\theta)} l \right\}, \quad (2.26)$$

where $\pi(l, A) = Al^\alpha - w(l, A)l$ is the firm's gross profit. Denote the solution to (2.26) l^* . Now if $\pi(l, A)$ is (weakly) increasing in l , then l^* would definitely be the solution to the problem when vacancy costs are sunk:

$$\max_l \pi(l, A). \quad (2.27)$$

Brügemann (2014) points out that wages given by (2.25) imply that the firm's gross profit $\pi(l, A)$ is not monotonic in l . Furthermore, he claims that there is no simple restrictions on exogenous parameters that would achieve monotonicity, except for the trivial $\bar{w} = 0$. Therefore, in a setting with diminishing returns such wages cannot be consistent with any sharing mechanism that produces private efficiency on and off the equilibrium path.

Brügemann (2014) suggests using the following wages that are privately efficient

on and off the equilibrium path:

$$w = \bar{w}A^\tau l^{\alpha-1}, \quad 0 \leq \tau < 1. \quad (2.28)$$

These wages are flexible with respect to employment and partially rigid with respect to productivity shocks.

In my model, which includes capital in the production function $y = F(k_{-1}, l)$, I implement the following wage schedule for the Cobb-Douglas form:

$$w(k_{-1}, l, A) = \bar{w}_1 A k_{-1}^\alpha l^{-\alpha} + \bar{w}_2 u, \quad \bar{w}_1 \geq 0, \quad \bar{w}_2 \geq 0, \quad (2.29)$$

with \bar{w}_1 and \bar{w}_2 being flexibility parameters and u denotes workers' value of leisure. These wages includes features of wages in both Michailat (2012) and Brügemann (2014). On one hand, as in Michailat (2012), they contain a constant term which does not depend on the firm's number of workers. This makes them more rigid than wages in Brügemann (2014). On the other hand, as in Brügemann (2014), they include a term which depends on the number of workers but also on capital since capital takes part in the production process. This makes wages more flexible than wages in Michailat (2012).

Notice that in equation (2.29) I effectively set wage elasticity to productivity shocks τ equal to 1. Thus, I do not distinguish between wage rigidity with respect to aggregate productivity, capital or labour. Instead, I consider how wages depend on the average labour productivity.

The assumed form of wages is admittedly somewhat ad hoc, and it would be interesting to see, if wages are split by a surplus-sharing rule, how credit frictions affect the splitting process. Chapter 3 investigates this question. The main challenge is that under credit frictions wages affect firms' net worth, and this significantly

complicates the solution to the surplus-sharing rule.

The advantage of using wages given (2.29) is that by varying parameters \bar{w}_1 and \bar{w}_2 one could see how these affect the model results. When \bar{w}_1 is high, wages vary a lot with labour productivity, so they are more flexible in this sense. When \bar{w}_1 is low, wages are more rigid. I discuss how this affects the demand for capital.

This section concludes the description of the model. The next section discusses the calibration strategy and quantitative applications of the model.

2.3.5 Deterministic steady-state equilibrium

In the simulations that follow, for a range of positive \bar{c} , I have been able to find only one deterministic steady-state equilibrium in which both the credit constraint and the profit constraint bind. The equilibrium is described by eight equations which jointly determine steady-state values of capital, employment, debt, the price of capital, labour market tightness, the interest rate, wages and unemployment $\{K^*, L^*, B^*, q^*, \theta^*, R^*, w^*, U^*\}$.

1. Equilibrium interest rate is determined by the household's discount factor:

$$R^* = \frac{1}{\beta^h}. \quad (2.30)$$

2. Firms borrow up to their credit limit:

$$B^* = \frac{q^*(1 - \delta_k)K^*}{R^*}. \quad (2.31)$$

3. Firms invest as much as possible into capital. As a result, firms' capital demand

is determined by their net worth:

$$K^* = \frac{(1 - \bar{c}) \left[AF(K^*, L^*) - w(K^*, L^*, A)L^* - \frac{c\delta_l}{\varphi(\theta^*)}L^* \right]}{q^* \left(1 - \frac{1-\delta}{R^*} \right)} \quad (2.32)$$

Note that firms' net worth in steady state is their revenue after wage and minimum dividend payments. If compared to the dynamic capital demand equation (2.22), the terms $q^*(1 - \delta_k)K^*$ and R^*B^* are missing in (2.32) because in steady state the value of land is exactly offset by the required debt repayment.

4. Labour demand (or job-creation condition) is determined by setting the marginal product of labour equal to the marginal cost of hiring an additional worker:

$$AF_L(K^*, L^*) = w(K^*, L^*, A) + w_L(K^*, L^*, A)L^* + \frac{c}{\varphi(\theta^*)}[1 - \beta(1 - \delta_l)]. \quad (2.33)$$

5. Capital supply, which in equilibrium should be equal to capital demand:

$$K^{*\frac{1}{\nu}} = q^* \left(1 - \frac{1 - \delta_k}{R^*} \right). \quad (2.34)$$

6. Wage function:

$$w(K^*, L^*, A) = \bar{w}_1 AK^{*\alpha} L^{*-\alpha} + \bar{w}_2 u. \quad (2.35)$$

7. The law of motion for employment, where ΔL is set to be equal to 0:

$$f(\theta^*)U^* = \delta_l L^*. \quad (2.36)$$

Equation (2.36) says that the number of unemployed workers who find jobs should be equal to the number of employed workers who lose their jobs.

8. The adding-up constraint:

$$U^* + (1 - \delta_l)L^* = \bar{L}. \quad (2.37)$$

One could obtain the Beveridge curve from equation (2.36) by substituting U for L :

$$U^* = \frac{\delta_l \bar{L}}{\delta_l + (1 - \delta_l)\theta^*\varphi(\theta^*)}. \quad (2.38)$$

2.4 Quantitative applications

2.4.1 Calibration

The calibration strategy mirrors the calibration strategy in Chapter 1 on the capital side.

The model time period is taken to be equal to one week. The productivity shock evolves according to the following geometric AR(1) process:

$$\ln A_t = \rho \ln A_{t-1} + \epsilon, \quad \epsilon \sim N(0, \sigma^2). \quad (2.39)$$

Following King and Rebelo (1999) I assume quarterly values of ρ and σ to be 0.979 and 0.0072, respectively, which implies weekly autoregressive parameter of 0.998 and weekly standard deviation of 0.002.⁵

When calibrating discount factors I follow Iacoviello (2005) and set $\beta^h = 0.999$ and $\beta = 0.998$ which corresponds to his quarterly values of 0.99 and 0.98, respectively. These values imply an internal rate of return for a firm which is twice as large

⁵Given the shock process $E[\ln A_{t+m} | \ln A_t] = \rho^m E[\ln A_t]$. Thus, $\rho_{week} = \rho_{quarter}^{1/13}$. Also, since $Var(\ln A_t) = \frac{\sigma^2}{1-\rho^2}$ keeping variances of quarterly and weekly processes constant requires $\sigma_{week}^2 = \sigma_{quarter}^2 \frac{1-\rho_{quarter}^2}{1-\rho_{week}^2}$.

as the rate of return faced by the household.

I assume that the production function is of the Cobb-Douglas form and that capital share of output is equal to $1/3$, $F(k, l) = k^{1/3}l^{2/3}$. Capital depreciation rate is assumed to be equal to 10% a year, or 0.19% a week.

I set the value of \bar{c} , the minimum dividend share of gross profit, equal to 30% which is a median of the dividend payout ratio (ratio of dividends paid to net income) across S&P500 firms over the last 10 years.⁶

House and Shapiro (2008) obtain estimates of investment supply elasticities in the interval between 6 and 14. In steady state this investment elasticities imply the same capital supply elasticities. I set ν to be equal to 10, which is a midpoint of their estimates.

On the labour side, the calibration strategy is as follows. I assume a conventional matching function of the Cobb-Douglas form, $M = \chi U^\psi V^{1-\psi}$. Based on the evidence in Petrongolo and Pissarides (2001), I set matching elasticity ψ to be equal to 0.6. I calibrate matching efficiency χ in the same way as Elsby and Michaels (2013): I target a steady-state job-finding rate equal to 0.1125, which is consistent with a monthly rate of 0.45 reported by Shimer (2005). At the same time, I target a steady-state value of labour market tightness of 0.72 as in Pissarides (2009). Since $f = \chi\theta^{1-\psi}$, this implies that χ should be equal to 0.1283.

I set the value of the job destruction rate δ_l to be equal to 0.0078, which is consistent with estimates in Shimer (2012). As in Michailat (2012), I set the labour force \bar{L} to be equal to 1.

I calibrate the cost of opening a vacancy, c , to be such that per worker hiring cost c/φ is equal to 14% of the quarterly worker compensation estimated by Silva and Toledo (2009), that is $c/\varphi = 0.14 \cdot 13Ew$.⁷ Since $\varphi = \chi\theta^{-\psi} = 0.16$ in steady

⁶http://www.factset.com/websitefiles/PDFs/dividend/dividend_9.28.15

⁷Note that the model time period is one week, so to get quarterly wages I have to multiply

state, this implies that $c/Ew = 0.29$. Thus, c should be equal to 29% of an average worker wage.

Following Hall and Milgrom (2008) I calibrate the value of leisure u to be equal to 70% of average labour productivity.

Finally, I calibrate the wage flexibility parameters \bar{w}_1 and \bar{w}_2 in the following way. I experiment with values of \bar{w}_1 , which is responsible for how much wages react to labour productivity. Then, for each value of \bar{w}_1 I calibrate \bar{w}_2 so that steady-state job-finding rate is equal to 11.25%. This implies that steady-state unemployment rate is equal to 6.5%. Various combinations of \bar{w}_1 and \bar{w}_2 are shown in Table 2.4.1 along with the other calibrated parameter values.

weekly wages by 13.

Table 2.4.1: Calibrated model parameters

Description	Parameter	Value
Firm's discount factor	β	0.998
Households' discount factor	β^h	0.999
Capital share	α	1/3
Labour share	γ	2/3
Minimum dividend share	\bar{c}	0.3
Capital supply elasticity	ν	10
Matching elasticity	ψ	0.6
Matching efficiency	χ	0.1283
Job destruction rate	δ	0.0078
Labour force	\bar{L}	1
Autoregressive parameter	ρ	0.998
Volatility	σ	0.002
Vacancy cost	c	0.1
Value of leisure	u	0.32
Wage parameter 1	\bar{w}_1	0.3
Wage parameter 2	\bar{w}_2	0.649
Vacancy cost	c	0.09
Value of leisure	u	0.28
Wage parameter 1	\bar{w}_1	0.5
Wage parameter 2	\bar{w}_2	0.4525
Vacancy cost	c	0.08
Value of leisure	u	0.22
Wage parameter 1	\bar{w}_1	0.7
Wage parameter 2	\bar{w}_2	0.268
Vacancy cost	c	0.05
Value of leisure	u	0.14
Wage parameter 1	\bar{w}_1	0.9
Wage parameter 2	\bar{w}_2	0.0746

2.4.2 Interaction between search and credit frictions

Before I move on to investigating how wage flexibility affects the model outcomes, it is useful to compare the results of the model with both frictions to results of the model when one of the frictions is turned off. To do so, I examine impulse response functions of different models to an unexpected, persistent negative productivity shock. Note

that, once the shock realizes, firms in the model know that it is going to last according to the process described in the calibration section. As in Chapter 1 I take the size of the shock to be equal to one standard deviation. Figure 2.4.1 presents the shock and its evolution over time.

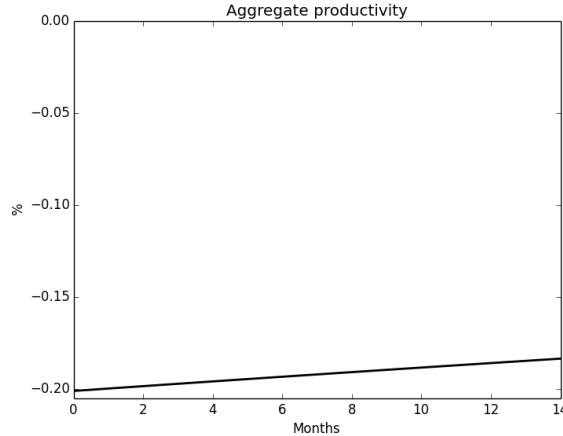


Figure 2.4.1: One standard deviation negative productivity shock.

The three models that I consider are a) the one with both search and credit frictions, b) one with search frictions only, and c) one with credit frictions only. The first model, with both frictions, is the one described in this Chapter. In the second model, with search frictions only, firms still face the credit constraint but they do not face the profit constraint. This implies that they are constrained in terms of borrowing at rate R but they still can borrow as much as they want from the household at higher rate $\frac{1}{\beta}$. This economy is in some sense the "first-best" economy, as explained in Chapter 1, but with added search frictions and rigid wages. I calibrate the model with search frictions only in the same way as I calibrate the one with both frictions.

The third model that I consider is the one with credit frictions only. This one is similar to one of the model specifications from Section 4.5 in Chapter 1 with inelastic labour supply. So here wages are completely flexible, and thus adjust to

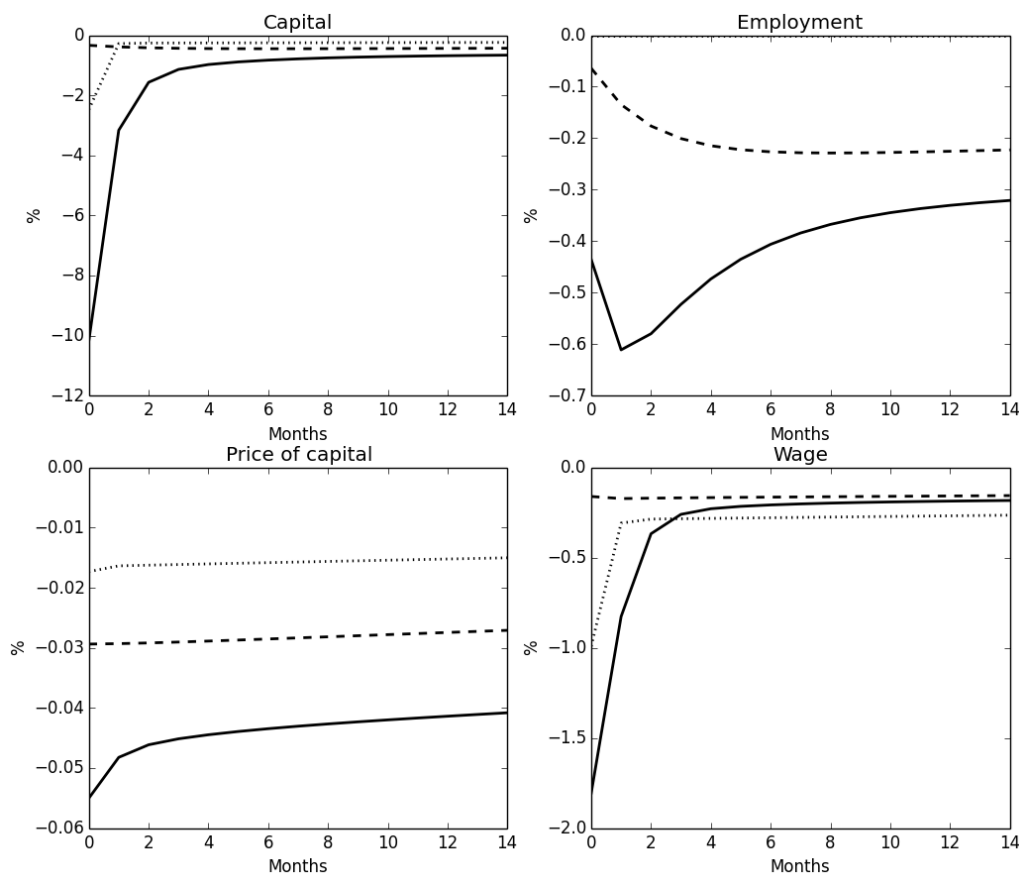


Figure 2.4.2: Impulse responses to productivity shock in deviations from steady state. Solid line - credit and search frictions, dashed line - search frictions, dotted line - credit frictions.

equate labour supply to labour demand. The only difference from Chapter 1 is the uncertainty over aggregate productivity shocks.

In this section I consider impulse responses when $\bar{w}_1 = 0.5$. So wages are somewhere in between being completely flexible, $\bar{w}_1 = 1$, and completely rigid, $\bar{w}_1 = 0$. I check that, qualitatively, changes between the models remain the same for other values of \bar{w}_1 which lie between 0 and 1 reported in Table 3.6.1.

Figure 2.4.2 depicts impulse responses of capital, employment, capital price and wages in different models, whereas Figure 2.4.3 depicts impulse responses of unemployment, labour market tightness and job-finding rate. Solid lines stand for the model with both frictions, dashed line for the one with search frictions only, and



Figure 2.4.3: Impulse responses to productivity shock in deviations from steady state. Solid line - credit and search frictions, dashed line - search frictions, dotted line - credit frictions.

dotted line for the one with credit frictions only. Note that in Figure 2.4.3 there are no dotted lines because there is no unemployment in the model with credit frictions only.

It can be seen from the graphs that responses of all variables are much larger in the model with both frictions compared to those with only one friction. This is because there is a significant interaction between the two frictions. From capital friction into labour demand through the marginal product of labour, and from labour friction to capital demand through wages.

Consider the model with credit frictions only - the dotted lines in Figure 2.4.2. In this model the response of capital is much smaller than it is in the model with both frictions. This is because wages adjust freely and firms' employment remains constant. Intuitively, when the labour supply is fixed and wages are fully flexible, the net worth of a firm does not fall by much when a negative productivity shock hits because wages adjust. Therefore, capital demand also does not decline a lot, and neither does its price. The overall financial accelerator effect is small. Note that because wages are equal to the marginal product of labour, which falls on impact considerably less than under both frictions, wages also do not initially fall by as much as they do under both frictions. This feature is related to wage flexibility and

is discussed in more detail in the next section.

Consider now the model with search frictions only - the dashed lines in Figures 2.4.2 and 2.4.3. In this model capital is determined by the frictionless Jorgensonian first-order condition, and it is thus at its "first-best" level. So, as the marginal product of capital falls because of the fall in aggregate productivity, capital decreases as well. But because there is no financial accelerator effect, the decline in capital is relatively small. Despite the fact that wages only partially adjust to the decline of the marginal product of labour, the decline in employment is still small and is approximately equal to the decrease in productivity. The other labour market variables respond accordingly.

Now consider the model with search and credit frictions now - the bold lines. When compared to the case with search friction only, the model predicts much larger responses of its variables to the shock. With the financial accelerator the effects on variables are significantly amplified.

The model with search and credit frictions also generates more propagation than the model with search frictions only. Unemployment rate stays higher under search and credit frictions for longer. This is because under both frictions firms delay hiring until it becomes cheaper in terms of its effect on net worth.

When compared to the model with credit frictions only, the model with both frictions generates larger responses because when wages are not fully flexible the firm's net worth decreases more than it does under full wage flexibility. This affects capital demand and, through complementarity of capital and labour, the number of hires.

While this discussion is useful in terms of understanding the differences between the three models, understanding how each friction works and how it impacts the other friction, I now turn to examining how wage flexibility affects the model outcomes.

2.4.3 Role of wage flexibility

We saw that the model with both frictions generates much larger responses of variables to productivity shocks. The reason is that, on one hand, credit frictions affect firms' labour demand because capital and labour are complementary in the production process, and on the other hand, search frictions affect capital demand because wages play an important role in shaping firms' net worth. Now consider how different wage flexibilities affect model outcomes.

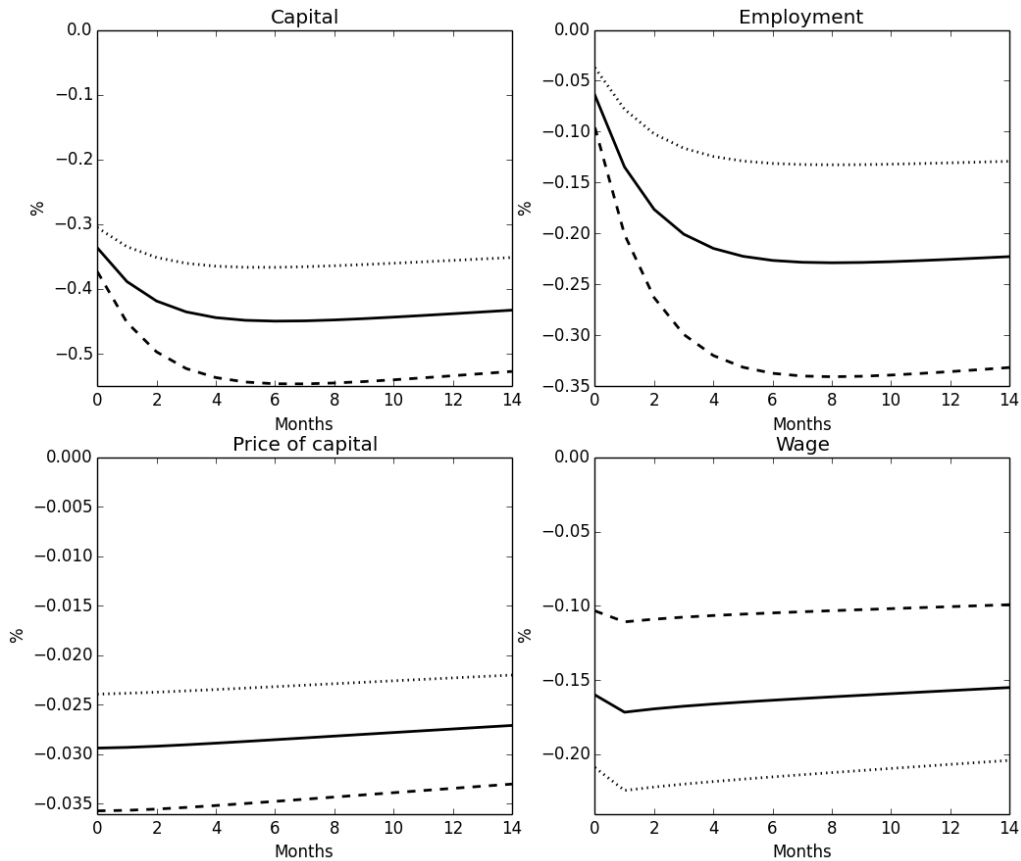


Figure 2.4.4: Impulse responses to productivity shock in deviations from steady state under search frictions only. Solid line $\bar{w}_1 = 0.5$, dashed line $\bar{w}_1 = 0.3$, dotted line $\bar{w}_1 = 0.7$.

Let me start by considering how wage flexibility affects capital demand and other variables under search frictions only. Figures 2.4.4 and 2.4.5 show the impulse responses. Solid lines indicate the responses when wages are medium rigid, dashed

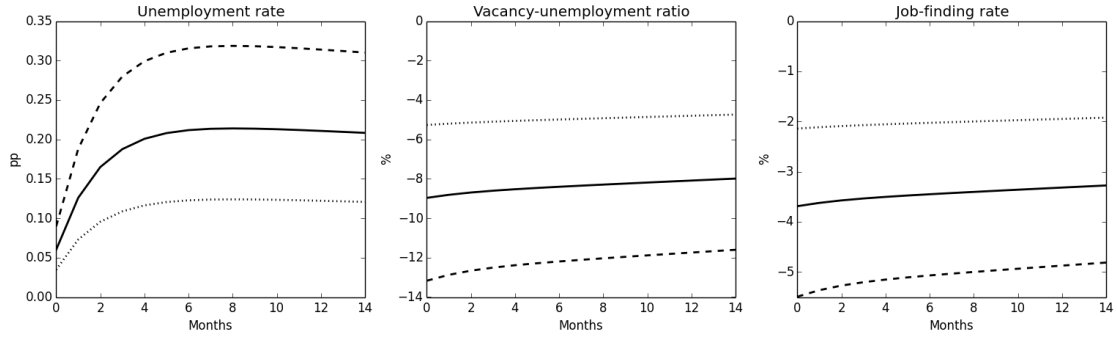


Figure 2.4.5: Impulse responses to productivity shock in deviations from steady state under search frictions only. Solid line $\bar{w}_1 = 0.5$, dashed line $\bar{w}_1 = 0.3$, dotted line $\bar{w}_1 = 0.7$.

lines when wages are most rigid, dotted lines when wages are most flexible.

As one would expect, when wages are more flexible they respond most to the shock, leading to a smaller decline of employment. The effect in turn spills over to capital demand, the capital price and the other labour market statistics in Figure 2.4.3. When wages are more rigid, they respond less to the shock, and so the drop in employment and the other variables is large.

Now consider the implications in the model with search and credit frictions. The impulse responses are shown in Figures 2.4.6 and 2.4.7. It could be seen that the more rigid wages are, the more capital and employment respond to the shock. This indicates that, indeed, wage flexibility significantly affects how economy responds to productivity shocks. When wages are more rigid, net worth falls more as a result of a negative shock leading to a larger decrease in capital demand. This effect persists back into labour market through the marginal product of labour.

In contrast to the model with search frictions only, the effects of different wage flexibilities are significantly amplified. This is due to the financial accelerator effect. When capital is determined by net worth (Figure 2.4.6) it falls much more than it does when it is determined by frictionless first-order condition for capital (Figure 2.4.4). This leads to larger responses of employment and the other labour market

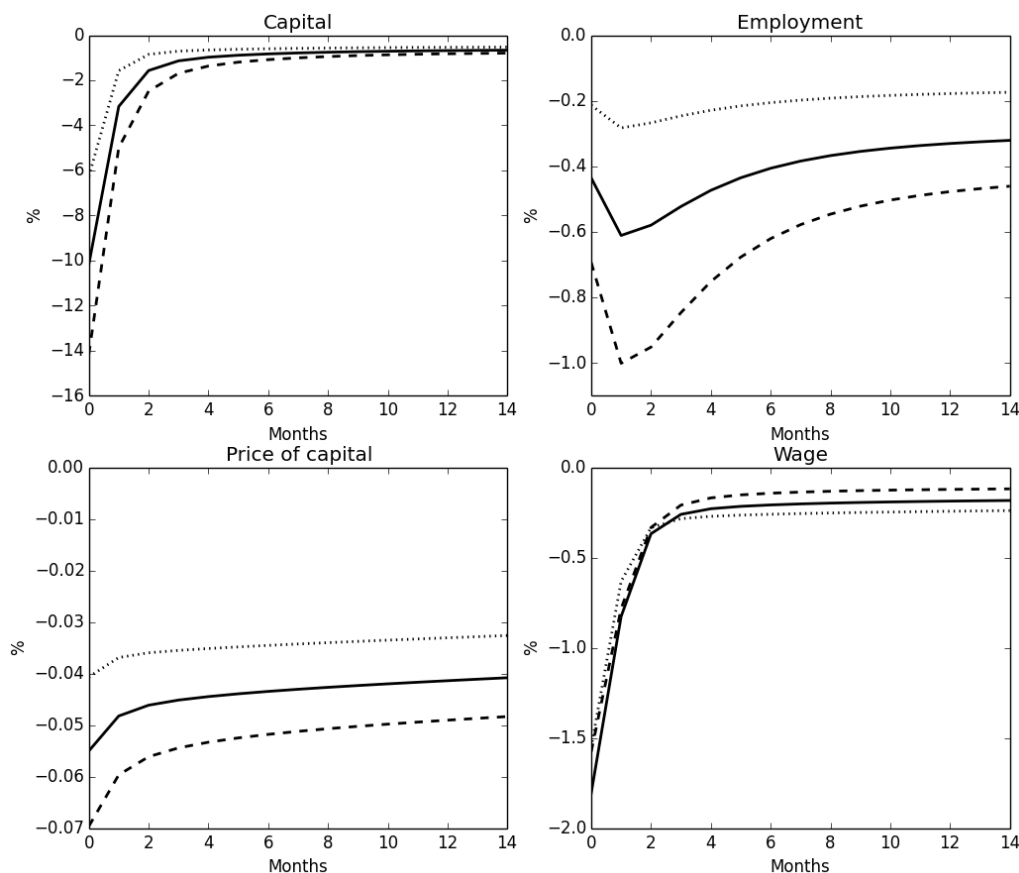


Figure 2.4.6: Impulse responses to productivity shock in deviations from steady state under search and credit frictions. Solid line - $\bar{w}_1 = 0.5$, dashed line - $\bar{w}_1 = 0.3$, dotted line - $\bar{w}_1 = 0.7$.

variables.

Now consider wages under search and credit frictions presented in the bottom right panel of Figure 2.4.6. It could be seen that equilibrium wages are almost invariant to wage flexibility. Looking closer, one could observe that medium rigid wages (solid line) fall on impact more than both more rigid (dashed line) and more flexible (dotted line) wages. The reason for that is that change in wages is determined by their flexibility parameters and labour productivity. More rigid wages fall by less than medium rigid wages because they depend more on constant unemployment benefit and less on labour productivity. On the other hand, more flexible wages, which depend more on labour productivity, fall by less than medium rigid wages



Figure 2.4.7: Impulse responses to productivity shock in deviations from steady state under search and credit frictions. Solid line - $\bar{w}_1 = 0.5$, dashed line - $\bar{w}_1 = 0.3$, dotted line - $\bar{w}_1 = 0.7$.

because the fall in labour productivity is smaller under more flexible wages.⁸

Overall, it could be concluded that the change in wages is non-monotonic in wage flexibility. This is in contrast to the results of the model with search frictions only where more flexible wages always fall by less than more rigid ones as a result of a shock. This result may potentially explain why we see wages fall little in data. This is because wages may respond to changes in labour productivity which are endogenous to flexibility of wages.

2.5 Conclusion

In this paper I have introduced a model in which the representative firm faces two types of frictions. In the labour market, it faces standard search and matching frictions when hiring workers. In the credit and capital markets, it face credit frictions in the form of collateral constraints. I then study the interaction of the two frictions. Credit frictions affect the firm's labour demand through complementarity of capital and labour as factors of production. When the firm's net worth decreases, its capital and labour demands decrease. Because of the financial accelerator the rise in

⁸In fact, when wage flexibility increases the model converges to the model with credit frictions only. This result is presented in Appendix.

unemployment is much greater than in the model with search frictions only.

There is also an effect going from search frictions to capital demand which is via wages. The wage bill plays an important role in forming the firm's net worth. Therefore, wage rigidities affect changes in net worth resulting from productivity shocks. More rigid wages result in greater decreases in net worth than more flexible wages. As a result firms' capital demand decreases more, and the effect persists into the labour market.

While studying how wage flexibility affect impulse responses of macroeconomic variables I also find that responses of wages to different wage flexibilities are non-monotonic, which could potentially explain why wages are observed to fall little in data.

In this Chapter, I established that wages indeed have a significant influence on the firm's net worth, and thus on its capital demand. The next step would be to examine how wages look like if they are determined not by a general rule, by rather by firms and workers sharing surpluses created by their employment relationships. This will shed light on how different their shares are, if different at all, depending on whether the firm is credit-constrained or not. This question is investigated in Chapter 3.

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Appendix

A Flexible wages

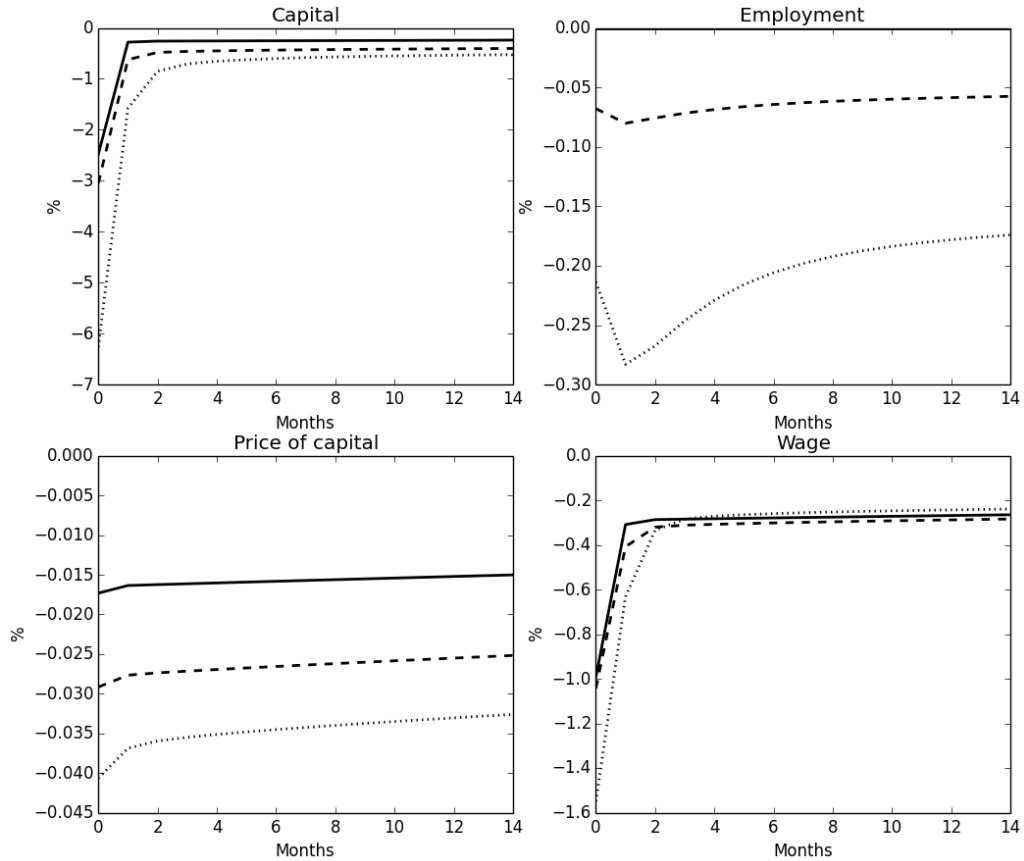


Figure 2.5.1: Impulse responses to productivity shock in deviations from steady state. Solid line - credit frictions only, dashed line - search and credit frictions $\bar{w}_1 = 0.9$, dotted line - search and credit frictions $\bar{w}_1 = 0.7$.

In Figure 2.5.1 dotted and dashed lines depict the impulse responses in the model with search and credit frictions under different wage flexibilities, $\bar{w}_1 = 0.7$ and $\bar{w}_1 = 0.9$ respectively, solid lines present the impulse responses in the model with credit frictions only. It could be seen that the more flexible wages are, the more impulse responses look like the ones under credit frictions only. As for wages, more flexible wages (dashed line) fall less on impact than less flexible (dotted line) because of the

smaller fall in labour productivity.

B Simulations

In order to get the numerical impulse responses from Section 2.4 I first solve for the deterministic steady state ($\{\epsilon_t\} = \{0\}$). I then use Dynare⁹ to solve for linear policy rules $K(K_{-1}, L_{-1}, B_{-1}, A_{-1}, \epsilon)$, $L(K_{-1}, L_{-1}, B_{-1}, A_{-1}, \epsilon)$, $B(K_{-1}, L_{-1}, B_{-1}, A_{-1}, \epsilon)$, as well as $q(K_{-1}, L_{-1}, B_{-1}, A_{-1}, \epsilon)$ implied by a first-order¹⁰ Taylor series approximation around the deterministic steady state.

Using these policy functions, I am then able to simulate what happens if the economy is hit by an unexpected shock, and I can also calculate the values of all the other variables presented on the graphs in the main text.

⁹Dynare (<http://www.dynare.org>) is a set of Matlab code files that linearizes a model around its deterministic steady state and solves for the coefficients of linearized policy functions.

¹⁰I also experiment with second-order approximations. Results are very similar to those reported.

Chapter 3. Wages and Credit Frictions

In this paper I use a model where firms face search and credit frictions to investigate wages determined by surplus sharing in such environment. I find that credit frictions affect the surplus-sharing mechanism in such a way that they increase the worker's effective bargaining power. That is, the firm and the worker negotiate wages as if the worker had a higher bargaining power. The reason for this is that under search and credit frictions the firm values workers more than under pure search frictions because output they produce increases the firm's net worth which, as a result of credit frictions, determines its ability to invest in capital. However, the effective worker's bargaining power appears to be endogenous to the firm's capital holdings and the number of employees. The more capital the firm has, the less the firm is financially constrained, and the lower wages its workers are able to extract. Due to endogeneity of the worker's effective bargaining power, the effect of credit frictions on wages is ambiguous.

3.1 Introduction

The times of financial distress are usually characterized by disruptions in credit markets which naturally persist into firms' capital investment. While it has been shown that decreased capital investment affects the labour market because capital and labour are complementary in the production process, it is perhaps an interesting question whether it also affects wages determined by surplus sharing between firms and workers. The channel through which credit frictions may affect wages determined by surplus sharing is likely to be through the firm's surplus of a match. It is reasonable to assume that this surplus should somehow be affected by the firm's financial position. So far, the literature has not been able to answer this question, as will be discussed further.

In this paper I use the model from Chapter 2 where firms face search frictions in the labour market and credit frictions in the capital market to examine wages determined by surplus sharing under search and credit frictions. Due to the diminishing marginal returns to labour, firms and workers share the marginal surplus as in Stole and Zwiebel (1996). I find that credit frictions affect the surplus-sharing mechanism in such a way that they increase the worker's effective bargaining power. That is, the firm and the worker negotiate wages as if the worker had a higher bargaining power. The reason for this results is that under search and credit frictions the firm values the worker more than under pure search frictions because output she produces increases the firm's net worth which determines its ability to invest into capital.

However, the effective worker's bargaining power appears to be endogenous to the firm's capital holdings and the number of employees. The more capital the firm has, the less it becomes financially constrained, and the lower wage the worker is able to extract. Due to the endogeneity of the worker's effective bargaining power, the effect of credit frictions on wages is ambiguous.

This paper is related to several strands of literature. Firstly, it is related to the literature which discusses model with both search and matching and credit frictions, discussed in more detail in Chapter 2. Secondly, because I consider wages determined by surplus sharing between a large firm and its workers, it is related to papers that discuss the underlying game theory mechanism. Thirdly, this paper has common features with papers that embed surplus sharing in large firms into search and matching environment. I now discuss each these strands in turn.

The first strand of literature considers models with two types of frictions: search and matching frictions and credit frictions. As already discussed in Chapter 2, some of the papers model credit frictions as frictions affecting capital market, Garín (2015), Zanetti (2015), Mumtaz and Zanetti (2016), and some of them model credit frictions affecting directly labour markets, Petrosky-Nadeau and Wasmer (2013), Petrosky-Nadeau (2014), Monacelli et al. (2011).

This strand of literature significantly contributed to the motivation for this work. To my knowledge, this is the first paper that investigates how surplus sharing, and the resulting wages, are affected by the fact that the firm is constrained in its investment into capital. The latter is important because it is a natural characteristic of a recession.

Garín (2015) uses Nash-bargained wages in a model with search and credit frictions. However, he assumes that firms take wages as given when making a decision about the number of hires. Wages under this assumption do not take into account the effect of hiring an extra worker on wages of incumbent workers. This effect is present in wages in this paper.

Zanetti (2015) and Mumtaz and Zanetti (2016) consider Nash-bargained wages under search and matching friction as in Blanchard and Galí (2010), where the firm's surplus from a match is equal to hirings costs because matching is instantaneous. In

contrast, I consider a model where matching a job to a worker takes time, and the firm's surplus of the match takes into account the effect of hiring an extra worker on the firm's profit and its constraints.

In Petrosky-Nadeau and Wasmer (2013) and Petrosky-Nadeau (2014) capital market is not modeled explicitly. In their setting, Nash bargained wages are only affected by credit constraints through increased costs of hiring. In Petrosky-Nadeau and Wasmer (2013) wages under credit constraints include a new procyclical component such that, when labour market tightness increases, the worker's bargaining position improves, which results in more volatile wages. Petrosky-Nadeau (2014) finds the opposite effect. In his model, the terms of credit contracts are counter-cyclical. This results in counter-cyclical hiring costs. Thus, wages include a new term which results in a strengthened bargaining position of the firm during an up-turn. Therefore, the response of wages to shocks are dampened compared to the standard search and matching model.

In contrast, this paper benefits from modeling capital demand explicitly because the effect of credit frictions on hiring costs is endogenous since it comes directly from the firm's problem in the capital market. Moreover, in my model wages are not only different because of hiring costs, but also because the firm's surplus from a match is different under credit frictions. Although, so far I make conclusions only about steady-state values of wages, in the future, this setup will help to shed light on wage volatility under credit constraints as well.

In Monacelli et al. (2011) wages are based on surplus-sharing and are determined by the Nash bargaining solution. However, the underlying model, where firms strategically borrow up to the credit limit in order to bargain lower wages, is quite different from my model, where firms borrow to finance capital investment.

The second strand of literature, to which this paper is related, discusses game

theory mechanisms which determine surplus-sharing rules in large firms, which I consider in this paper. In a setting with diminishing returns to labour, the size of the firm matters for wage determination. The reason for this is that the marginal product of labour is not constant, so the firm's surplus of a match depends on the margin on the number of employees. Stole and Zwiebel (1996) develop a non-cooperative bargaining game where one firm employs several workers. Due to diminishing marginal returns to labour, the firm knows that, if its negotiations with a given worker break down, it will have to renegotiate wages with all of the remaining workers. Therefore, the firm negotiates with each individual worker as if she marginal. Brügemann et al. (2015) later show that the game itself in Stole and Zwiebel (1996) does not support wages that coincide with the workers' Shapley values, as claimed in the original paper, and propose an alternative bargaining game.

The last strand of literature which this work is related to is papers that consider search and matching models in various settings with large firms and use the Stole and Zwiebel (1996) bargaining solution for wage determination. This strand includes, for example, papers by Cahuc and Wasmer (2001), Elsby and Michaels (2013) who consider endogenous job separations, Acemoglu and Hawkins (2014) who focus on the time-consuming side of search frictions and others. I use a similar to these papers wage environment in the labour market, but consider how it is affected by frictions in the capital market.

The rest of the paper is organized as follows. I summarize the model from Chapter 2 which I use to investigate wages under search and credit frictions in Section 3.2. Section 3.3 discusses how wages based on surplus sharing are determined in the environment which exhibits diminishing marginal returns to labour under pure search frictions. In Section 3.4 I consider how wages are determined under search and credit frictions. Section 3.5 presents a numerical example which summarizes findings from

Section 3.4. Section 3.6 concludes.

3.2 Model summary

3.2.1 Firm's problem

Consider the framework discussed in Chapter 2 where a typical firm maximizes its expected lifetime profit:

$$\begin{aligned} \Pi(k_{-1}, b_{-1}, l_{-1}, A) = \max_{k, b, l} \left\{ AF(k_{-1}, l) - w(k_{-1}, l, A)l - \frac{c}{\varphi(\theta)}(l - (1 - \delta_l)l_{-1}) + b - Rb_{-1} - \right. \\ \left. - q(k - (1 - \delta_k)k_{-1}) + \beta E\Pi(k, b, l, A'|A) \right\} \end{aligned} \quad (3.1)$$

subject to the credit constraint:

$$Rb \leq Eq'(1 - \delta_k)k, \quad (3.2)$$

and the profit constraint

$$(1 - \bar{c}) \left[AF(k_{-1}, l) - w(k_{-1}, l, A)l - \frac{c}{\varphi(\theta)}(l - (1 - \delta_l)l_{-1}) \right] + b - Rb_{-1} - q(k - (1 - \delta_k)k_{-1}) \geq 0. \quad (3.3)$$

The firm produces output according to the production function $AF(k_{-1}, l)$, where A denotes the aggregate productivity level and capital takes one period to install. The firm buys capital in the capital market at market price q . Each period capital depreciates at rate δ_k . Thus, the firm's total capital investment is $q(k - (1 - \delta_k)k_{-1})$.

The firm hires workers in the labour market which exhibits standard search and matching frictions. The firm has to pay the hiring cost $\frac{c}{\varphi(\theta)}$ per each workers hired. θ denotes labour market tightness and $\varphi(\theta)$ is a vacancy-filling rate. Job-worker

matches get destroyed at exogenous rate δ_l at the end of each period. The firm's pays wages $w(k_{-1}, l, A)$ to its workers, so its total wage bill is $w(k_{-1}, l, A)l$.

In the credit market, the firm may sign a debt contract with the household and borrow b units of output. Next period it has to repay the debt at interest rate R determined by equilibrium in the credit market.

Due to the contracting problem described in more detail in Chapter 2 the firm faces a credit constraint. The credit constraint (3.2) requires the firm to use capital as collateral for its debt. It says that the expected market value of collateral next period should at least cover the firm's debt repayment.

The firm also faces a profit constraint. Each period the firm has to pay a minimum dividend to the household. The minimum dividend should be equal to the share \bar{c} of the firm's gross profit $\left[AF(k_{-1}, l) - w(k_{-1}, l, A)l - \frac{c}{\varphi(\theta)}(l - (1 - \delta_l)l_{-1}) \right]$. The profit constraint (3.3) says that the firm's profits after paying minimum dividends should not be negative. This implies that the only way how the firm may borrow from the household is by signing a debt contract.

3.2.2 Some model features

In this section I summarize some of the model features that will be useful for explaining wage determination mechanisms in the next sections.

Consider first the credit market where a representative household supplies funds to firms. I assume that the household is more patient than firms, $\beta < \beta^h$. The household is willing to lend as long as the interest rate is greater or equal to $\frac{1}{\beta^h}$.

On the demand side, the firm is willing to borrow as long as the interest rate is lower or equal to $\frac{1}{\beta}$. If the profit constraint does not bind much more tomorrow than it does today, the firm is willing to borrow up to the maximum because it is

relatively impatient. This means that the credit constraint (3.2) binds.¹

The competitive credit market ensures that in equilibrium the household makes zero profit. Thus, the equilibrium interest rate is

$$R^* = \frac{1}{\beta^h}. \quad (3.4)$$

We shall see that the interest rate will appear later in the wage equation.

In the capital market, capital suppliers face increasing marginal costs $S(K)$. The marginal benefit of supplying a unit of capital is the difference between its price today and its price tomorrow, corrected for depreciation. When optimizing, capital suppliers equate marginal benefit to marginal cost:

$$S(K) = q - \frac{Eq'(1 - \delta_k)}{R}. \quad (3.5)$$

The firm's demand for capital depends on whether or not the profit constraint binds. If the profit constraint does not bind the firm has always the option to borrow more from the household as its shareholder at higher rate $\frac{1}{\beta}$. So, the firm's capital demand is determined by the frictionless first-order condition with respect to capital. The interesting case is when the profit constraint does bind which means that the firm's capital demand is determined its net worth:

$$k = \frac{1}{q - \frac{Eq'(1-\delta_k)}{R}} \left[(1 - \bar{c}) \left[AF(k_{-1}, l) - w(k_{-1}, l, A)l - \frac{c}{\varphi(\theta)}(l - (1 - \delta_l)l_{-1}) \right] + q(1 - \delta_k)k_{-1} - Rb_{-1} \right]. \quad (3.6)$$

As a result, when the firm's net worth decreases the firm's demand for capital de-

¹For more details see Section 2.3.2 in Chapter 2.

creases as well. This effect is large because when the demand for capital decreases its price decreases as well. The reason is that the capital market has to clear. The decline in the price of capital further reduces the net worth.

Additionally, as the firm's capital demand decreases today, the firm's expected net worth tomorrow decreases as well. This is due to the fact that the firm is able to produce less output tomorrow. As the net worth tomorrow decreases, so does tomorrow's demand for capital, and thus, the expected capital price. This tightens the credit constraint today. Because the firm is able to borrow less today, its capital demand today decreases even further.

Credit frictions affecting the firm's capital demand persist into the firm's labour demand through complementarity of factors of production.

The relationship between net worth and capital demand is important for wages based on a surplus-sharing rule, which I consider in this Chapter. The reason is that in the presence of credit frictions the firm values its workers not only because they produce output. In addition to that, by increasing the firm's production workers also relax the profit constraint. This increases the firm's net worth which allows it increase its capital investment. This makes wages under search frictions different from wages under both search and credit frictions, as the reader will see in the following sections.

I now turn to discussing how wages are determined in details. I start with wages that the firm pays when it is not constrained. Wages under pure search frictions are going to act as benchmark when I consider wages under credit constraints.

3.3 Wages under search frictions

Consider first how wages are determined if there are no credit frictions - that is, if the firm does not face the profit constraint. I will maintain the assumption that the firm faces the credit constraint, so it is restricted in how much it can borrow at rate R , but it can borrow an unlimited amount from the household as its shareholder at higher rate $\frac{1}{\beta}$. The firm's problem is then given by the value function (3.1) and the credit constraint (3.2).

I assume that wages are determined by surplus sharing between firms and workers. Due to diminishing marginal product of labour, they share the marginal surplus as in Stole and Zwiebel (1996), rather than total surplus as in the standard Mortensen and Pissarides (1994) model. The intuition for this is as follows. Due to search and matching frictions in the labour market, it is costly for workers to find jobs and for firms to fill vacancies. Because of that, each employment relationship creates a rent which is shared between a firm and a worker. Moreover, the rent of each particular employment relationship depends on the margin on the number of employees: The more workers the firm employs the smaller this rent becomes as a result of the diminishing marginal product of labour. Therefore, in the process of negotiating with a given worker the firm knows that if this negotiations break down it will have to renegotiate wages with all of the remaining workers. As a result of this, it negotiates with each worker as if she was marginal.

Consider the marginal surplus of a match to the firm, $J(k_{-1}, l, A)$. At the time of bargaining the firm has already paid hiring costs, so these are sunk. Thus, the marginal surplus is

$$J(k_{-1}, l, A) = AF_l(k_{-1}, l) - w(k_{-1}, l, A) - w_l(k_{-1}, l, A)l + \beta E \frac{c}{\varphi(\theta')} (1 - \delta_l), \quad (3.7)$$

which comprises the marginal product of labour less the wage plus savings on future hirings costs. The term $w_l(k_{-1}, l, A)l$ captures how much the wages of inframarginal workers change if an additional worker is hired.

The surplus of a match to a worker is the difference between her value of being employed, $W(k_{-1}, l, A)$, and her value of being being unemployed, Υ . If employed, each period the worker receives her wage. Next period, the match could be exogenously destroyed with probability δ_l , in which case the worker joins the unemployment pool and gets the value of being unemployed. With probability $(1 - \delta_l)$, however, the worker stays employed and continues obtaining the value of being employed. Therefore, the value of being employed in a firm with capital k_{-1} and l workers is:²

$$W(k_{-1}, l, A) = w(k_{-1}, l, A) + \beta E[\delta_l \Upsilon' + (1 - \delta_l)W(k, l', A'|A)]. \quad (3.8)$$

If the worker is unemployed she receives an unemployment benefit u in the current period. Next period, she faces a probability $f(\theta')$ to find a job and become employed, and thus receives the value of employment. With probability $(1 - f(\theta'))$ she stays unemployed and continues to obtain the value of unemployment:

$$\Upsilon = u + \beta E[(1 - f(\theta'))\Upsilon' + f(\theta')W(k, l', A'|A)]. \quad (3.9)$$

Wages are then the outcome of Stole and Zwiebel (1996) surplus-sharing solution, which generalizes Nash bargaining to a setting with diminishing returns:

$$(1 - \eta)[W(k_{-1}, l, A) - \Upsilon] = \eta J(k_{-1}, l, A), \quad (3.10)$$

²Here I assume that workers' discount factor is the same as firms', β , rather than the household's β^h . The equality of the discount factors allows to cancel terms in the process of wage derivations, which is especially useful in the next section for wages under both frictions. This allows to keep wages as simple and as tractable as possible.

where η denotes the worker's bargaining power.

3.3.1 Differential equation for wages under search frictions

In order to obtain a wage differential equation under pure search frictions from (3.10) one should follow the steps below.

First, forwarding (3.10) one period ahead gives:

$$(1 - \eta)[W(k, l', A') - \Upsilon'] = \eta J(k, l', A'). \quad (3.11)$$

Consider the firm's future surplus of a match $J(k, l', A')$. In the next period, the firm will choose the number of workers to hire according to the first-order condition of the problem (3.1)-(3.2) with respect to labour, forwarded one period ahead:

$$AF_l(k, l') = w(k, l', A') + w_l(k, l', A')l' + \frac{c}{\varphi(\theta')} - \beta E \frac{c}{\varphi(\theta'')} (1 - \delta_l). \quad (3.12)$$

From equation (3.12) it follows that the future marginal surplus of a match will be equal to the future marginal hiring cost:

$$J(k, l', A') = \frac{c}{\varphi(\theta')}. \quad (3.13)$$

Therefore, substituting in for $J(k, l', A')$ in equation (3.11) gives the following simple expression for the future value of being employed

$$W(k, l', A') = \Upsilon' + \frac{\eta}{1 - \eta} \frac{c}{\varphi(\theta')}. \quad (3.14)$$

Now one can use this expression to rewrite current values of employment (3.8)

and unemployment (3.9), which become

$$W(k_{-1}, l, A) = w(k_{-1}, l, A) + \beta E[\Upsilon' + (1 - \delta_l) \frac{\eta}{1 - \eta} \frac{c}{\varphi(\theta')}], \quad \text{and} \quad (3.15)$$

$$\Upsilon = u + \beta E[\Upsilon' + \frac{\eta}{1 - \eta} \frac{c}{\varphi(\theta')} f(\theta')]. \quad (3.16)$$

Subtracting one from the other gives the following surplus of a match to a worker

$$W(k_{-1}, l, A) - \Upsilon = w(k_{-1}, l, A) - u + \beta \frac{\eta}{1 - \eta} E \frac{c}{\varphi(\theta')} [1 - \delta_l - f(\theta')]. \quad (3.17)$$

Finally, substituting the firm's surplus (3.7) and the workers' surplus (3.17) into the bargaining solution (3.10) gives an expression for the bargained wage:

$$w(k_{-1}, l, A) = \eta \left[AF_l(k_{-1}, l) - w_l(k_{-1}, l, a)l + \beta E f(\theta') \frac{c}{\varphi(\theta')} \right] + (1 - \eta)u. \quad (3.18)$$

Wages are increasing in workers' bargaining power, the marginal product of labour, unemployment benefit and marginal hiring costs. On the other hand, wages decrease with $w_l(k_{-1}, l, A)l$. This term is explained by the fact that if wage negotiations with a particular worker break down, the firm will have to renegotiate wages with the remaining workers. Moreover, as the marginal product of labour decreases with the number of workers hired, so do wages ($w_l(k_{-1}, l, A) < 0$). This means that, if the worker walks away, the firm will have to increase wages of the remaining workers. The worker knows about this, and thus the firm has to pay her a higher wage.

Result (3.18) is a simple generalization of the differential equation for wages in Elsby and Michaels (2013) to a setting with capital as second factor of production. I will later compare this wage equation to the one under both frictions.

3.3.2 Solution to differential equation for wages under search frictions

The differential equation for wages (3.18) can be solved once one assumes a form of the production function and sets a boundary condition. As in Chapter 2, I assume that the production function is of the Cobb-Douglas form, and that it exhibits constant returns to scale:

$$F(k_{-1}, l) = k_{-1}^{\alpha} l^{1-\alpha}. \quad (3.19)$$

As for the boundary condition, I assume the same condition as in Cahuc and Wasmer (2001):

$$\lim_{l \rightarrow 0} w(k_{-1}, l, A)l = 0, \quad (3.20)$$

which says the wage bill should not explode as the number of workers approaches zero.

The solution to (3.18) then takes the following form:

$$w(k_{-1}, l, A) = \eta \left[\frac{(1-\alpha)}{1-\eta\alpha} A k_{-1}^{\alpha} l^{-\alpha} + \beta E f(\theta') \frac{c}{\varphi(\theta')} \right] + (1-\eta)u. \quad (3.21)$$

Wages increase with the marginal product of labour, marginal hiring costs and the size of unemployment benefit.

Given a matching functions $M(U, V)$, where U is the number of unemployed workers in the economy, and V is the number of vacancies, the job-finding rate is $f(\theta) = \frac{M(U, V)}{U}$, and the vacancy filling rate is $\varphi(\theta) = \frac{M(U, V)}{V}$. Therefore, the ratio of the two gives $V/U = \theta$. Thus, the wage equation (3.21) could be written as

$$w(k_{-1}, l, A) = \eta \left[\frac{(1-\alpha)}{1-\eta\alpha} A k_{-1}^{\alpha} l^{-\alpha} + \beta c E \theta' \right] + (1-\eta)u, \quad (3.22)$$

which implies that wages linearly increase with labour market tightness.

This concludes the description of wages under search frictions only. I now turn to the main part of the paper and look into how wages are formed in the presence of both search and credit frictions.

3.4 Wages under search and credit frictions

In this section I follow the same steps as in the previous one but draw attention to the differences implied by credit frictions. First, I consider the firm's surplus of a match followed by discussing the worker's surplus of a match. Then, I derive a differential equation for wages under search and credit frictions. I proceed by discussing wages in steady state.

Consider the firm's surplus of a match. Under credit frictions, the firm's problem is given by (3.1)-(3.3). Thus, as hiring costs are sunk at the time of surplus-sharing, the firm's marginal surplus of a match is given by:

$$J(k_{-1}, l, A) = (1 + \mu(1 - \bar{c})) [AF_l(k_{-1}, l) - w(k_{-1}, l, A) - w_l(k_{-1}, l, A)l] + \beta E \frac{c}{\varphi(\theta')} (1 - \delta_l)(1 + \mu'(1 - \bar{c})). \quad (3.23)$$

The firm's marginal surplus (3.23) is different from the surplus under pure search frictions (3.7) due to the presence of the Lagrange multipliers $\mu \geq 0$ and $\mu' \geq 0$. These state that the firm values the worker not only because she produces output, but also because the increased production relaxes the profit constraint. To put it another way, output produced by an extra worker increases the firm's net worth, which allows the firm to buy more capital. Thus, if the profit constraint binds, the marginal surplus of a match to the firm is larger than when it does not bind. I

discuss how μ is determined and what it depends on later in this section.

Now consider the surplus of a match to a worker, which is the difference between the value of employment $W(k_{-1}, l, A)$ and unemployment Υ . These remain the same as under pure search frictions. The value of employment is given by (3.8), and the value of unemployment is given by (3.9).

The Stole and Zwiebel (1996) bargaining solution implies that the surplus is shared according to the following rule:

$$(1 - \eta)[W(k_{-1}, l, A) - \Upsilon] = \eta J(k_{-1}, l, A). \quad (3.24)$$

3.4.1 Differential equation for wages under search and credit frictions

In order to derive a differential equation for wages under search and credit frictions I follow the same steps as in Section 3.3.1.

First, by forwarding the bargaining solution (3.24) I obtain:

$$(1 - \eta)[W(k, l', A') - \Upsilon'] = \eta J(k, l', A'). \quad (3.25)$$

Next period, the first-order condition with respect to labour implies that the firm's marginal surplus should be equal to the marginal hiring costs:

$$J(k, l', A') = (1 + \mu'(1 - \bar{c})) \frac{c}{\varphi(\theta')}. \quad (3.26)$$

Thus, substituting for $J(k, l', A')$ in (3.25) gives the following expression for the

future value of employment to a worker:

$$W(k, l', A') = \Upsilon' + \frac{\eta}{(1-\eta)} \frac{c}{\varphi(\theta')} (1 + \mu'(1 - \bar{c})), \quad (3.27)$$

When substituting in for $W(k, l', A')$ the value of unemployment to a worker becomes

$$\Upsilon = u + \beta E \left[\Upsilon' + f(\theta') \frac{\eta}{(1-\eta)} \frac{c}{\varphi(\theta')} (1 + \mu'(1 - \bar{c})) \right], \quad (3.28)$$

and the value of employment becomes

$$W(k_{-1}, l, A) = w(k_{-1}, l, A) + E\beta \left[\Upsilon' + (1 - \delta_l) \frac{\eta}{(1-\eta)} \frac{c}{\varphi(\theta')} (1 + \mu'(1 - \bar{c})) \right]. \quad (3.29)$$

The difference between the two gives the surplus of a match to the worker:

$$W(k_{-1}, l, a) - \Upsilon = w(k_{-1}, l, A) - u + \beta \frac{\eta}{1-\eta} E(1 + \mu'(1 - \bar{c})) \frac{c}{\varphi(\theta')} [1 - \delta_l - f(\theta')]. \quad (3.30)$$

Finally, the bargaining solution requires the worker's surplus to be equal to a fraction $\frac{\eta}{1-\eta}$ of the firm's surplus $J(k_{-1}, l, A)$. Multiplying equation (3.23) by $\frac{\eta}{1-\eta}$ and equating it to (3.30) results in the following differential equation for wages under search and credit frictions:

$$w(k_{-1}, l, A) = \frac{1}{1 + \eta\mu(1 - \bar{c})} \left[\eta(1 + \mu(1 - \bar{c})) [AF_l(k_{-1}, l) - w_l(k_{-1}, l, A)l] + \beta\eta c E(1 + \mu'(1 - \bar{c}))\theta' + (1 - \eta)u \right], \quad (3.31)$$

which coincides with (3.18) in the absence of the profit constraint ($\mu = \mu' = 0$).

The challenge of solving equation (3.31) is that, at the time of bargaining, the Lagrange multiplier on the profit constraint is endogenous. That is, the extent to

which the profit constraint binds depends on whether the firm hires the worker it is negotiating wages with or not. Specifically, the value of μ is determined by the first-order condition of problem (3.1)-(3.3) with respect to capital:³

$$\begin{aligned} \beta E(1 + \mu'(1 - \bar{c})) [A' F_k(k, l') - w_k(k, l', A') l'] = (1 + \mu) \left[q - \frac{(1 - \delta_k) E q'}{R} \right] - \\ - \beta(1 - \delta_k) \text{cov}(\mu', q'). \end{aligned} \quad (3.32)$$

Thus, the wage differential equation under search and credit frictions not only includes the term $w_l(k_{-1}, l, A)l$, but also $w_k(k, l', A')l$, which significantly complicates its solution. For this reason, I consider wages in deterministic steady state in this Chapter. The investigation of the dynamics of wages out of steady state is left for future research.

3.4.2 Wages in deterministic steady state

In this section I discuss the differential equation for wages under search and credit frictions in deterministic steady state and compare it to the differential equation for wages under pure search frictions.

Consider a (deterministic) steady-state version of (3.31):

$$\begin{aligned} w(k, l, A) = \frac{\eta(1 + \mu(1 - \bar{c}))}{1 + \eta\mu(1 - \bar{c})} [(1 - \alpha) A F_l(k, l) - w_l(k, l, A)l + \beta c\theta] + \\ + \frac{(1 - \eta)u}{1 + \eta\mu(1 - \bar{c})}. \end{aligned} \quad (3.33)$$

In contrast, the steady-state version of the differential equation under pure search

³The Lagrange multiplier on the credit constraint is substituted out using the first-order condition with respect to b .

frictions (3.18) is given by:

$$w(k, l, A) = \eta[AF_l(k, l) - w_l(k, l, a)l + \beta c\theta] + (1 - \eta)u. \quad (3.34)$$

There are two main difference between (3.33) and (3.34). The first difference is a multiplier $\frac{\eta(1+\mu(1-\bar{c}))}{1+\eta\mu(1-\bar{c})}$ in the first term, which is increasing in μ .⁴ This multiplier increases wages under search and credit frictions compared to wages under pure search frictions. The intuition is that a constrained firm value workers more than an unconstrained firm because output that workers produce in the constrained firm raises its net worth. Thus, workers are able to extract higher wages as if they had a higher bargaining power.

The second difference is the latter term, $\frac{(1-\eta)}{1+\eta\mu(1-\bar{c})}$, which is decreasing in μ . So wages under credit frictions put less weight on the unemployment benefit compared to wages under pure search frictions. This indicates that credit frictions change firms' and workers' bargaining weights. Denoting the first term by $\tilde{\eta}$, it appears that $1 - \tilde{\eta}$ is, in fact, equal to the second term:

$$1 - \tilde{\eta} = 1 - \frac{\eta(1 + \mu(1 - \bar{c}))}{1 + \eta\mu(1 - \bar{c})} = \frac{1 + \eta\mu(1 - \bar{c}) - \eta(1 + \mu(1 - \bar{c}))}{1 + \eta\mu(1 - \bar{c})} = \frac{1 - \eta}{1 + \eta\mu(1 - \bar{c})}. \quad (3.35)$$

Hereafter, I refer to $\tilde{\eta}$ as the worker's effective bargaining power.

The worker's effective bargaining power $\tilde{\eta}$ has two main characteristics. First, it is increasing in the Lagrange multiplier μ meaning that the more constrained the firm is, the more valuable its workers become, and thus the stronger workers' bargaining position becomes, $\tilde{\eta} \geq \eta$. The second characteristic is that the worker's effective bargaining is endogenous in the sense that it depends on the firm's capital and the

$$\frac{4d\left[\frac{\eta(1+\mu(1-\bar{c}))}{1+\eta\mu(1-\bar{c})}\right]/d\mu}{\eta\frac{(1-\bar{c})(1-\eta)}{(1+\eta\mu(1-\bar{c}))^2}} = \eta\frac{(1-\bar{c})(1+\eta\mu(1-\bar{c}))-\eta(1-\bar{c})(1+\mu(1-\bar{c}))}{(1+\eta\mu(1-\bar{c}))^2} = \eta\frac{(1-\bar{c})(1+\eta\mu(1-\bar{c})-\eta-\eta\mu(1-\bar{c}))}{(1+\eta\mu(1-\bar{c}))^2} = \eta\frac{(1-\bar{c})(1-\eta)}{(1+\eta\mu(1-\bar{c}))^2} > 0.$$

number workers at the time of wage negotiations. The reason for this is that μ is endogenous, as explained in the previous section.

Consider how μ depends on k and l . From equation (3.32) the steady-state value of μ could written as

$$\mu = -\frac{\beta(AF_k(k, l) - w_k(k, l, A)l) - q\left(1 - \frac{(1-\delta_k)}{R}\right)}{(1 - \bar{c})\beta(AF_k(k, l) - w_k(k, l, A)l) - q\left(1 - \frac{(1-\delta_k)}{R}\right)}. \quad (3.36)$$

For the Cobb-Douglas production function with constant returns to scale $y = Ak^\alpha l^{1-\alpha}$ it is useful to rewrite (3.36) in terms of the capital-labour ratio κ :

$$\mu(\kappa, A) = -\frac{\beta(A\alpha\kappa^{\alpha-1} - \omega(\kappa, A) - q\left(1 - \frac{(1-\delta_k)}{R}\right))}{(1 - \bar{c})\beta(A\alpha\kappa^{\alpha-1} - \omega(\kappa, A)) - q\left(1 - \frac{(1-\delta_k)}{R}\right)}, \quad (3.37)$$

where I used the following notation: If wages are denoted by $w(k, l, A) = \omega(\kappa, A)$, where $\kappa = \frac{k}{l}$, then $w_k(k, l, A)l = \omega'(\kappa, A)$ and $w_l(k, l, A)l = -\kappa\omega'(\kappa, A)$.

It is not entirely clear from equation (3.37) how μ depends on κ . However, it is reasonable to assume that the Lagrange multiplier on the profit constraint μ depends negatively on the capital-labour ratio. This is because the more capital the firm has for a given number of workers, the less profit-constrained it should be. This claim is later verified using a numerical example. Therefore, since the worker's effective bargaining power $\tilde{\eta}$ depends positively on μ , it is also decreasing in the capital-labour ratio.

Consider now the effect of $\tilde{\eta}$ on wages. Since the worker's effective bargaining power increases when the firm becomes more constrained, it is clear that its direct effect on wages is positive. This effect dictates that wages should increase when the firm is experiencing financial difficulties because workers become more valuable. However, there is a second effect that the worker's endogenous bargaining weight has

on wages. This effect appear through the term $w_l(k, l, A)l$, which becomes different in the presence of credit frictions compared to the case of pure search frictions. The reason for this is precisely the change in bargaining weights, which are endogenous to employment under credit frictions.

The term $w_l(k, l, A)l$ tells how wages of the remaining workers change if an additional worker is hired. In the pure search frictions case, this term is negative due the diminishing marginal product of labour ($w_l(k, l, A) < 0$). The intuition is that, if negotiations with a given worker break down, the marginal product of labour of the remaining workers will increase, and so must their wages. Therefore, the higher is the term $-w_l(k, l, A)l$, the more the marginal product of the remaining workers will increase, the higher wage the firm must pay to the given worker.

Under search and credit frictions, the term $w_l(k, l, A)l$ is also negative, as will be verified in a numerical example, but it is different because it takes into account the change in workers' effective bargaining power.

Imagine that there exist values of k and l for which the change in the wage bill (if an extra worker is hired) under search and credit frictions $w_l^{sc}(k, l, A)l$ is equal to the change in wage bill change under pure search frictions $w_l^s(k, l, A)l$. Now consider how they compare if the firm's capital increases. Under pure search frictions, the increase in capital implies that the marginal product of labour is higher, so $-w_l^{sc}(k, l, A)l$ becomes more positive. This has a positive effect on wages.

In contrast, under search and credit frictions the increase in capital also implies that workers' effective bargaining power decreases because the profit constraint binds less. We shall see that the bargaining power effect outweighs the marginal product effect in the numerical example. Thus, the change in the firm's wage bill if an extra worker is hired is smaller under search and credit frictions than under pure search frictions. Mathematically, if for given k_1 and l_1 $-w_l^{sc}(k_1, l_1, A)l = -w_l^s(k_1, l_1, A)l_1$,

then $-w_l^{sc}(k_2, l_1, A)l_1 < -w_l^s(k_2, l_1, A)l_1$ for $k_2 > k_1$. Thus, the term $-w_l^{sc}(k_2, l_1, A)l_1$ makes wages smaller under search and credit frictions.

Consider now the opposite scenario. Imagine that the firm's capital decreases. Then, under pure search frictions the marginal product of labour decreases, so the term $-w_l^s(k, l, A)l$ becomes less positive, which has a negative effect on wages. Under search and credit frictions, the worker's effective bargaining power increases, therefore, in case of the breakdown of negotiations with a given worker, the firm will have to pay higher wages to the remaining workers. So the term $-w_l^{sc}(k, l, A)l$ becomes more positive. Mathematically, if for given k_1 and l_1 $-w_l^{sc}(k_1, l_1, A)l_1 = -w_l^s(k_1, l_1, A)l_1$, then $-w_l^{sc}(k_3, l_1, A)l_1 > -w_l^s(k_3, l_1, A)l_1$ for $k_3 < k_1$.

Thus, for different values of k and l the term $w_l(k, l, A)l$ might have different effect on wages. Under some combinations of capital and labour it may result in higher wages compared to the pure search frictions case. Under some combinations of capital and labour it may result in lower wages compared to the pure search frictions case.

To sum up, wages under search and credit frictions are affected by endogenous effective workers' bargaining power. The bargaining power has a direct implications for wages which is that the worker's position in wage negotiations strengthens. It also has an indirect effect on wages through the term $w_l(k, l, A)l$ which states how much wages of the remaining workers change if negotiations with a given worker break down. This term has an ambiguous effect on wages.

In the next section, I consider a numerical example which illustrates the effects discussed in this section.

3.5 A numerical example

3.5.1 Solution method

I have not been able to derive an analytical solution to the differential equation for wages under credit and search frictions (3.33). One can verify that simple analytical forms, such as isoelastic, do not solve (3.33). I therefore explore numerical solutions to this equation, imposing the same boundary condition as for the differential equation for wages under pure search frictions.

Firstly, in order to simplify the solution to (3.33) I use the Cobb-Douglas production function $y = Ak^\alpha l^{1-\alpha}$ and rewrite the equation in terms of the capital-labour ratio κ . This gives the following ordinary differential equation:

$$\omega(\kappa, A) = \frac{\eta(1 + \mu(1 - \bar{c}))}{1 + \eta\mu(1 - \bar{c})} [(1 - \alpha)Ak^\alpha + \kappa\omega(\kappa, A) + \beta c\theta] + \frac{(1 - \eta)u}{1 + \eta\mu(1 - \bar{c})}. \quad (3.38)$$

I then use (3.37) to substitute for μ . Derivations and the resulting differential equation are presented in Appendix A.

As for the boundary condition, note that the condition (3.20) could be rewritten in terms of the capital-labour ratio as well:

$$\lim_{\kappa \rightarrow \infty} \frac{\omega(\kappa, A)}{\kappa} = 0. \quad (3.39)$$

Because the boundary condition (3.20) should hold for any value of capital, conditions (3.20) and (3.39) imply the same solution for wages.

Finally, the results presented in the next section hold for a reasonable set of parameter values reported in Appendix B.

3.5.2 Numerical results

Consider firstly the values of the Lagrange multiplier on the profit constraint $\mu(\kappa, A)$ and the values of the worker's effective bargaining power $\tilde{\eta}(\kappa, A)$ implied by the solution to (3.38). These are presented in Figure 3.5.1 with μ (left panel) and $\tilde{\eta}$ (right panel) on the vertical axis and κ on the horizontal axis. The considered range of values for κ ensures, first, that the profit constraint binds for all values of κ in this range, and second, that the resulting wages are potential equilibrium candidates.⁵

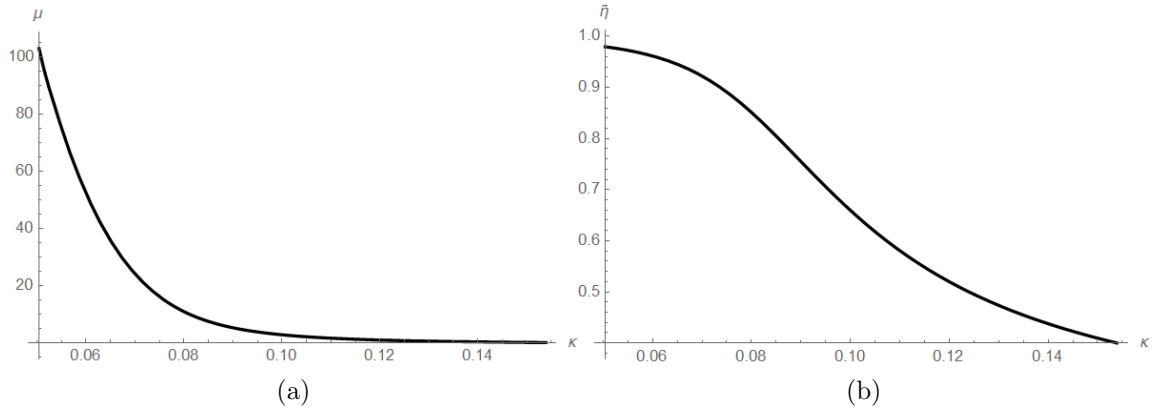


Figure 3.5.1: The Lagrange multiplier on the profit constraint $\mu(\kappa)$ (left) and the workers' effective bargaining power $\tilde{\eta}(\kappa)$ (right).

It could be seen from Figure 3.5.1 that both μ and $\tilde{\eta}$ are decreasing in the capital-labour ratio. The reason is that for a given number of workers, if the firm has more capital (resulting in higher κ), the profit constraint binds less. Therefore, as κ increases, the Lagrange multiplier on the profit constraint decreases.

It was shown in the previous section that the worker's effective bargaining power $\tilde{\eta}$ is an increasing function of μ . Thus, in Figure 3.5.1b as capital-labour ratio increases, μ decreases and $\tilde{\eta}$ decreases as well. When μ reaches zero, $\tilde{\eta}$ reaches 0.4, which is a chosen value for the worker's bargaining power η .

⁵Workers are willing to accept these wages because these are higher than the value of unemployment benefit.

This shows the direct effect of $\tilde{\eta}$ on wages. When a firm is more constrained, its marginal surplus of a match increases. This allows the workers to negotiate higher wages as if they had a higher bargaining power.

Consider now the second effect of the worker's endogenous effective bargaining weight which is contained in the term $w_l(k, l, A)l$. Note that rewritten in terms of the capital-labour ratio $w_l(k, l, A)l$ becomes $-\kappa\omega'(\kappa, A)$. Figure 3.5.2 depicts $-w_l(k, l, A)l = \kappa\omega'(\kappa) > 0$ as a function of κ . The solid line represents the values of this term under search and credit frictions, whereas the dashed line represents its values under pure search frictions.

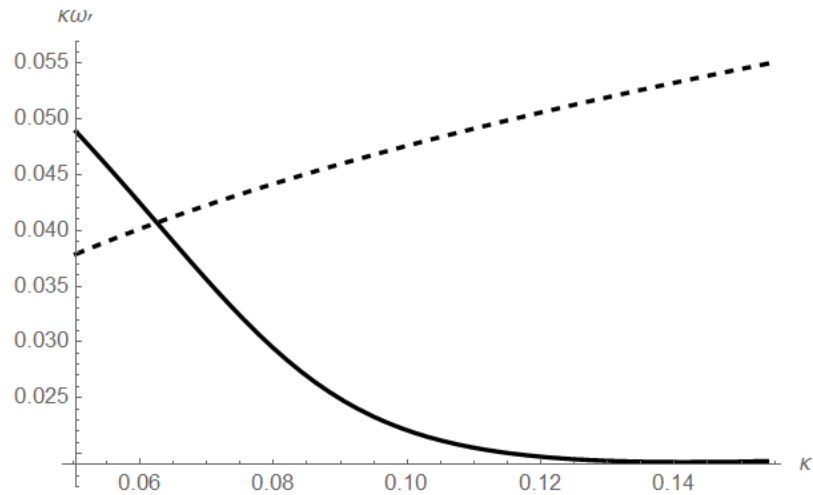


Figure 3.5.2: $\kappa\omega'(\kappa)$ under search and credit frictions (solid) and under pure search frictions (dashed).

The values of $\kappa\omega'(\kappa)$ coincides for $\kappa \approx 0.065$. To the right of this value, the worker's effective bargaining power decreases, so if the firm's negotiations with a given worker breaks down, the remaining workers will have less bargaining power, and the change in the firm's wage bill would be smaller than it would have been under pure search frictions.

To the left of the intersection of the two curves in Figure (3.5.2), the worker's effective bargaining power increases. The change in the firm's wage bill if a worker

walks away will be larger than it would have been under pure search frictions because the remaining workers' effective bargaining power increases.

Now consider a combined influence of the two effects on wages presented in Figure 3.5.3. Figure 3.5.3 depicts wages under search and credit frictions (solid) and wages under pure search frictions (dashed) as functions of the capital-labour ratio.

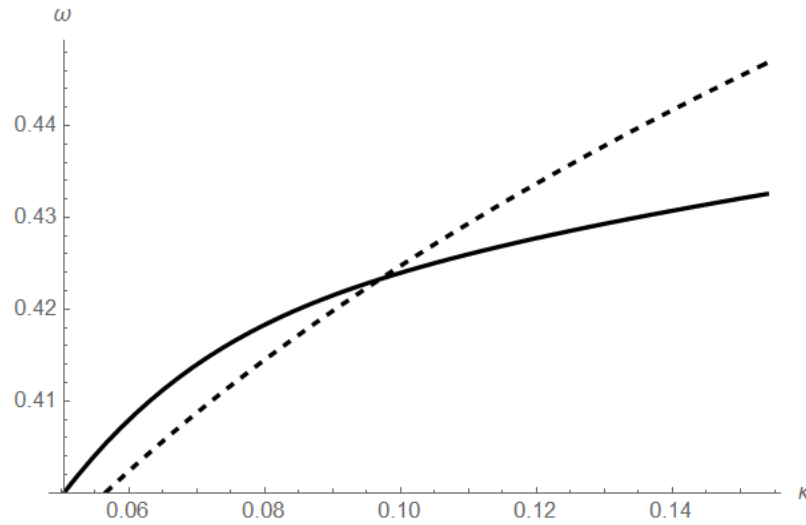


Figure 3.5.3: Wages under search and credit frictions (solid) and under pure search frictions (dashed).

One could see that for different values of the capital-labour ratio different effects dominate. When κ is smaller than around 0.095, the firm's profit constraint binds considerably, and the worker's bargaining power is very high. Thus, they are able to negotiate higher wages than under pure search frictions. When κ is higher than 0.095, the firm is able to pay lower wages than under pure search frictions because, if the worker walks away, the change in the wages of the remaining workers will be much smaller than under pure search frictions due to a decrease in their bargaining weight.

Overall, it could be concluded that the effect of credit frictions on wages is ambiguous.

3.6 Conclusions and discussion

In this paper I considered wages that are determined by surplus sharing between firms and workers in the environment with both search and credit frictions. In the model firms face credit constraints which affect their ability to invest in capital. They also face standard search and matching frictions in the labour market.

I have shown that financial constraints affect wages by changing bargaining weights of firms and workers but the overall effect on wages is ambiguous.

There are several extensions to this paper that are left for future work. The first one is to solve for steady-state equilibrium with wages determined by surplus sharing and compare the steady-states equilibrium values of labour market tightness, unemployment, vacancies to those under pure search frictions.

The second extension would be to look into out-of-steady-state effects on wage solution. Consider the differential equation for wages under search and credit frictions (3.31) and the differential equation for wages under pure search frictions (3.18). The two equations coincide when the worker's bargaining power is equal to zero. When $\eta = 0$ both equation imply that wages are equal to the unemployment benefit. Of course, if the worker does not have any bargaining power then the firm has no incentive to pay her anything more that what she receives being unemployed.

Now consider the case where the worker's bargaining power is equal to 1. Then under search and credit frictions the worker receives

$$w(k_{-1}, l, A) = F_l(k_{-1}, l, A) - w_l(k_{-1}, l, A) + \beta c E \theta' \frac{1 + \mu'(1 - \bar{c})}{1 + \mu(1 - \bar{c})}. \quad (3.40)$$

Note that in a deterministic steady state where $\mu = \mu'$ this wage again coincides with the wage the worker receives under pure search frictions. In this case, the worker receives all of the firm's surplus of the match. Out of steady state, wages are higher

relative to steady state if the profit constraint binds tomorrow more than today. In this case, paying hiring costs tomorrow is very costly for the firm because these costs significantly decrease its future net worth. The firm is better off hiring more workers today, its labour demand increases, and the firm has to pay higher wages. On the other hand, if the profit constraint binds more today than tomorrow, it is relatively cheap to hire workers tomorrow, so today's labour demand decreases, and so do wages.

For a non-trivial case with $0 < \eta < 1$ the out-of-steady-state effects on wage solution are not clear. These require extra attention and are left for future research.

The final extension of this paper would be to examine out-of-steady-state equilibrium transition dynamics of the economy with wages based on surplus-sharing.

One may wonder how this paper relates to the research that focuses on the idea of financially constrained firms borrowing from workers through decreasing their wages. Perhaps the most well-known paper that exploits this idea is the one by Michelacci and Quadrini (2009).⁶ In their model heterogeneous firms sign optimal long-term wage contracts with workers which they hire in the competitive labour market. Each firm starts with an amount of initial wealth which is not enough to pay for desired capital investment. Thus, it borrows from external investors on a credit market which is subject to frictions. These frictions result in the firm being financially constrained. However, as it accumulates more and more wealth over time, its need for external borrowing declines. At some point the credit constraint stops binding, and the firm becomes unconstrained. In the described scenario firms prefer to pay lower wages at the beginning of their life cycle, when they are small and constrained,

⁶Another paper which is related to this idea is the one by Wang (2015). The model in that paper is based on a mechanism very similar to the one described in Michelacci and Quadrini (2009). For empirical evidence of constrained firms borrowing from workers see, for example, Guiso et al. (2013) who use the degree of development of financial markets in Italy to understand the role of firms as internal credit markets.

in exchange for higher wages later on, when they are large and unconstrained. Thus, in this sense, firms borrow from workers by offering an increasing wage profile.

The work in this Chapter, and its future extensions, differs from Michelacci and Quadrini (2009) along several dimensions. The model in this thesis characterises the decisions of a representative firm that is always credit-constrained in steady state. Wages are determined by splitting joint firm-worker surpluses which arise due to the presence of search frictions in the labour market. In deterministic steady-state environment the resulting wages are, in contrast to Michelacci and Quadrini (2009), constant over time, there is no borrowing from workers in steady state. However, these wages are different from wages that workers would receive if there were no credit frictions. This allows me to compare these two wage functions as functions of capital-to-labour ratios in this paper, and will allow me to compare them in steady-state equilibrium in the future.

In my work, despite wages being constant in steady state, firms' borrowing from workers is possible out of steady state. This is because wages do depend on realizations of the productivity shock in my model. This is not only because the latter affects workers' productivity, but also because it affects firms' net worth. Depending on whether the shock today is good or bad wages adjust in some way. These wage adjustments could potentially be interpreted as firms' borrowing from workers.

In contrast, in Michelacci and Quadrini (2009) under optimal contracts and i.i.d. shocks neither employment nor wages depend on productivity shocks. This happens because firms are insured from bad shocks by risk-neutral investors. Firms may make smaller debt payments in bad times and larger debt payments in good times. Hence, Michelacci and Quadrini (2009) focus on steady-state transition of firm from being constrained to being unconstrained. The change in wages comes from the variation in firms' life cycle in their paper. As opposed to this, I consider changes in wages that

come from the variation in aggregate productivity. My focus is on constrained firms, their labour policy, and on how wages based on surplus-sharing adjust in response to productivity shocks.

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Appendix

A Wage derivation

Assume the Cobb-Douglas production technology $F(k_{-1}, l) = k_{-1}^\alpha l^{1-\alpha}$. In steady state μ satisfies

$$(1 + \mu(1 - \bar{c}))\beta(A\alpha k^{\alpha-1}l^{1-\alpha} - w_k(k, l, A)l) = (1 + \mu) \left[q - \frac{(1 - \delta)q}{R} \right], \quad (3.41)$$

so

$$\mu = - \frac{\beta(A\alpha k^{\alpha-1}l^{1-\alpha} - w_k(k, l, A)l) - q \left(1 - \frac{(1-\delta)}{R} \right)}{(1 - \bar{c})\beta(A\alpha k^{\alpha-1}l^{1-\alpha} - w_k(k, l, A)l) - q \left(1 - \frac{(1-\delta)q}{R} \right)}. \quad (3.42)$$

Wage in steady state

$$w(k, l, A, \mu) = \frac{\eta(1 + \mu(1 - \bar{c}))}{1 + \eta\mu(1 - \bar{c})} [(1 - \alpha)Ak^\alpha l^{-\alpha} - w_l(k, l, A)l + \beta c\theta] + \frac{(1 - \eta)u}{1 + \eta\mu(1 - \bar{c})}. \quad (3.43)$$

$$1 + \eta\mu(1 - \bar{c}) = 1 - \frac{\eta(1 - \bar{c}) \left[\beta(A\alpha k^{\alpha-1}l^{1-\alpha} - w_k(k, l, A)l) - q \left(1 - \frac{(1-\delta)}{R} \right) \right]}{(1 - \bar{c})\beta(A\alpha k^{\alpha-1}l^{1-\alpha} - w_k(k, l, A)l) - q \left(1 - \frac{(1-\delta)}{R} \right)}. \quad (3.44)$$

Common denominator

$$1 + \eta\mu(1 - \bar{c}) = \frac{(1 - \eta)(1 - \bar{c})\beta(A\alpha k^{\alpha-1}l^{1-\alpha} - w_k(k, l, A)l) - (1 - \eta(1 - \bar{c}))q \left(1 - \frac{(1-\delta)}{R} \right)}{(1 - \bar{c})\beta(A\alpha k^{\alpha-1}l^{1-\alpha} - w_k(k, l, A)l) - q \left(1 - \frac{(1-\delta)}{R} \right)}. \quad (3.45)$$

So

$$\frac{1}{1 + \eta\mu(1 - \bar{c})} = \frac{(1 - \bar{c})\beta(A\alpha k^{\alpha-1}l^{1-\alpha} - w_k(k, l, A)l) - q \left(1 - \frac{(1-\delta)}{R} \right)}{(1 - \eta)(1 - \bar{c})\beta(A\alpha k^{\alpha-1}l^{1-\alpha} - w_k(k, l, A)l) - (1 - \eta(1 - \bar{c}))q \left(1 - \frac{(1-\delta)}{R} \right)}. \quad (3.46)$$

Now

$$\eta(1+\mu(1-\bar{c})) = \eta \frac{\eta(1-\bar{c}) \left[\beta(A\alpha k^{\alpha-1} l^{1-\alpha} - w_k(k, l, A)l) - q \left(1 - \frac{(1-\delta)}{R}\right) \right]}{(1-\bar{c})\beta(A\alpha k^{\alpha-1} l^{1-\alpha} - w_k(k, l, A)l) - q \left(1 - \frac{(1-\delta)}{R}\right)}. \quad (3.47)$$

Common denominator

$$\eta(1+\mu(1-\bar{c})) = \frac{-\eta\bar{c}q \left(1 - \frac{(1-\delta)}{R}\right)}{(1-\bar{c})\beta(A\alpha k^{\alpha-1} l^{1-\alpha} - w_k(k, l, A)l) - q \left(1 - \frac{(1-\delta)}{R}\right)}. \quad (3.48)$$

Therefore,

$$\frac{\eta(1+\mu(1-\bar{c}))}{1+\eta\mu(1-\bar{c})} = \frac{-\eta\bar{c}q \left(1 - \frac{(1-\delta)}{R}\right)}{(1-\eta)(1-\bar{c})\beta(A\alpha k^{\alpha-1} l^{1-\alpha} - w_k(k, l, A)l) - (1-\eta(1-\bar{c}))q \left(1 - \frac{(1-\delta)}{R}\right)}. \quad (3.49)$$

Thus, the wage

$$\begin{aligned} w(k, l, A) &= \frac{-\eta\bar{c}q \left(1 - \frac{(1-\delta)}{R}\right) [(1-\alpha)A\kappa^\alpha l^{-\alpha} - w_l(k, l, A)l + \beta c\theta]}{(1-\eta)(1-\bar{c})\beta(A\alpha k^{\alpha-1} l^{1-\alpha} - w_k(k, l, A)l) - (1-\eta(1-\bar{c}))q \left(1 - \frac{(1-\delta)}{R}\right)} + \\ &+ \frac{(1-\bar{c})\beta(A\alpha k^{\alpha-1} l^{1-\alpha} - w_k(k, l, A)l) - q \left(1 - \frac{(1-\delta)}{R}\right)}{(1-\eta)(1-\bar{c})\beta(A\alpha k^{\alpha-1} l^{1-\alpha} - w_k(k, l, A)l) - (1-\eta(1-\bar{c}))q \left(1 - \frac{(1-\delta)}{R}\right)} (1-\eta)u. \end{aligned} \quad (3.50)$$

Let $w(k, l) = \omega(\kappa)$, where $\kappa = \frac{k}{l}$. Thus, $w_k(k, l)l = \omega'(\kappa)$ and $w_l(k, l)l = -\kappa\omega'(\kappa)$.

$$\begin{aligned} \omega(\kappa) &= \frac{-\eta\bar{c}q \left(1 - \frac{(1-\delta)}{R}\right) [(1-\alpha)A\kappa^\alpha + \kappa\omega'(\kappa) + \beta c\theta]}{(1-\eta)(1-\bar{c})\beta(A\alpha\kappa^{\alpha-1} - \omega'(\kappa)) - (1-\eta(1-\bar{c}))q \left(1 - \frac{(1-\delta)}{R}\right)} + \\ &+ \frac{(1-\bar{c})\beta(A\alpha\kappa^{\alpha-1} - \omega'(\kappa)) - q \left(1 - \frac{(1-\delta)}{R}\right)}{(1-\eta)(1-\bar{c})\beta(A\alpha\kappa^{\alpha-1} - \omega'(\kappa)) - (1-\eta(1-\bar{c}))q \left(1 - \frac{(1-\delta)}{R}\right)} (1-\eta)u. \end{aligned} \quad (3.51)$$

B Parameter values for numerical example

The time period is taken to be equal to one week. I choose values for $\beta, \alpha, \bar{c}, \delta_k, \delta_l$ according to the calibration strategy in Chapter 2.

I set the value of β equal to the firm's internal rate of return, $\beta = 0.998$, as in Iacoviello (2005). I assume a conventional capital share of output, equal to $1/3$. I set the value of \bar{c} , the minimum dividend share of gross profit, equal to 30% which is a median of the dividend payout ratio (ratio of dividends paid to net income) across S&P500 firms over the last 10 years.⁷ I choose capital depreciation rate to be equal to 10% a year, or 0.19% a week. I set the value of the job destruction rate δ_l to be equal to 0.0078, which is consistent with estimates in Shimer (2012).

I normalize the productivity level A equal to 1. The interest rate R is set to its equilibrium rate equal to the inverse of the household's discount factor, 0.999 as in Iacoviello (2005).

To parameterize workers' bargaining power η I calibrate the model with search frictions only with wages determined by bargaining to match an elasticity of wages to output per worker of 0.985, as in Pissarides (2009). This results in η being around 0.4.

In Chapter 2 I calibrated the cost of opening a vacancy c to be equal to 29% of wages. Here, I solve for wages as a function of capital-labour ratios. The resulting wages range from 0.4 to 0.44 for κ in range from 0.06 to 0.15. I keep c to be equal to around 29% of this wage range.

Hall and Milgrom (2008) suggest a calibration of the value of leisure u of around 70% of average labour productivity. In my model average labour productivity can be written as $A\kappa^\alpha$, which changes for different values of κ . I set u equal to 0.4 which not too far from 70% of the average product of labour for this range of capital-labour

⁷http://www.factset.com/websitefiles/PDFs/dividend/dividend_9.28.15

ratios.

Finally, to illustrate the point of wage ambiguity I set the value for labour market tightness equal to 1.05 and the value for the price of capital to 380. These two parameters are equilibrium objects. The two chosen values are not too far from potential equilibrium values, as indicated by the calibration of the model with search frictions only.

Table 3.6.1: Parameter values

Description	Parameter	Value
Firm's discount factor	β	0.998
Capital share	α	1/3
Minimum dividend share	\bar{c}	0.3
Job destruction rate	δ_l	0.0078
Capital depreciation rate	δ_k	0.002
Productivity level	A	1
Interest rate	R	1.00077
Vacancy cost	c	0.1
Value of leisure	u	0.4
Capital price	q	380
Vacancy-unemployment ratio	θ	1.05