ABSTRACT

Title of dissertation: INFORMATIONAL FRICTIONS IN MACROECONOMICS

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This dissertation explores the role of informational frictions in macroeconomics and highlights how these frictions influence micro-level decisions which, when aggregated, result in more volatile macroeconomic fluctuations.

Chapter 1 explores the role of costly information and pricing complementarities in explaining persistent effects of monetary policy and behavior of prices at the micro level. In a setting where firms plan on when to acquire costly information, strategic complementarities in pricing generates planning complementarities. This results in a sluggish response of prices to monetary policy shocks. The calibrated model matches frequent and large price changes along with substantial non-neutralities. The chapter analyzes the effectiveness of monetary policy in the US since the 1970's and finds that it was relatively less effective in the 1970's compared to the subsequent decades.

Chapter 2 explores the role of dispersed opinions about economic conditions in reinforcing economic fluctuations and the role of policy to curtail these fluctuations.

Output fluctuations arising from optimism and pessimism are often believed to be inconsistent with rational expectations. I show that dispersed information together with strategic complementarity, can give rise to endogenous cycles of pessimism and optimism which amplify these fluctuations. In the model, agents try to infer both true fundamentals and what others perceive the fundamentals of the economy to be, from both private signals and common prices. More precisely, agents Forecast the forecasts of others. Correlated forecast errors mimic pessimism and optimism. These endogenously generated correlated forecast errors interact with production and investment decisions to cause volatile swings in both current and future output. Three key results emerge. First, an economy with dispersed information features amplified output fluctuations relative to the full information economy. Importantly, prices reinforce sentiments which in turn generate more volatile and persistent fluctuations. Second, in an otherwise neoclassical economy, dispersed information implies that the perception of aggregate demand matters in the output decision of firms. Finally, these fluctuations are inefficient and aggregate demand management through pro-cyclical payroll taxes or counter-cyclical sales subsidies can be reduce volatility and improve welfare without the need for the policy maker to have an informational advantage over the private sector.

INFORMATIONAL FRICTIONS IN MACROECONOMICS

by

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Dedication

To my family

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Table of Contents

List of Tables

List of Figures

Chapter 1: Costly Information, Planning Complementarities and the Phillips Curve

1.1 Introduction

Price stickiness is often assumed in macroeconomic models as a way of generating a Phillips curve relationship, i.e. a positive relationship between inflation and output in the short run. Previous literature, dating as far back as Phelps (1970) and Lucas Jr. (1972), has stressed the importance of informational frictions to explain this relationship.[1] More recently, the *sticky information* literature (Mankiw and Reis, 2002; Reis, 2007), following Calvo (1983), the literature has assumed that only a fraction of firms get endowed with up-to-date information exogenously every period. As a result of this *sticky* information, only a fraction of firms have full information when making their pricing decision and this results in a delayed response of prices to shocks.

Recent micro level pricing studies suggest that prices are not sticky. Bils and Klenow (2004) and Klenow and Kryvtsov (2008) show that the median non-housing

[1] Other recent approaches to similar problems include work by Mackowiak and Wiederholt (2009) who draw on the concept of Rational Inattention to explain the sluggish movement of prices. Woodford (2002) shows how dispersed information can result in persistent effects of monetary policy.

consumer price changes once every 4.3 months. Conditional on a price change, the mean size of a price change is large in absolute terms, at about 11 percent. However, many macro level studies (Christiano et al., 1999; Uhlig, 2005) find support for a role of sticky prices in the monetary transmission mechanism. Uhlig (2005) finds that only about 25 percent of the long-run response of the U.S GDP price deflator to a monetary policy shock occurs within the first year after the shock.

The sticky information approach, by exploiting dispersed information, squares well with the macroeconomic evidence on the effects of nominal shocks but struggles to explain frequently changing and volatile prices at the micro level as it assumes an exogenous arrival rate of new information. In this paper, I present an endogenous mechanism which is able to rationalize frequent and large price changes at the micro level amid sluggish response in the aggregate price level to monetary shocks. The driving force behind this mechanism centers around the strategic motives of firms in choosing when to acquire new information. In the model, firms are allowed to change prices at zero cost but face a positive cost in updating their information, as in Reis (2007).

The basic mechanism can be summarized as follows. Strategic complementarity in pricing results in a strategic complementarity in the decision about when to acquire new information about the state of monetary policy, but **not** in the decision to acquire information about idiosyncratic productivity. This complementarity in planning results in a delay in the acquisition of information about monetary policy, but not about idiosyncratic productivity.[2]

[2]This mechanism is distinct from the model of delay presented in Caballero (1999) which em-

When firm i gets new information about a positive nominal shock, its price response is not only dependent on the true state of monetary policy, but is also contingent on how other firms react to this nominal shock. If other firms do not adjust their prices, then it is not optimal for firm i to unilaterally increase its price, as relative prices matter for profit. In a symmetric staggered stationary equilibrium, a firm that chooses to observe new information about a recent monetary shock, is forced to temper its price changes to compensate for the large fraction of firms that remain uninformed about this shock and whose prices therefre have not adjusted. This, in turn, diminishes the benefit of obtaining information about the shock today, as the firm will only find it optimal to fully act on this information in the future when a majority of firms have updated their information. As the firm faces an upfront cost of acquiring information today while the benefit is accrued only in the future, the firm has an incentive to delay the acquisition of information.

In addition, strategic complementarity in planning reinforces this decision to delay information acquisition about monetary policy. If all other firms update information about the monetary state infrequently, then every firm has an incentive to delay the acquisition of costly information about the monetary state. The aggregate price moves sluggishly and does not track money supply well when a large mass of firms remains uninformed. A sluggish aggregate price, together with pricing complementarity, imply that the difference between the firm's current price and its target price is small. Thus, the loss from remaining uninformed and mis-pricing is small. Small losses from mis-pricing and delayed benefits from acquiring information give

phasizes information externalities in causing delay.

rise to planning complementarities which in turn lead to delayed information acquisition about the monetary state. Overall, prices increase less than proportionately on impact after a monetary shock. Prices catch up eventually when all firms update and incorporate the information about the monetary shock into their prices.

However, strategic complementarity in planning does not extend to a firm's decision to update its information about idiosyncratic productivity. A firm's benefit from being informed about their idiosyncratic state is not contingent on other firms' information about its individual productivity. Firms, thus optimally, update information about their idiosyncratic productivity often, and prices respond fully to the size of these firm-specific shocks. I show that, even if both the cost of acquiring information about monetary policy and the idiosyncratic productivity and the volatility of monetary shocks and idiosyncratic productivity are the same, firms will optimally prioritize acquiring new information about their idiosyncratic productivity. Thus, prices will reflect new information about idiosyncratic shocks more frequently. Consequently, prices change often and by large amounts in response to idiosyncratic shocks, while responding sluggishly to monetary policy shocks.

The sticky information literature, by assuming an **exogenous arrival rate** for new information, cannot generate this planning complementarity and hence struggles to explain this differential rate of adjustment to idiosyncratic productivity and monetary shocks. Consequently, the literature fails to offer an explanation that can account for both micro and macro pricing facts.

The importance of information processing in firms can be seen in Radner (1992), who points out that a large proportion of the workforce employed in American firms is employed for the purpose of information processing. There is substantial direct and indirect evidence that prices set by firms do not reflect *up to date* information. Klenow and Willis (2007) find that price changes in CPI data reflect older information, while Zbaracki et al. (2004) find that the costs of acquiring and processing information are quantitatively about six times larger than *physical* costs of changing prices. This evidence justifies the choice of focusing on the cost of processing information rather than the cost of changing prices.

I show that pricing models with costly information perform better than the standard time dependent or state dependent menu cost models in matching features of micro data and at the same time account for the sluggish reaction of prices to monetary shocks. Klenow and Kryvtsov (2008) conclude that existing standard pricing models in macroeconomics cannot match both macro and micro facts convincingly. Time-dependent models (Calvo, 1983; Yun, 1996) can explain the sluggish price level only if firms change prices infrequently and by *small* amounts. These requirements are at odds with micro data. The other popular alternative are state-dependent *menu* cost models (Dotsey et al., 1999; Golosov and Lucas Jr., 2007). Golosov and Lucas Jr. (2007) calibrate such a model to match micro pricing facts and conclude that menu cost models are unable to generate persistent real responses to monetary shocks. Moreover, Midrigan (2008) points out that menu cost models struggle in generating small price changes. This is at odds with the empirically observed distribution of price changes, which has been found to have numerous small changes. Some recent *next generation* state dependent models like Midrigan (2008) and Gertler and Leahy (2008) get around this problem by assuming *economies of* *scope* and Poisson shocks, respectively. The model in this paper generates a large number of small price changes by not requiring firms to incur a physical cost of changing prices. A calibrated model matches the micro pricing facts, it is still able to generate a persistent response of output to monetary shocks.[3]

Mankiw and Reis (2010) highlight that: (1) firms change prices *all the time*, (2) firms set price schedules over time rather than prices at each instant and (3) sometimes these schedules are flat. This paper is able to capture all these aspects. I show that the model is capable of accounting for firms setting price plans over time as in Burstein (2006). Unlike Burstein (2006), I do not impose a physical cost of changing price schedules.

From a methodological point of view, the paper presents a model where aggregation is tractable. Modeling the information acquisition decision as choosing the duration of inattentiveness allows one to utilize the tractability that hierarchical information structures offer in aggregating models with dispersed information (Townsend, 1983). The resulting hierarchical information structure allows an easy and exact solution to the problem of *forecasting the forecasts of others*. This problem in general results in infinite dimensional state spaces which makes it difficult to find exact solutions. In addition, the solution to my model can be split into two parts, allowing the information choice to be solved as a deterministic control problem. This is similar methodologically to recent papers like Reis (2011), who decomposes a ra-

[3]Hellwig and Venkateswaran (2009) also present a model with endogenous information choice. On calibrating their model to match micro pricing moments, their model is also unable to generate substantial persistence in the response of output to a monetary shock. However, they allow firms to learn from past market generated information. In addition, the firms are not allowed to receive independent signals about only one type of shock. This reduces the persistence of the effect of monetary policy.

tional inattention problem into a stochastic part and a fully deterministic control problem.

In addition to the calibration exercise, I conduct an empirical study in the spirit of Lucas Jr. (1973). Instead of looking at cross-country differences in the effectiveness of monetary policy, I examine how the effectiveness of monetary policy has changed since 1969. The 1970's were a period of many monetary 'mistakes' (Romer and Romer, 2002). Monetary policy actively tried to exploit the Phillips curve relation and was more discretionary than in the subsequent decades. The model predicts that the central bank has a tradeoff between the extent of discretionary policy changes and effectiveness of monetary policy. If there are frequent shocks to monetary policy which are also large in magnitude, firms would choose to remain inattentive for shorter durations. As a result, the time before which all firms get informed about a shock to monetary policy will be short. The aggregate price will reflect monetary shocks as soon as all firms update their information and hence the effect of monetary shocks on output dies out quicker. Estimating the model consistent Phillips Curve separately over subsamples covering the 1970's and the subsequent decades enables estimation of the duration for which firms choose to remain inattentive to changes in monetary policy. By looking at the estimates of this duration over the two subsamples, I can infer that firms updated their information more frequently during the 1970's than in later decades. Hence monetary policy had a more persistent effect on output in later decades than during the 1970's.

The paper is arranged as follows. In the next section, I present the basic model. Section 3 contains a description of the equilibrium under costly and costless information. Section 4 contains the main analytical results. Section 5 presents the calibration strategy and discusses the quantitative performance of the calibrated model. I present the results of the empirical study in Section 6. I conclude in Section 7.

1.2 Model

The model combines features from Golosov and Lucas Jr. (2007), Hellwig and Veldkamp (2009) and Reis (2007). Time is continuous.[4] The economy consists of a representative household and a unit mass of ex-ante identical monopolistically competitive firms indexed by $i \in [0, 1]$ which produce differentiated goods. Each of the households are infinitely lived and consume goods produced by each monopolistically competitive firm. I do not model entry or exit of firms, and hence the mass of firms remains constant over time.

The economy is subject to two kinds of shocks: a monetary shock and firm-specific idiosyncratic shocks. The monetary authority is assumed to follow a money supply rule, which implies that the log of nominal money supply follows a Brownian motion with drift μ variance σ_m^2

$$d \ln M(t) = \mu dt + \sigma_m dW(t) \tag{1.1}$$

where $W(t)$ is a standard Brownian motion. μ is the steady state rate of inflation.

[4]The use of continuous time is to avoid multiple equilibria in the choice of the optimal planning horizons with strategic complementarity. See Hellwig and Veldkamp (2009) for details.

σ_m^2 can be thought as the degree of discretion of the monetary authority; a higher σ_m^2 corresponds to a higher degree of discretion as it means larger deviations from the deterministic component of the rule.

Firm specific productivity shocks $Z_i(t)$ are assumed to be i.i.d across firms and follow a mean reverting Ornstein-Uhlenbeck process with zero drift, rate of mean reversion η and variance σ_z^2 as in Golosov and Lucas Jr. (2007):

$$d \ln Z_i(t) = -\eta \ln Z_i(t) dt + \sigma_z B_i(t) \tag{1.2}$$

where $B_i(t)$ is a standard Brownian motion such that for $j \neq j'$, B_j and $B_{j'}$ are independent. Each B_j is also independent of W. Earlier work such as Lach and Tsiddon (1992) and Golosov and Lucas Jr. (2007) argue for the need of an idiosyncratic shock, since the average size of price changes is large and thus cannot be fully explained by average inflation, which is not large.

1.2.1 Representative Household's Problem

The household enjoys utility from consumption of a final good, leisure and from holding real balances.[5] Disutility from labor enters utility in a linear fashion as in Hansen (1985). The representative household's problem can be written as

[5] The log utility specification for real balances allows analytical tractability. Without this functional form, the analytical solution would not be possible. However, the predictions of the model do not change qualitatively.

choosing the sequence $\left\{C(t), n(t), M^D(t)/P(t)\right\}_{t=0}^{\infty}$ to maximize

$$E_0 \left\{ \int_0^{\infty} e^{-\rho t} \left[\frac{C(t)^{1-\gamma} - 1}{1 - \gamma} - \alpha n(t) + \ln \left(\frac{M^D(t)}{P(t)} \right) \right] dt \right\}$$

subject to its budget constraint

$$M(0) \geq E_0 \left\{ \int_0^{\infty} Q(t)[P(t)c(t) + R(t)M^D(t) - \omega(t)n(t) - \Pi(t)]dt \right\} \tag{1.3}$$

where $Q(t)$ is the shadow price of nominal cash flows, and $\Pi(t)$ includes the nominal profits from firms and lump sum transfers. $R(t)$ is the nominal interest rate and satisfies the following:

$$Q(t) = e^{\int_t^{t+dt} R(s)ds} E_t\{Q(t + dt)\}$$

$C(t)$ is the *Dixit-Stiglitz* consumption aggregator

$$C(t) = \left[\int_0^1 c_i(t)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

which aggregates consumption of a continuum of goods indexed $i \in [0, 1]$.

The first-order conditions with respect to $C(t)$, $n(t)$ and $M^D(t)$ can be written as

$$e^{-\rho t}C(t)^{-\gamma} = \lambda Q(t)P(t) \tag{1.4}$$

$$e^{-\rho t}\alpha = \lambda Q(t)\omega(t) \tag{1.5}$$

$$e^{-\rho t}\frac{1}{M^D(t)} = \lambda Q(t)R(t) \tag{1.6}$$

where λ is the multiplier on (1.3) and is independent of time. Utility maximization yields iso-elastic demand curves for each of the differentiated goods of the form

$$c_i(t) = C(t)\left(\frac{P_i(t)}{P(t)}\right)^{-\epsilon} \tag{1.7}$$

1.2.2 Firm's Problem

There is a continuum of ex-ante identical monopolistically competitive firms indexed by $i \in [0,1]$, each of whom produce a differentiated variety. Firm i's production technology can be described by a non-increasing returns to scale production function

$$y_i(t) = AZ_i(t)L_i(t)^{\theta} \tag{1.8}$$

where $A > 0$ is a constant and $\theta \in (0,1]$. $Z_i(t)$ is the firm-specific idiosyncratic productivity shock and $L_i(t)$ is the amount of labor that the firm hires in an economy-wide labor market from households at a wage rate $\omega(t)$ at date t.[6] Firm i's nominal

[6] I abstract from capital in the production process. Including capital accumulation decisions complicates the analysis by potentially raising issues of *Forecasting the Forecasts of Others* as in

profit, excluding the cost of information acquisition, at date t can be written as

$$\pi_i(t) = P_i(t)c_i(t) - \omega(t)L_i(t) = P_i(t)c_i(t) - \omega(t)\left(\frac{c_i(t)}{AZ_i(t)}\right)^{\frac{1}{\theta}}$$

Each firm faces a fixed labor cost F_m , if they decide to acquire information about the monetary shock and F_z if they *plan* about the idiosyncratic state. The firm then chooses its process of prices $\{P_i(t)\}_{t=0}^{\infty}$, and a process of planning dates $\{D_i^m(t), D_i^z(t)\}_{t=0}^{\infty}$ to maximize its expected discounted profits:

$$E_0^i\left\{\int_0^{\infty} Q(t)\pi_i(t)dt - F_m\int_0^{\infty} Q(t)\omega(t)dD_i^m(t) - F_z\int_0^{\infty} Q(t)\omega(t)dD_i^z(t)\right\} \quad (1.9)$$

taking as given $\{P(t), Q(t), \omega(t), C(t)\}_{t=0}^{\infty}$ and its information set at date 0. $dD_i^k(t) = 1$ refers to the firm's decision to plan about state $k \in \{m, z\}$ at date t, and $dD_i^k(t) = 0$ otherwise.

1.3 Equilibrium

Money Market Equilibrium. Money market equilibrium requires that $M^D(t) = M(t)$. The interest rate ensures that the demand for money equals the supply. Appendix A.1.1 shows that the nominal interest rate is constant in equilibrium.

$$R(t) = \rho + \mu - \frac{\sigma_m^2}{2} \ , \forall t \quad (1.10)$$

Townsend (1983). Despite not modelling capital explicitly, I can evaluate the effect of fixed factors by varying the returns to scale to labor. $\theta < 1$ or decreasing returns can be interpreted as the presence of a fixed factor.

Labor Market Equilibrium. Equations (1.5) and (1.6) imply the following relationship between equilibrium the wage rate $\omega(t)$ and money supply $M(t)$:

$$\omega(t) = \alpha R(t) M(t) \tag{1.11}$$

Thus, $\ln \omega(t)$ is also a Brownian motion with variance σ_m^2:

$$d \ln \omega(t) = d \ln M(t) = \mu dt + \sigma_m dW(t) \tag{1.12}$$

Goods Market Equilibrium. Equations (1.4) and (1.6), along with equation (1.10), imply that consumption depends on real money balances.

$$C(t) = \left[\frac{RM(t)}{P(t)} \right]^{\frac{1}{\gamma}} \tag{1.13}$$

Since there is no capital in the economy, the entire output is allocated either to consumption or to the resource cost of updating information. The resource constraint can be written as

$$Y(t) = C(t) + \chi(t) \tag{1.14}$$

where

$$\chi(t) = \frac{\omega(t) \left[F_m d\Gamma_t^m(t) + F_z d\Gamma_t^z(t) \right]}{P(t)}$$

represents the resource cost associated with the cost incurred by firms in updating their information. $d\Gamma_t^m(t)$ and $d\Gamma_t^z(t)$ represent the mass of firms updating their information about the aggregate and idiosyncratic states respectively and are formally defined in Section 1.3.2.

1.3.1 Costless Information

This is the case when $F_m = F_z = 0$ and will be referred to as the Costless Information case. Since information is free, firms update their information set at each instant and hence know the realizations of all shocks through the present.

Lemma 1. *In the Full Information case, firm i sets the optimal profit maximizing price given by*

$$\ln P_i^f(t) = \zeta \ln Z_i(t) + r \ln P^f(t) + (1-r) \ln M(t) \tag{1.15}$$

where $\zeta = \frac{-1}{\theta(1-\epsilon)+\epsilon}$ *and* $r = 1 - \frac{1+\gamma\theta-\theta}{\gamma(\theta(1-\epsilon)+\epsilon)}$

Proof. See A.1.2. □

In equation (1.15), r measures the strength of the strategic complementarity in pricing. Note that $\frac{\partial \ln P_i}{\partial \ln P} = r > 0$. If $r = 0$, then $\ln P_i(t) = \zeta \ln Z_i(t) + \ln M(t)$, i.e. there is no strategic complementarity. Strategic complementarity in pricing implies that a firm wants to set its price close to the average price.

Steady State: A steady state in this economy is defined by the case where there are no monetary or idiosyncratic productivity shocks, i.e., $\ln Z_i(t) = 0, \forall i \in$

$[0, 1]$ and $d \ln M(t) = \mu dt$. All real variables are constant in the steady state and nominal variables grow at a constant rate. In the steady state, all firms would want to set the same price at each instant such that

$$d \ln P_i^{ss}(t) = d \ln P^{ss}(t) = \mu dt$$

i.e. all prices increase at the rate of growth of money supply.

Proposition 1. *Classical Dichotomy: In the Full Information case, prices track nominal money balances. Monetary policy is neutral.*

Proof. Appendix A.1.3 shows that the aggregate price level tracks money supply perfectly. Equations (1.13) and (1.14) then demonstrate that real output is unaffected by a change in money supply. □

Each firm sets its price to exactly match the target price defined in equation (1.15). Prices reflect up-to-date information about the aggregate and idiosyncratic state at all points in time. Hence, prices adjust proportionally to changes in money supply such that nominal shocks have no effect on real output even in the short run; both the long and short run Phillips curves are vertical. Strategic complementarity in pricing has no impact in this costless information setting.

1.3.2 Costly Information

When information is costly to acquire, a firm's problem can be thought of as one in which it is trying to track a target price. In general, each firm knows the stochastic process of its target price, which is a function of the exogenously specified processes for the monetary and idiosyncratic productivity shocks. Since the firm observes all realizations of the exogenous shocks in the costless information case, it always knows its target price exactly. However, this is not the case when F_m and F_z are positive, i.e. when firms need to expend resources to collect and process new information. In such a setting, it is natural to think of a firm's decision as one in which it chooses to update its information intermittently. In the costly information case, the firm only observes a particular history of shocks when it incurs the associated cost, and hence can only observe its stochastically evolving target price when it updates its information. The expected lifetime loss to the firm from not updating its information set at each instant can be written as:

$$\begin{aligned} \mathcal{L} \;=\; & E_0^i \left\{ \int_0^\infty Q(t) \left[\pi(P_i^f(t); P(t), M(t), Z_i(t)) - \pi(P_i(t); P(t), M(t), Z_i(t)) \right] dt \right. \\ & \left. + \int_0^\infty Q(t)\omega(t)[F_m dD_i^m(t) + F_z dD_i^z(t)] \right\} \end{aligned} \tag{1.16}$$

Maximizing the objective in equation (1.9) is equivalent to minimizing the expression in equation (1.16). A second order Taylor expansion of the loss function in (1.16) around $P_i^f(t) - P_i(t) = 0$ yields

$$E_0^i \left\{ \int_0^\infty e^{-\rho t} [\ln P_i(t) - \ln P_i^f(t)]^2 dt + \int_0^\infty e^{-\rho t} [\mathcal{C}_m dD_i^m(t) + \mathcal{C}_z dD_i^z(t)] dt \right\} \tag{1.17}$$

where $\mathcal{C}_m = \frac{\alpha(\epsilon-1)F_m}{\epsilon}\left[\frac{\theta-\theta\epsilon+\epsilon}{\theta}\right]$ and $\mathcal{C}_z = \frac{\alpha(\epsilon-1)F_z}{\epsilon}\left[\frac{\theta-\theta\epsilon+\epsilon}{\theta}\right]$. $\mathcal{C}_k,\ k = m, z$ can be interpreted as the cost in terms of labor of acquiring and processing information about the state k.

The first term in equation (1.17) can be labeled as the *loss from mis-pricing* and can be interpreted as the loss from setting prices in an uninformed state. The second and third terms represent the resource costs that a firm must bear when it *plans*/collects and processes new information about the aggregate and idiosyncratic state respectively. Incurring the costs to update information reduces the loss from mis-pricing.

For ease of exposition in solving the model, define the following

Define: $p_i(t) = \ln P_i(t), p(t) = \ln P(t), m(t) = \ln M(t), z_i(t) = \ln Z_i(t)$ and $p^*(t) = rp(t) + (1-r)m(t)$.

From the first order condition of the minimization problem specified in equation (1.17), firm i that last planned at $(\hat{\tau}_m, \hat{\tau}_z)$ will set price:

$$p_i(t) = E\{\overline{p}(t) \mid \mathcal{I}_{\hat{\tau}_m}, \mathcal{I}^i_{\hat{\tau}_z}\} = E\{p^*(t) \mid \mathcal{I}_{\hat{\tau}_m}\} + \zeta E\{z_i(t) \mid \mathcal{I}^i_{\hat{\tau}_z}\} \tag{1.18}$$

where $\mathcal{I}_{\hat{\tau}_m} = \{m(s)\}_{s\leq\hat{\tau}_m}$ and $\mathcal{I}^i_{\hat{\tau}_z} = \{z_i(s)\}_{s\leq\hat{\tau}_z}$ and $\overline{p}_i(t) \equiv p^*(t) + \zeta z_i(t)$ is the target (log) price that firm i wants to set to maximize profit. Since the aggregate price is determined endogenously and depends on the decisions of all other firms, the target price is also endogenously determined. This distinguishes the current model from other research such as Bonomo et al. (2010) where this target price is exogenously specified. As mentioned earlier, the firm observes the true target price only when it simultaneously acquires information about both the monetary and idiosyncratic state. Acquiring information about the monetary state only allows the firm to calculate the first component of the target price, while acquiring information about its idiosyncratic productivity reveals the second component. On dates when the firm does not update either type of information, the firm sets a price equal to the expected target price given its most recent information.

As information acquisition is costly, firms choose not to update their information at each date. At date t, the economy is characterized by a cross sectional distribution of firms, $\Gamma_t(\tau_m, \tau_z)$, with different vintages of information. $\Gamma_t(\tau_m, \tau_z)$ denotes the fraction of firms that updated their information about monetary shocks prior to date τ_m and about their own idiosyncratic productivity prior to date τ_z. Accordingly, the marginal distribution, $\Gamma_t^m(\tau_m) \in [0, 1]$, refers to the mass of firms at time t that acquired information about monetary policy prior to date τ_m while $\Gamma_t^z(\tau_z)$ represents the mass of firms that last acquired information about their idiosyncratic productivity prior to date τ_z. $d\Gamma_t^m(\tau)$ and $d\Gamma_t^z(\tau)$ denote the marginal densities, i.e. the mass of firms that acquire information exactly at date τ about monetary policy and their idiosyncratic productivity respectively.[7] The evolution of $\Gamma_t^k(\tau)$, $k = m, z$ can be written recursively as

$$\Gamma_{t+dt}^k(\tau) = \Gamma_t^k(\tau) - \int_{-\infty}^{\tau} \mathcal{D}_t^k(s) d\Gamma_t^k(s) \text{ ,}\forall t \leq \tau \text{ and } k = m, z \tag{1.19}$$

[7]In other words, the fraction $1 - \Gamma_t^m(\tau_m)$ is the fraction of firms that know all the realizations of the aggregate state up to date $\tau_m : \{m_s\}_{s \leq \tau_m}$ and $1 - \Gamma_t^z(\tau_z)$ has the analogous interpretation for the idiosyncratic state.

where $\mathcal{D}_t^k(s)$ is the probability that a firm that most recently acquired information about the aggregate (idiosyncratic)state at date s will acquire information about the aggregate (idiosyncratic) state again at date t.

Because of the dispersed information about the state of monetary policy, a firm's decision depends on all the past realizations of the aggregate and idiosyncratic state, making the dimensionality of the firm's problem infinite.[8] I use the method of undetermined coefficients to find an analytical solution to the firm's problem in (A.1.4).[9] I set $\mu = 0$ in the subsequent analysis for ease of exposition.[10]

Lemma 2. *In equilibrium, the following are true:*

1. *The aggregate (log) price $p(t)$ follows the following process:*

$$p(t) = \sigma_m \int_{-\infty}^{t} \frac{[1-\Gamma_t^m(\tau)](1-r)}{1-r+r\Gamma_t^m(\tau)} dW(\tau) \tag{1.20}$$

2. *The aggregate component of the target price $p^*(t)$ follows:*

$$p^*(t) = \sigma_m \int_{0}^{t} \frac{1-r}{1-r+r\Gamma_t^m(\tau)} dW(\tau) \tag{1.21}$$

3. *The firm's expectation of $p^*(t)$ follows:*

$$E\{p^*(t) \mid \mathcal{I}_{\hat{\tau}_m}\} = \sigma_m \int_{0}^{\hat{\tau}_m} \frac{1-r}{1-r+r\Gamma_t^m(\tau)} dW(\tau) \tag{1.22}$$

[8]See Townsend (1983), Nimark (2010) for more details.

[9]The basic approach to finding the solution is a two step procedure. First, I solve for the optimal price a firm sets conditional on any information set as shown in equation (1.18). I then solve for the firm's optimal choice of when to update information to minimize its loss from being uninformed. The solution to this step takes the form of a deterministic control problem.

[10]All proofs in the appendix do not set $\mu = 0$. I revert to $\mu > 0$ in Section 1.4.2.

Proof. See A.1.4 □

The target price at time t is given by

$$\overline{p}_i(t) = \sigma_m \int_{-\infty}^{\hat{\tau}_m} \frac{1-r}{1-r+r\Gamma_t^m(\tau)} dW(\tau) + \zeta\sigma_z \int_{-\infty}^{t} e^{-\eta(t-\tau)} dB_i(\tau) \tag{1.23}$$

Thus, firm i that last updated its information about monetary shocks at $\hat{\tau}_m$ and about its idiosyncratic productivity at $\hat{\tau}_z$, will set a price

$$p_i(t) = E\{\overline{p}_i(t) \mid \mathcal{I}_{\hat{\tau}_m}, \mathcal{I}^i_{\hat{\tau}_z}\} = \sigma_m \int_{-\infty}^{\hat{\tau}_m} \frac{1-r}{1-r+r\Gamma_t^m(\tau)} dW(\tau) + \zeta z_i(\hat{\tau}_z) e^{-\eta(t-\hat{\tau}_z)} \tag{1.24}$$

Next, we need to solve for the optimal planning horizon for each firm. In the present setup, firms choose when to update their information, so they optimally decide their information set at each instant. The difference between the forecasted and actual target price of the firm is:

$$p_i(t) - \bar{p}_i(t) = \sigma_m \int_{\hat{\tau}_m}^{t} \frac{1-r}{1-r+r\Gamma_t^m(\tau)} dW(\tau) + \zeta \left[z_i(\hat{\tau}_z) e^{-\eta(t-\hat{\tau}_z)} - z_i(t) \right] \tag{1.25}$$

The first term is the forecast error in the aggregate component of the target price given that forecasts are formed with respect to the information set $\mathcal{I}_{\hat{\tau}_m}$. The second term is the forecast error in the idiosyncratic component of the target price where the forecasts are made with respect to $\mathcal{I}^i_{\hat{\tau}_z}$.

Since $W(t)$ and $B_i(t)$ are standard Brownian motions with unit variance, equations (1.25) and (1.17) imply that the loss from mis-pricing can be expressed as

$$L(t,\hat{\tau}_m,\hat{\tau}_z) = \sigma_m^2 \int_{\hat{\tau}_m}^{t} \left[\frac{1-r}{1-r+r\Gamma_t^m(\tau)}\right]^2 d\tau + \zeta^2 \frac{\sigma_z^2}{2\eta}\left[1-e^{-2\eta(t-\hat{\tau}_z)}\right] \equiv L_1(t,\hat{\tau}_m) + L_2(t,\hat{\tau}_z)$$

Since the loss function can be separated into a purely aggregate part and a purely idiosyncratic part, the problem of when to update information about each state can be solved as two separate problems. The first and second terms are the variances of the forecast errors associated with forecasting the aggregate component and the idiosyncratic component of the target price respectively.

The equilibrium in this incomplete information economy is a stationary Bayesian-Nash Equilibrium. Two structures of equilibria arise naturally: synchronized and staggered. In the synchronized equilibrium, all firms choose to update their information about state k, $k = m, z$ at the same date. In the staggered stationary equilibrium, only a fixed fraction of firms *plans* about the aggregate and idiosyncratic state at each date. Existing research suggests that price changes are not synchronized. For example, Lach and Tsiddon (1992) find a lack of synchronization of price changes in Israel. Thus, I focus on the **stationary staggered equilibrium**, which is empirically more relevant. Some implications of the synchronized equilibrium are discussed in footnote 12.

I concentrate on the pricing problem of firm i. Assume that all other firms acquire information about the aggregate state every T_m periods and about their idiosyncratic state every T_z periods. Thus, the proportion of firms acquiring information about the aggregate state over any interval is given $\frac{1}{T_m}dt$ and about the idiosyncratic state is $\frac{1}{T_z}dt$. As a result, in the staggered equilibrium

$$\Gamma_t^k(\tau) = \begin{cases} 20 & \text{if } \tau < t - T_k \\ 1 - \frac{t-\tau}{T_k} & \text{if } t - T_k \leq \tau < t \end{cases}$$

for $k = m, z$.

To select the optimal timing of future updates to its information about the monetary state, a firm that acquires information about monetary policy today minimizes the lifetime loss from being inattentive:

$$\mathcal{L}_1(\hat{\tau}_m) = \min_{\tau'_m \geq \hat{\tau}_m} \int_{\hat{\tau}_m}^{\tau'_m} e^{-\rho(s-\hat{\tau}_m)} L_1(s, \hat{\tau}_m) ds + e^{-\rho(\tau'_m - \hat{\tau}_m)} [\mathcal{C}_m + \mathcal{L}_1(\tau'_m)] \tag{1.26}$$

where

$$L_1(t, \hat{\tau}) = \begin{cases} \sigma_m^2 \int_{\hat{\tau}}^t \frac{(1-r)^2}{(1-r\frac{(t-\tau)}{T_m})^2} d\tau & \text{if } \hat{\tau} \geq t - T_m \\ \sigma_m^2 \int_{t-T_m}^t \frac{(1-r)^2}{(1-r\frac{(t-\tau)}{t})^2} ds + \sigma_m^2 (t - T_m - \hat{\tau}) & \text{if } \hat{\tau} < t - T_m \end{cases}$$

Similarly the problem on when to next acquire information on the idiosyncratic state conditional on obtaining information today can be written as

$$\mathcal{L}_2(\hat{\tau}_z) = \min_{\tau'_z \geq \hat{\tau}_z} \int_{\hat{\tau}_z}^{\tau'_z} e^{-\rho(s-\hat{\tau}_z)} L_2(s, \hat{\tau}_z) ds + e^{-\rho(\tau'_z - \hat{\tau}_z)} [\mathcal{C}_z + \mathcal{L}_2(\tau'_z)] \tag{1.27}$$

where

$$L_2(t, \hat{\tau}) = \frac{\zeta^2 \sigma_z^2}{2\eta} \left[1 - e^{-2\eta(t-\hat{\tau})}\right]$$

The solution to the firm's problem can be seen as a threshold for these error variances. The loss from being inattentive depends on the variance of the forecast error. This variance is increasing in the duration since the firm last updated. Once the threshold error variance for a state is reached, the firm chooses to incur the fixed cost of acquiring information about that state and resets the forecast error variance associated with that component of the target price to zero. It then conditions its forecasts of the target price on this newly expanded information set. The threshold is chosen such that if the firm did not incur the fixed cost to update its information set, the loss from mis-pricing would result in larger overall losses than the cost of obtaining information and reducing the forecast error.

Proposition 2. ***Optimal Planning Horizon***: *Each firm chooses to update its information set about monetary policy shocks every T_m^* periods and about its idiosyncratic productivity every T_z^* periods.*

1. *The unique optimal horizon for planning about the monetary shock T_m^* is implicitly defined by*

$$\mathcal{C}_m = \sigma_m^2 T_m^* \int_0^{T_m^*} (1-r)e^{-\rho s} \frac{T_m^* - s}{T_m^* - rs} ds \tag{1.28}$$

2. *The unique optimal horizon for planning about the idiosyncratic productivity shock T_z^* is implicitly defined by*

$$\mathcal{C}_z = \frac{\zeta^2 \sigma_z^2}{2\eta} \int_0^{T_z^*} e^{-\rho\delta}(e^{-2\eta\delta} - e^{-2\eta T_z^*}) d\delta \tag{1.29}$$

Proof. A.1.5 derives the expression for the T_m^*. The expression for T_z^* can be derived using the same procedure. □

Proposition 3. *If* $\mathcal{C}_k > 0$*, then* $T_k^* > 0$ *for* $k = m, z$*.*

Proof. Plugging $T_k^* = 0$ into the RHS of equation (1.28) or equation (1.29) yields zero on the RHS, which is a contradiction; i.e. it is never optimal for a firm to update its information about the aggregate state or the idiosyncratic state at each instant unless doing so is costless. □

1.4 Analytical Results

1.4.1 Differential adjustment to nominal and idiosyncratic productivity shocks

For this section, I set $\eta = 0$. As a result both, the money supply and the idiosyncratic shock processes follow a drift-less Brownian motion

$$\begin{aligned} \ln M(t) &= \sigma_m dW(t) \\ \ln Z_i(t) &= \sigma_z dB_i(t) \end{aligned}$$

Equation (1.29) in the limit as $\eta \to 0$ can be written as

$$\mathcal{C}_z = \zeta^2 \sigma_z^2 \int_0^{T_z^*} e^{-\rho\delta}(T_z^* - \delta)d\delta \tag{1.30}$$

Lemma 3. *T_m^* is increasing in the strength of the strategic complementarity, i.e.* $\frac{\partial T_m^*}{\partial r} > 0, \forall r \in [0, 1]$

Proof. Lemma 3 can be verified by applying the Implicit Function Theorem on equation (1.28). □

Proposition 4. *For $r \in (0, 1)$ and normalizing $\zeta = 1$, if $\sigma_m = \sigma_z = \sigma$ and $\mathcal{C}_m = \mathcal{C}_z = \mathcal{C}$, then $T_m^* > T_z^*$*

Note that equation (1.29) is of the same form as equation (1.30) with r set to 0 (with $|\zeta|$ normalized to 1). Lemma 3 then implies that $T_z^* < T_m^*$. Proposition 4 implies that firms choose to incorporate new information about idiosyncratic shocks into prices more often than information about aggregate shocks, even when both shocks are equally volatile and when the costs associated with updating information about each shock are equivalent. Empirical evidence suggests that idiosyncratic productivity is highly volatile relative to aggregate shocks. Thus, in the calibrated model firms update their information about idiosyncratic productivity at a substantially higher frequency. Hence, the model is able to explain differential adjustment of prices to idiosyncratic and aggregate shocks.

Strategic complementarity in pricing spills over into information acquisition decisions about monetary policy and causes a *delay* in information acquisition about monetary shocks. This can be seen as a combination of two forces.

When a firm gets new information about monetary policy, it realizes that a large fraction of firms remain uninformed and price according to their old information. In the Full Information Case, the arrival of new information about a monetary shock warrants a full price change. However, with costly information, a firm takes into account the behavior of the mass of uninformed firms and tempers its pricing response.[11]This diminishes the value of obtaining information about the shock today, as an informed firm's pricing behavior is constrained in the present by the mass of uninformed firms. The firm faces an upfront cost of acquiring information today while the benefit is accrued only in the future. This incentivizes the firm to delay its acquisition of information about monetary shocks.

The loss from being uninformed is increasing in the fraction of firms that are informed, i.e.

$$\frac{\partial L_1(t,\hat{\tau})}{\partial(1-\Gamma_t^m(\tau))} > 0 \ ,\forall \tau \in (\hat{\tau}, t] \text{ iff } r > 0$$

From the point of view of firm i, the staggered nature of information acquisition[12] implies that a large fraction of firms is less informed than itself. As long as all other firms remain uninformed about monetary policy, then no firm has an incentive to update its information. The large mass of uninformed firms implies that the aggregate price level moves sluggishly and does not track the money supply well.

[11]This is similar to the older literature on strategic complementarity such as Haltiwanger and Waldman (1985)

[12]The synchronized equilibrium also has similar properties. However, instead of a unique optimal planning horizon for the monetary state, there exists a closed interval on the real line of optimal planning horizons, out of which any planning horizon can be the equilibrium. This multiplicity of equilibria is due to the complementarity $r \in (0,1]$. The optimal planning horizon for the idiosyncratic state remains unique as there is no complementarity associated with that aspect of pricing. For more details on the multiplicity of equilibria, see Hellwig and Veldkamp (2009). Even so, it can be shown that the optimal planning horizon about idiosyncratic state is still shorter than that for the aggregate monetary state. In a synchronized equilibrium, all firms adjust prices simultaneously and hence when firms choose to update their information, prices reflect this information fully. This is in contrast to the staggered equilibrium where prices only gradually adjust because firms who observe the new information have to temper their response to it to account for those who have not acquired it yet. Thus, if there is a monetary policy shock at a time between two planning dates, the output stays at the high level and does not decline till the next planning date at which point, it then falls to the natural level as the aggregate price adjusts proportionally to the change in the monetary policy.

Given the importance of relative prices to firms' profits, a firm wants to price close to the average. Since the aggregate price moves sluggishly, the gap between a firm's current price and its target price is small in terms of the loss from being uninformed.

Small losses from mis-pricing and delayed benefits from acquiring information give rise to planning complementarities which delay information acquisition about monetary shocks. The aggregate price does not respond on impact to a monetary shock but catches up eventually when all firms update and incorporate the information about the monetary shock into their prices.

Prices will reflect new information about idiosyncratic productivity more quickly than information about monetary shocks because of the *beauty contest* nature of the price setting problem. A firm seeks to limit the gap between their price and the average price as profits depend on relative prices. At the same time, the firm also wants to set its price to reflect the true state of monetary policy. Thus, the firm's pricing response to a monetary shock is contingent not just on the monetary policy but also on how other firms respond to the nominal shock. Strategic complementarity in price setting feeds into strategic complementarity in information acquisition. As aforementioned, a firm wants to update its information about the monetary state often if other firms do so too, and vice versa. However, a firm's price response to its own idiosyncratic productivity shock is not contingent on the actions of others, as each firm is too small and cannot affect the average price. The absence of strategic complementarities in pricing with respect to idiosyncratic productivity implies that prices respond fully to idiosyncratic shocks. Moreover, this absence of strategic complementarity in pricing extends to a lack of complementarities associated with planning over the idiosyncratic state. Hence, there is no delay in information acquisition about idiosyncratic productivity unlike the case of information acquisition regarding monetary policy.

1.4.2 Non Zero Long Run Inflation and Static Indexation

For this section, I relax the assumption that $\mu = 0$. Instead, I set $\mu > 0$ which implies a positive constant rate of long run wage inflation. Since this section deals with indexation of prices to long run inflation, I abstract away from idiosyncratic productivity shocks and set $z_i(t) = 0$ for all t. With $\mu > 0$, the price that firm i sets at time t when it last updated its information at date $\hat{\tau}_m$ can be seen as a modified version of equation (1.24):[13]

$$p_i(t) \quad = \quad E\{\overline{p}_i(t) \mid \mathcal{I}_{\hat{\tau}_m}\} = \sigma_m \int_{-\infty}^{\hat{\tau}_m} \frac{1-r}{1-r+r\Gamma_t^m(\tau)} dW(\tau) + \mu t$$

In such a setting, prices change all the time but only reflect new information at discrete intervals. Thus, rather than setting prices, firms set price plans which they reset every time they update information. Since long run inflation is positive, firms set a price schedule by which they index their prices to long run inflation in between dates of information acquisition.

A large portion of the literature on the New Keynesian Phillips curve, which make use of the Calvo-Yun type sticky price setup, assumes that the non-adjusting firms index their prices to past lags of inflation or average inflation [Erceg et al.

[13]This expression is derived in A.1.4.

(2000) etc.]. Indexation is often motivated as rule-of-thumb behavior followed by the non-adjusting firms. This model gives micro foundations to such an assumption.

The inflation indexation is an attempt by a firm to minimize its forecast error. During periods when a firm chooses not to update its information, the firm sets price plans which require it to raise their prices at a rate commensurate with long run inflation. Each firm's price is indexed to long run inflation and hence changes all the time. However, monetary policy is still effective in the short run because new information is only incorporated into prices gradually, implying that the aggregate price still moves sluggishly in response to a monetary shock.

1.4.3 Mean Reverting Idiosyncratic Productivity Shocks

Mankiw and Reis (2010) point out three features which are prominent in analyzing price paths observed in the data:

1. Prices change all the time, on average every three to four months.

2. Many price changes follow what seem like predetermined patterns that follow simple algorithms, and actual resetting of price plans based on new information seems less frequent.

3. There are many horizontal segments, reflecting short-lived intervals when nominal prices are unchanged.

In addition to the three facts, empirical studies have found that relative price changes are transitory. The basic model specified in Section 2 is capable of generating pricing behavior which is consistent with all the above features.[14] Empirical evidence suggests that idiosyncratic shocks are very volatile and hence firms would tend to update their information about idiosyncratic shocks frequently. Prices incorporate new information more often, which results in large jumps in prices from firms resetting their price plans (See Figure 1.1).

In the present setup, firms update their information about idiosyncratic productivity every T_z^* [defined in equation (1.29)] periods and reset price plans every time they do so. They set prices according to this new plan at every subsequent date until they update their information again. Prices change at each date but only incorporate new information at discrete intervals which correspond to resetting of price plans at the arrival of new information. This is consistent with Blinder et al. (1998), who find evidence of managers' adjusting their price plans.

Suppose firm i updates its information about idiosyncratic productivity today at $t = t_0$ and observes that $z(t_0) = z_0$ is above the mean. Since the marginal cost schedule is lower this period, the firm can afford to set a lower relative price and attract more demand. However, since productivity is mean reverting, the firm expects productivity to fall back to the average at a rate η. The firm chooses not to update its information between t_0 and $t_0 + T_z^*$. During this period, it sets prices according to a simple pricing plan that it determined at t_0. The price plan stipulates that prices be raised over time towards the average price to compensate for the increasing marginal costs, and is set to track the target price as closely as

[14]For ease of exposition, I again set $\mu = 0$ since μ does not affect relative prices. I also set the variance of nominal shocks to zero.

possible. The price plan can be written as

$$p_i(t) = \zeta z_0 e^{-\eta(t-t_0)} \text{for } t_0 \leq t \leq T_z^* \tag{1.31}$$

At $t_0 + T_z^*$, the firm updates information about its idiosyncratic productivity and resets the price plan. Suppose it observes that $z(t_0 + T_z^*) = z_1$. It then resets the price plan to

$$p_i(t_0 + T_z^* + dt) = \zeta z_1 e^{-\eta dt} \text{ for } 0 \leq dt < T_z^*$$

As can be seen from the price plan in equation (1.31), the model predicts that relative price changes are transitory which is consistent with empirical findings.

The model struggles at generating flat price paths. In terms of the model, flat price paths wouldoccur when at the planning date t_0, the firm observes $z(t_0) = 0$ at the planning date. Under a diffusion process, the probability of drawing $z = 0$ is a zero measure event. Thus, without introducing any other friction, observing a flat price path is a zero measure event. A possible solution to this problem might be found in a rational inattention setup as in Woodford (2008). In such a setting, rationally inattentive firms might choose to set prices out of a discrete set even if the shock process is continuous. This would complicate the aggregation problem and is left for future research.

1.4.4 Volatility of Monetary Policy Shocks and Effectiveness of Monetary Policy

Proposition 5. *For $r \in (0,1)$, duration for which firms remain inattentive about state $k \in \{m, z\}$ is decreasing in the volatility of that particular state. In other words*

$$\frac{\partial T_k^*}{\partial \sigma_k^2} < 0 \ , \ \forall k \in \{m, z\}$$

Proof. The proof is provided for the relation between T_m^* and σ_m^2. The proof for T_z^* is analogous.

Define

$$F_m(T_m, \sigma_m^2) = \sigma_m^2 T_m \int_0^{T_m} (1-r)e^{-\rho s} \frac{T_m - s}{T_m - rs} ds - \mathcal{C}_m$$

Equation (1.28) can be seen as the case $F_m(T_m^*, \sigma_m^2) = 0$. Appendix A.1.5 shows that $\partial F_m(T_m^*, \sigma_m^2)/\partial T_m^* > 0$. A simple application of the Implicit Function Theorem then implies:

$$sign\left[\frac{dT_m^*}{d\sigma_m^2}\right] = -sign\left[\frac{\partial F_m(T_m^*, \sigma_m^2)}{\partial \sigma_m^2}\right] = - \text{ , since } 0 < r < 1$$

□

Proposition 5 has direct implications for the effectiveness of monetary policy. The longer firms choose to stay inattentive about changes to monetary policy, the more persistent is the effect of monetary policy on output. This is because the effect of monetary policy on output dies out as soon as all firms learn about the monetary policy change. If each firm updates its information frequently then all firms will become informed about a change in policy more quickly, which then reduces the effect of that change in monetary policy.

Thus, the model predicts that a monetary authority has a tradeoff between effectiveness of monetary policy and the level of discretion it can use in the conduct of monetary policy. A more discretionary monetary policy, in terms of the model, corresponds to a higher variance of monetary policy σ_m^2. The model predicts that monetary policy will have more persistent effects on output in regimes with lower σ_m^2. Another way to look at this prediction is that periods with high inflation volatility (high σ_m^2) correspond to periods where the persistence of the effect of monetary policy on output is lower. I use this prediction of the model to analyze the effectiveness of monetary policy in the US since the 1970's in Section 6.

1.5 Calibration and Numerical Results

As is standard in the New Keynesian literature (e.g. Gertler and Leahy (2008)), I simulate the model around a zero long run inflation steady state, i.e. I set μ to 0. I draw on existing literature for the values of the preference parameters ρ, γ, α, and ϵ. The discount rate ρ is set to 0.04, following an annual calibration. The risk aversion parameter γ is set to to 2. The elasticity of substitution parameter ϵ is set to 10. The disutility of labor α is set to 9, which implies that roughly 33 percent of the unit time endowment is allocated to labor in steady state. The exponent on labor in the production function is calibrated to the standard value of $\theta = 2/3$. This calibration yields $r = 0.7917$ which measures the strength of the strategic complementarity in price setting. Woodford (2003) suggests values of r between 0.75 and 0.9. I set $\sigma_m = 0.0248$, which corresponds to the standard deviation of annual inflation in the Klenow-Kryvtsov data set. To calibrate the cost of acquiring and processing information, I draw on existing literature. Studies such as Chevalier et al. (2003) estimate menu costs to be of the order of about 0.75 per cent of a firm's revenue. Zbaracki et al. (2004) report that information processing costs are 6 times as large as menu costs. Thus, I set the cost of acquiring information as $0.75 \times 6 = 4.5$ percent of steady state revenue. Without any strong reason to set the cost of acquiring information about monetary shocks differently from that about the idiosyncratic state, I set $\mathcal{C}_m = \mathcal{C}_z$.

This leaves two parameters to be calibrated: the variance σ_z^2 and the rate of mean reversion η of idiosyncratic productivity shocks. I follow a calibration strategy similar to Golosov and Lucas Jr. (2007). First, I shut down the monetary policy shock. Thus, all price changes are in response to idiosyncratic shocks. I calibrate $\hat{\sigma}_z^2 \equiv \zeta^2 \sigma_z^2$ and η by targeting the average number of price reviews per year and the average size of a price change conditional on a price increase.[15] I choose the parameters to minimize the sum of the squared differences between the values of the two targets in the data and the model generated counterparts.

Alvarez et al. (2010) report that the median number of price reviews per year in

[15] Equations (1.15) and (1.18) show that ζ determines the sensitivity of the profit-maximizing price to the idiosyncratic state variable and has the same effect as the variance of the idiosyncratic state variable. Therefore, I normalize $|\zeta|$ to one and only choose the variance of the idiosyncratic state variable. Thus, I calibrate $\hat{\sigma}_z^2 \equiv \zeta^2 \sigma_z^2$ instead of just σ_z^2.

the US is between 2 and 3. I target the median number of price reviews a year to be 3. Golosov and Lucas Jr. (2007) find that the average price change conditional on a price increase is 0.095 for regular price changes in the Klenow and Kryvtsov (2008) dataset. Minimizing the squared difference yields $\hat{\sigma_z} = 0.58$, which corresponds to a quarterly standard deviation of 0.14, and $\eta = 0.6652$, which corresponds to a persistence of 0.85 in a quarterly AR(1). I simulate the model with the shortest time period being a month to calculate the average size of price changes. The calibration exercise yields 0.086 as the average price change conditional on an increase and 3.9 price reviews a year. Details of the calibration are summarized in Table 1.

I calculate the optimal planning horizon about the monetary state from equation (1.28). The half-life of the response of output is approximately 5.2 years.[16] Thus, unlike Golosov and Lucas Jr. (2007), this model is able to generate substantial non-neutralities and a strongly persistent effect of a shock to monetary policy on output.

Golosov and Lucas Jr. (2007) note that the strength of the non-neutrality is increasing in the share of the fixed factor which can be measured as $1 - \theta$. This is because the degree of strategic complementarity (r) in price setting is decreasing in

[16] If I let $\mathcal{C}_m \neq \mathcal{C}_z$, then the model is able to match any characteristics of both the micro level data and sluggish responses of prices. On setting $\mathcal{C}_m$ to be 0.1 percent of steady state revenue, a positive shock to monetary policy results in a positive effect on output for approximately 6 quarters which is the length of the non-neutrality estimated in Christiano et al. (1999). It is not unreasonable to believe that the cost of acquiring information about monetary policy might be substantially smaller than the cost associated with the idiosyncratic state. Information has characteristics of a public good and becomes cheaper as more people choose to acquire it. All firms are looking for the same information about monetary policy while they only want information about their own idiosyncratic state. Thus, the higher demand for information about monetary policy makes the cost of acquiring information about it much lower. This can be also seen from the fact that one might learn about the state of monetary policy from reading a newspaper which is much cheaper than a firm hiring consultants to find out about its demand and cost advantages. Veldkamp and Wolfers (2007) formalize such an argument.

	Parameter	**Source/Target**
Disutility of Labor	$\alpha = 9$	Steady state labor = 0.34
Coefficient RRA	$\gamma = 2$	Std. in literature
Share of Labor in output	$\theta = \frac{2}{3}$	Std. in literature
Elasticity of substituiton across goods	$\epsilon = 10$	Std. in literature
Discount Rate	$\rho = 0.04$	Std. Annual Calibration
S.D. of monetary shock	$\sigma_m = 0.0248$	Annual standard deviation of inflation from Klenow and Kryvtsov (2008)
Cost of updating information	$\mathcal{C} = 0.0088$	Chevalier et al. (2003), Zbaracki et al. 2004
S.D. of idiosyncratic shock	$\hat{\sigma}_z = 0.58$	S.D. of price changes conditional on increase = 0.91, Golosov and Lucas(2007)
Persistence of idiosyncratic shock	$\eta = 0.6652$	# Price Reviews/year= 3 ,Alvarez et al. (2010)

Table 1.1: Baseline Parameter Values

$1 - \theta$. Even with $\theta = 0.99$, i.e, even if the importance of the fixed factor and the degree of the strategic complementarity ($r = 0.0872$) are much lower than suggested by Woodford (2003), the model still generates significant non-neutrality. The half-life of the effect on output for $\theta = 0.99$ is 2.8 years.

Figure 1.1 shows the paths of prices set by a firm under costly information compared to under costless information in response to a sample path of idiosyncratic shocks. Monetary Shocks are shut down in the figure, so prices are only reacting to changes in idiosyncratic productivity. The dashed line corresponds to the path of a firm's price under costless information. The solid line shows the reaction of price in an environment with costly information. The vertical lines indicate price reviews, i.e periods when firms update their information. The vertical dotted lines indicate a price review. The price jumps at the arrival of new information. Prices under costly information are certainly smoother but not very different from those under full information. The realized loss in profit from mis-pricing is not very large, since firms choose to update their information about idiosyncratic productivity frequently.

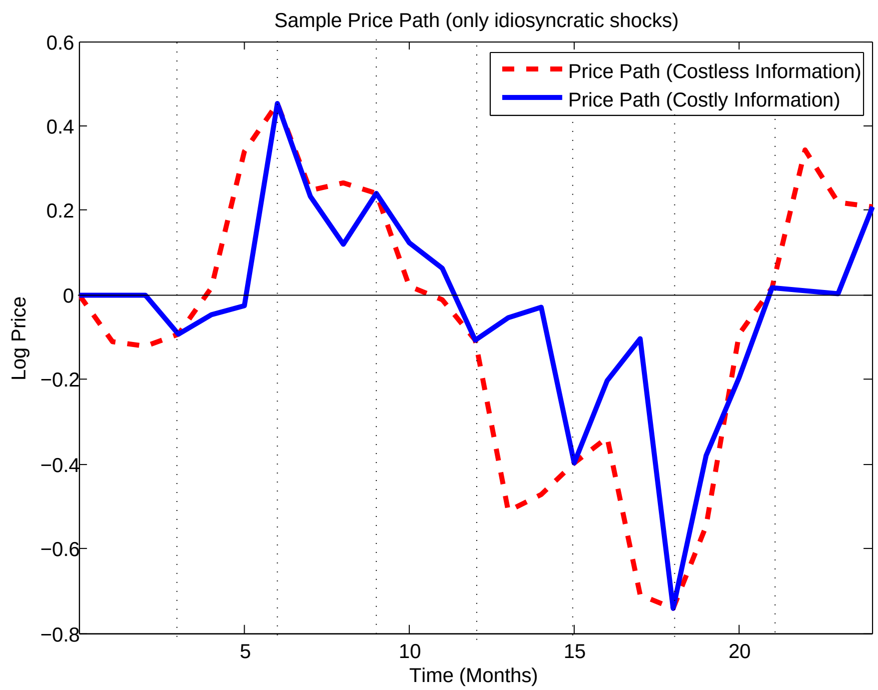

Figure 1.1: Price Paths of Representative firm in response to idiosyncratic shocks

The impulse response of output and the aggregate price to a positive monetary shock of one standard deviation magnitude can be seen seen in Figure 1.2. : The top panel depicts the response of aggregate price and the bottom panel depicts the deviation of output from the *natural level*. T^*, in the figure, is the optimal planning horizon for the monetary state. The staggered nature of the equilibrium results in

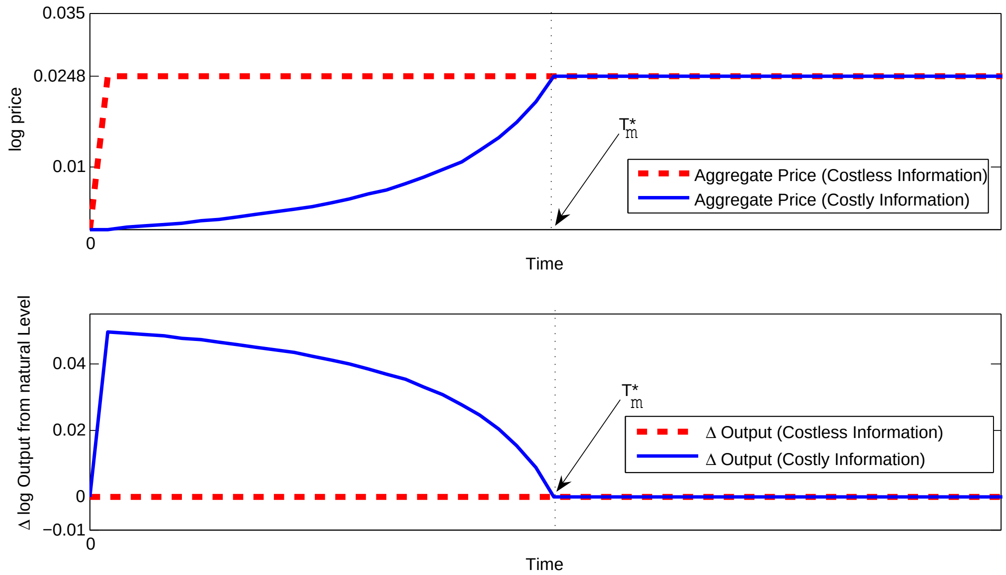

Figure 1.2: Impulse Response to a monetary expansion

a large fraction of firms being uninformed about the current true state. This allows for the model to generate sufficient heterogeniety in firm behavior. Midrigan (2008) points out that heterogeneity between firms in Golosov and Lucas Jr. (2007) is very limited and hence prices adjust very quickly to a shock in their model. Pricing and planning complementarities result in a sluggish response of the aggregate price to a monetary shock which results in output being different from the natural level. Once all firms update their information sets such that each of them has incorporated the shock to monetary policy into their prices, the aggregate price adjusts *fully* to the shock and output reverts back to the natural level.

Figure 1.3 shows how the aggregate price evolves under costly information in reaction to a sample path for monetary policy shocks as compared to the case with costless information. The dashed line is the path of (log) money supply. The dashed line also corresponds to the path of the aggregate (log) price in reaction to monetary policy shocks under costless information. The solid line corresponds to the response of the aggregate price under the costly information case. Changes in the

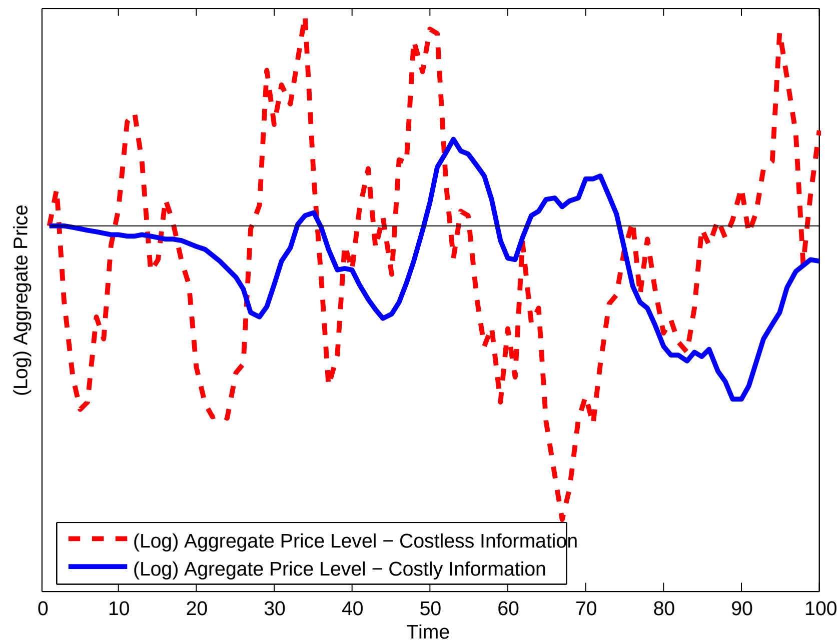

Figure 1.3: Sample Price Path of the aggregate price in response to a sequence of monetary policy shocks.

aggregate price are much smaller in the case with costly information compared to the case with costless information. Furthermore, the aggregate price responds slowly to shocks when information is costly. From Proposition 1, in a setting with costless information, the aggregate price tracks money supply perfectly. Thus, a fall(rise) in the dashed line in the figure corresponds to a contractionary (expansionary) shock to money supply. The solid line, representing the path of the aggregate price with costly information lags behind the dashed line. With costly information, the aggregate price starts to fall well after a contractionary shock to money supply and displays inertia as it continues to fall even after there has been an expansionary shock to monetary policy.

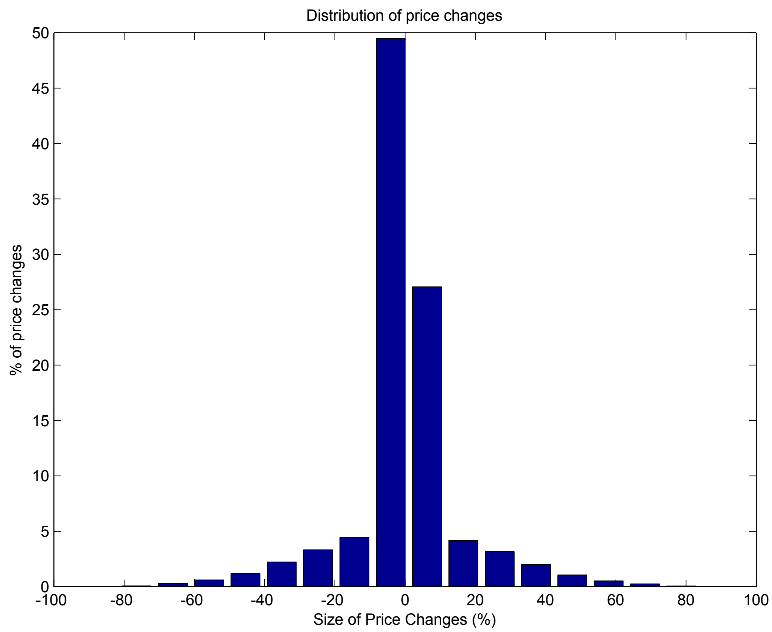

Figure 1.4: Distribution of size of price changes

Figure 1.4 plots the distribution of price changes along a sample price path. As Midrigan (2008) and Klenow and Kryvtsov (2008) point out, standard SDP models generate what has been termed the *missing middle*, i.e they fail to explain small price changes. Midrigan (2008) also points out that the distribution of price changes exhibits excess kurtosis. The current model is capable of generating this excess kurtosis and is able to explain a large number of both small price changes and at the same time, also large price changes. Since there is no cost associated with changing prices per se, the firm changes prices even when only small price changes are warranted. This is not the case in menu cost models.

1.6 Testing the Effectiveness of Monetary Policy

In this section, I look at changes in the effectiveness of US monetary policy since 1969. This empirical study is in the spirit of Lucas Jr. (1973). Instead of looking at cross-country differences in the effectiveness of monetary policy as in Lucas Jr. (1973), I examine changes in the effectiveness of US monetary policy over time. Economists generally agree that the 1970s, a period of high volatility in both output and inflation, was also a period in which monetary policy was highly discretionary and performed quite poorly, relative to both earlier and later periods (Romer and Romer, 2002). In terms of the current model, the 1970's would correspond to a higher σ_m^2 than in subsequent periods. Following Proposition 5, this would imply that $T_{1970's} > T_{post1970's}$, i.e. monetary policy had less persistent effects on output in the 1970's than in subsequent periods.

To test the above claim, I estimate T from equation (1.32) over separate subsamples. I use the 1970's as the first subsample and the subsequent years as the second subsample. Following Bernanke and Mihov (1998), I use the following two breakpoints for a change in policy regime between the 1970's and the subsequent period. The first breakpoint is 1979(Q3), which corresponds to the announcement of chairman Volcker's new operating regime. The second breakpoint is 1982(Q1), which Bernanke and Mihov (1998) claim as roughly corresponding to the abandonment of targeting of non-borrowed reserves in favor of interest rate control.

The model-consistent *Sticky Information Phillips Curve* (SIPC) can be written as:

$$\pi_t = \frac{p_t - E_{t-T}[p_t]}{T} + (1-r)\frac{y_t - E_{t-T}[y_t]}{T} + \sum_{j=0}^{T-1} E_{t-1-j}[\pi_t + (1-r)\Delta y_t] + \epsilon_t \quad (1.32)$$

where $T \in \{1, 2, 3, 4, ...\}$ is the number of quarters each firm chooses to wait before it updates its information about monetary policy, p_t is the log price at time t, y_t is the output gap, $r \in \mathbb{R}$ is the strength of the strategic complementarity in pricing, and ϵ_t is a measurement error which is i.i.d across time and is appended to the SIPC for estimation purposes. The full derivation is available in Appendix A.1.6. Compared to the standard version of the SIPC (Mankiw and Reis, 2002), this version of the model demonstrates an optimal relation between T and the number of lags that need to be included to estimate the Phillips Curve relationship consistently.[17]

To proxy for the forecasts of firms, I use data from the Survey of Professional Forecasters (Croushore, 1993). The survey provides individual responses of a panel of forecasters with up to four quarter ahead forecasts for the level of the price index and nominal GDP. Consistent forecasts of real GDP, inflation and the output gap are based on this information.[18] To calculate the values of the consolidated forecast in every quarter period, I calculate the median and mean forecasts within the cross-section of professional forecasters. Using the median or the mean does not alter results qualitatively.[19] The main limitation of the dataset is that forecasts are

[17] See A.2.1 for a detailed discussion.

[18] As in Coibion (2010), I assume that the forecasters know the CBO's measure of potential output at the time of forecasting. This enables me to calculate the forecasted output gap as the log difference between the forecasted real GDP and the measure of potential output for a particular quarter.

[19] Since the data in the period prior to the 1980's has some missing values for forecasts 4 quarters ahead of some of the variables of interest, I impute these values by averaging the data for the

available only up to 4 quarters ahead which limits the parameter space for T to $\{1, 2, 3, 4\}$.[20]

As a preliminary check, I estimate the parameters of equation (1.32) by the method of Maximum Likelihood Estimation.[21] Estimating the model over the whole sample yields an estimate of $\hat{T} = 4$. This corresponds to one price review every year,[22] which is consistent with other empirical findings such as Blinder et al. (1998). $T = 4$ is also consistent with the Mankiw and Reis (2002) estimate of the exogenous arrival rate of new information $(1 - \lambda = 0.25)$.

Estimating the model over the sub-samples separated by the first break point gives us the point estimates of $\widehat{T}_{t<1979(Q3)} = 3$ and $\widehat{T}_{t\geq 1979(Q3)} = 4$. Using the second break point also yields the estimates $\widehat{T}_{t<1982(Q1)} = 3$ and $\widehat{T}_{t\geq 1982(Q1)} = 4$. Thus, firms were updating information more frequently in the 1970's, a period which in terms of the model corresponds to a higher σ^2_m . Conversely, firms update information more slowly in periods where monetary policy is less discretionary and σ^2_m is low. Finally, I test the null hypothesis $H_0 : T_{t<1979(Q3)} \leq T_{t\geq 1979(Q3)}$ against the complement, and also test the null that $H_0 : T_{t<1982(Q1)} \leq T_{t\geq 1982(Q1)}$ against its complement.

Through a likelihood ratio test,[23] I am able to reject the hypothesis that $T_{t<1979(Q3)} \leq T_{t\geq 1979(Q3)}$ in favor of the alternative $T_{t<1979(Q3)} > T_{t\geq 1979(Q3)}$ at the 1%

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neighboring quarters which are available. Alternatively, dropping these entries does not alter the results qualitatively.

[20] $T = 1$ implies that each firm updates information every quarter and so there is no sticky information. In this case the SIPC relationship breaks down and equation (1.32) is not valid. Thus, I focus on $T \in \{2, 3, 4\}$

[21] See A.2.1 for details.

[22] The null hypothesis of $H_0 : T = 4$ v/s $H_a : T < 4$ cannot be rejected even with 90% confidence in a likelihood ratio test.

[23] see A.2.1 for details.

level of significance, i.e. firms updated their information more frequently prior to 1979(Q3). Moreover, using the second breakpoint, I am able to reject the hypothesis that $T_{t<1982(Q1)} \leq T_{t\geq 1982(Q1)}$ in favor of the alternative $T_{t<1982(Q1)} > T_{t\geq 1982(Q1)}$ at the 5% level of significance. These results are supportive of the model's predictions. Through the lens of the model, the aggregate price responded more quickly to monetary policy shocks in the 1970's than in subsequent decades, and hence monetary policy had a relatively less persistent effect on output in the 1970's.

1.7 Conclusion

Unlike the sticky information literature, the model presented in this paper is capable of explaining the differential adjustment of prices in response to monetary and idiosyncratic shocks. By relying on costly information rather than physical menu costs, the model is also consistent with the findings of Zbaracki et al. (2004), who find that information processing costs associated with pricing decisions are extremely important in determining pricing behavior. The model relies on sluggishness in prices to explain monetary non-neutrality. This sluggishness in price adjustment is caused by the decision of all firms to delay the acquisition of new information. This channel is shown to be particularly important by Klenow and Willis (2007) who find that price changes in the CPI reflect older information than would be predicted by a costless-flexible information environment.

One of the key implications of the model is that the persistence of output in response to a monetary shock should be decreasing in the degree of monetary policy discretion, or alternatively the level of inflation volatility. Using data from the 1970's and subsequent decades to estimate the model-consistent Phillips curve, I confirm the hypothesis that the persistence was indeed lower in the 1970's, which has been identified as a period where monetary policy was actively trying to exploit the Phillips curve relation and was highly discretionary.

The model is also able to account for features seen in micro pricing data as documented in Mankiw and Reis (2010). When calibrated to match frequently changing prices at the micro level, the model is still able to generate substantial non-neutralities. This is an improvement over standard menu cost models, as it is able to explain a sluggish response of the aggregate price to monetary shocks despite large and frequent price changes at the micro level to idiosyncratic shocks. In addition, the model does not suffer from the problem of the 'missing middle' and is able to generate a highly kurtotic distribution of price changes as has been documented by some researchers. The paper also provides micro-foundations for static indexation of prices, which is often assumed in the New Keynesian literature.

A shortcoming of the model is that it struggles to explain the existence of spells during which prices do not change at all. This is because price changes are costless, and thus firms are not deterred from changing prices by small amounts. Prices do not adjust continuously at low levels of inflation in the data. Adding menu costs to change prices does would resolve this problem but reintroduces the dilemma of the 'missing middle'. A possible resolution of this discrepancy can potentially be found in a rational inattention setup like in Woodford (2008), as mentioned earlier. This is left for future research.

Chapter 2: Dispersed Beliefs and Aggregate Demand Management

2.1 Introduction

There is a long tradition in economics of trying to explain business cycle fluctuations as arising from changes in expectations of economic agents. These ideas date back at least to the early twentieth century and are highlighted in the works of Pigou (1926) and Keynes (1936) who argue that investment decisions are subject to changes in sentiment, which in turn result in fluctuations in output. Pigou (1926) conjectured that limited information and linkages among agents in an economy heighten the incidence of correlated forecast errors across agents over time. These correlated forecast errors could be thought of as *waves of optimism and pessimism* which drive the economy as a whole to over or under-invest, causing fluctuations in output. In a similar vein, Keynes (1936) posited that institutions such as the stock market cause average opinion in the economy to become an object of speculation. Thus, fluctuations in average opinion, which may be unrelated to fundamentals of the economy, can affect investment behavior, giving rise to the notion of business cycle fluctuations driven by *animal spirits.* The basic idea behind these sentiment-driven business cycles can be summarized as follows: when consumers and firms become *optimistic* (*pessimistic*) about economic prospects, they consume and invest more (less), resulting in a boom (slump). I present a real business cycle model with fully rational agents in which pessimism and optimism in agents' beliefs are generated endogenously. More precisely, I introduce dispersed information in a standard incomplete markets heterogeneous-agent DSGE model with flexible prices and strategic complementarity in production amongst monopolistically competitive firms. In particular, this paper emphasizes how endogenously generated correlated forecast errors about economic fundamentals across agents interact with their investment behavior to cause volatile swings in both current and future output. Further, I show that these fluctuations are inefficient and that fiscal policy can be used to reduce these fluctuations and increase welfare.

In the model, agents do not directly observe the shocks hitting the economy; the presence of private information causes agents to hold heterogeneous beliefs about fundamentals. As a result, agents rely on both public and private information to infer the true fundamentals of the economy. Public signals are comprised of of market-consistent information such as wage rates and asset prices. In addition, strategic complementarity implies that each agent, in addition to inferring the true fundamentals, must also infer what others perceive the fundamentals to be. In other words, each agent must forecast the private information of other agents in order to deduce their actions. Each agent, however, internalizes that all others have the same forecasting problem, causing an infinite regress in expectations. This is known as the problem of *"forecasting the forecasts of others"* (Townsend, 1983). The infinite regress causes endogenous public signals such as prices to be less effective in conveying the true state of the economy, driving and perpetuating the existence of dispersed information.

Agents cannot infer the true fundamentals of the economy from the information they receive and must base their decisions on their forecast of true fundamentals. Since all agents learn from some common signals in the form of market consistent information, their forecasts are correlated. As a consequence their forecast errors are correlated. Since agents base their production and investment decisions on these correlated forecasts, there is a natural feedback effect on prices. Correlated forecast errors can be interpreted as pessimism and optimism. Importantly, the model shows that these correlated forecast errors can have both static and dynamic effects on the economy. The static effects arise from the firms' contemporaneous output decisions which are based on correlated forecasts of today's fundamentals. In contrast, the dynamic effects are due to households' intertemporal investment decisions, which interact with these correlated forecast errors to produce persistent fluctuations in the business cycle. These mechanisms are briefly summarized below.

Dispersed information coupled with strategic complementaries in firms' production decisions imply that actual output is a function of true fundamentals and average beliefs about aggregate demand. Correlated forecasts in the form of pessimistic beliefs about aggregate demand today lead to a decline in current output as firms cut back on production. These pessimistic beliefs can potentially be triggered by shocks completely unrelated to the true fundamentals. This phenomenon occurs because agents are unable to separate the effects of this noise from changes in true fundamentals. This is the static effect of dispersed information on the business cycle. In addition, dispersed information with strategic complementarity implies that agents react cautiously to information that they fear isn't shared by others. This results in prices not reflecting information about current conditions, and hence output to remains persistently low. Moreover, pessimistic beliefs about today's fundamentals also influence household investment decisions in two ways. First, lower aggregate economic activity directly impinges on the households' wealth, causing them to accumulate less capital. Second, since aggregate productivity is persistent, expectations of low aggregate economic activity today cause households to expect a low benefit from holding capital tomorrow. Consequently, households accumulate less capital, giving rise to a smaller capital stock that is available for production tomorrow. Hence, pessimism (optimism) about fundamentals today can lead to persistent declines (increases) in output. This dynamic effect of investment decisions interacting with correlated forecasts is the driving mechanism for persistent and volatile business cycle fluctuations in this model and is absent from the other literature on dispersed information. In summary, it is the noisy informational content of endogenous price signals and the bias embedded in them from correlated forecast errors that generate fluctuations based on sentiments.

I show that the volatility and persistence generated by dispersed information is inefficient. The planner can reduce this volatility and increase welfare by using countercyclical sales subsidies for firms and countercyclical investment subsidies for households financed by dividend taxation. This result holds even if the planner does not possess an informational advantage over private agents. By making taxes and subsidies contingent on the realization of aggregate output, the planner can provide firms with insurance against low profits by reducing the dependence of their output decision on average beliefs. In the absence of government intervention, agents put too much emphasis on prices as a source of information and too little weight on their private information at the time of making their decisions. This results in an information externality, and there is an inefficiently low amount of information produced. Prices do not reflect information about current conditions, as they aggregate decisions which do not respond strongly to new information. The inefficiently low informational content of prices reinforces the dispersed information by slows down agents' learning about fundamentals. Furthermore, pessimism and optimism caused by dispersed information have systematic effects on prices and cause over-investment in booms and under-investment in recessions. The planner can reduce this by instituting a countercyclical sales subsidy, which acts as a Pigouvian tax on the use of public information and incentivizes information production. The sales subsidy, by providing insurance, reduces the effective degree of strategic complementarity in the firms' output decisions, making it less sensitive to average perceptions. Additionally, it also reduces the reliance of agents on prices as sources of information. As a result, agents respond more to idiosyncratic exogenous information which provides a more accurate picture of true fundamentals than the asset prices which contain slow moving information. Since firms' decisions now respond to new information more quickly, prices, which aggregate these decisions, are less informationally inefficient. These findings confirm the Pigouvian conjecture that fluctuations arising from dispersed information are inefficient and may benefit from corrective action.

The importance of exploring belief-driven business cycles is highlighted by the common criticism of the lack of an internal propagation mechanism in standard business cycle models. These models cannot quantitatively account for post-war recessions without relying on a large technological regress as the driving force (Cogley and Nason, 1995). This criticism is particularly relevant since there is general consensus that the Great Recession was a result of the bursting of the housing bubble and was not driven by a severe decline in technology. Since bubbles represent non-fundamental movements, the Great Recession highlights the importance of exploring the plausibility of these sentiment driven business cycles as an alternative to the technology-based models.

However, it has been surprisingly hard to incorporate expectation-driven cycles into a a standard business cycle model with rational agents. Models with dispersed information in which agents face dynamic signal extraction problems as in the current setup have been proven difficult to work with. The infinite regress in expectations generates an infinite dimensional state space (Townsend, 1978). Townsend (1983) showed, in very stylized partial equilibrium examples, that economies with dispersed information where agents have to "forecast the forecasts of others" can exhibit "rapid oscillations in forecasts and decision variables in response to economic shocks"(Townsend, 1983, pg. 548). Recently, models with dispersed information as the cause of fluctuations have regained popularity with the influential work by Woodford (2002), Lorenzoni (2009) and Angeletos and La'O (2008, 2010). Angeletos and La'O (2008) explore a canonical business cycle model with firms making output decisions in an environment with dispersed information; they show that noise can be amplified and drive fluctuations independent of fundamentals of the economy. However, they simplify the problem by assuming that agents do not learn from endogenous signals such as prices.[1] In addition, they abstract from intertemporal decisions under dispersed information such as capital accumulation, and instead focus on the static effects of dispersed information. This paper complements the literature by adding learning from prices and an intertemporal investment decision. Lorenzoni (2009) shows that dispersed information in the presence of nominal rigidities can lead to noise driven fluctuations. The current paper highlights that perceptions about aggregate demand can cause fluctuations in current and future output in an environment with flexible prices. This opens the door for aggregate demand management policies even in the absence of nominal rigidities in a neoclassical model. This is an important result, as most aggregate demand management policies are seen as a response to some nominal rigidity in an economy. This result stands in contrast to Angeletos and La'O (2010) who find that fluctuations caused by noise under dispersed information are constrained efficient. The main reason why these fluctuations are not constrained efficient in this model is that agents are allowed to learn from prices. As a result, the planner can use subsidies to indirectly manipulate the informational content of prices as discussed earlier. Thus, the planner can influence the decisions of forward looking agents by affecting the information that they receive through prices. This result is reminiscent of Weiss (1980) and King (1982), who showed that when agents learn from prices, policy, in addition to affect-

[1]Recently many papers have introduced dispersed information as a friction in models. Amador and Weill (2010), Hellwig and Venkateswaran (2009), Hellwig and Venkateswaran (2012) and Graham and Wright (2010) are just some of the papers which have introduced learning from market information. However, most of these papers study the propagation of monetary policy shocks in these environments. Graham and Wright (2010) is the exception in this list as it shows that market-consistent information in a heterogeneous agent model with dispersed information does not reveal the true fundamentals to agents.

ing agents' economic decisions directly, can also affect the information content of prices by affecting the weights agents put on the endogenous and exogenous signals.

This paper is also related to the literature on information-based business cycle models. Early attempts by Phelps (1970) and Lucas Jr. (1972) showed that imperfect information about monetary shocks could result in fluctuations in output arising from monetary policy. However, the empirical relevance of this channel has been hotly debated, since these models require either a significant lack of information or a significant delay in access to information. Recently there has been a resurgence in the popularity of models that attempt to provide micro-foundations for why agents might make decisions under incomplete information. For example, the rational inattention literature (Sims, 2003; Mackowiak and Wiederholt, 2009) argues that limited ability to process information effectively implies that agents make decisions under noisy information even if all information is available freely. Alternatively, the sticky information literature (Mankiw and Reis, 2002; Reis, 2007) assumes that different agents in an economy get access to new information at different times. Acharya (2012) shows that planning complementarites may cause agents to optimally delay their acquisition of information that is publicly available. This paper provides another justification for the reduced form assumption of imperfect information by showing that with dispersed information, prices may fail to reveal all information about the true fundamentals of the economy, and hence agents make decisions under incomplete information.

Another related strand of literature has focused on full information models with news shocks to generate business cycle fluctuations. This literature studies the response of an economy to news about future economic fundamentals. This literature has faced difficulties, as Barro and King (1984), Cochrane (1994) and others have shown that the standard business cycle model generates a recession today in response to good news about future productivity. Optimism about the future leads to a contraction in output. These models have had limited success in generating the empirically observed co-movement in consumption, investment and employment. Recently, Jaimovich and Rebelo (2009) have shown how particular preference structures combined with variable capital utilization can yield positive co-movement in response to good news about tomorrow. This literature labels exogenous good (bad) news about the future as optimism (pessimism), while the current paper defines optimism (pessimism) as an endogenously generated belief which results in agents over-estimating (under-estimating) future productivity.[2]

This paper also addresses Hayek's (1945)[3] claim that markets are particularly effective at dealing with the limits to information and perception that are inherent in a market environment with a large number of participants. He argued that prices aggregate information efficiently and made dispersed information irrelevant. This paper shows that this is not always the case and highlights that prices in fact may exacerbate the problem by reinforcing dispersed information. Becasue prices do not aggregate information efficiently, the market needs to be supplemented by

[2]There is a separate literature that uses the idea of indeterminacy of equilibrium and sunspots in explaining business cycles fluctuations arising from "animal spirits". This literature provides an attractive way to explain expectation-driven business cycle fluctuation. However, in this paper, I impose restrictions which rule out the existence of sunspots and multiple equilibrium. Thus, my model provides a channel for belief-driven business cycles which is separate from the mechanisms described in the aforementioned literature. See Benhabib and Farmer (1999) for a survey on sunspots and indeterminacy.

[3]Hayek (1945)

government intervention to restore efficiency.

As discussed above, models with dispersed information featuring a dynamic signal extraction problem have been regarded as intractable, as they lead to infinite dimensional state spaces. This complexity has resulted in very few papers exploring the implications of dispersed information in a fully specified business cycle model. This paper draws on methods suggested by Whiteman (1983) and Kasa (2000), which transform the problem into the frequency domain to make the problem more tractable and helps in computing exact solutions. This can be seen as a methodological contribution of the current paper, which complements alternate approaches used to solve the infinite regress problem, such as Nimark (2010), Lorenzoni (2009) and others who attempt to find approximate solutions in the time domain.

The paper is organized as follows. Section 2.2 describes the model economy. Section 2.3 sets up the log-linearized model and solves for the equilibrium under the full information case and under a case with dispersed information. I then highlight how dispersed information changes the predictions of the standard model by presenting results from a calibration exercise. In Section 2.5, I explore the normative implications of dispersed information and present the solution to the problem of a constrained social planner. Next, I show how the constrained efficient allocations can be implemented by a policy maker who has no informational avantage over the private sector. I conclude in Section 2.6.

2.2 Model

The model is a slight variant of the standard business cycle model with monopolistic competition. There is a continuum of islands indexed by $i \in [0, 1]$. Each island consists of a continuum of consumers and a representative monopolistically competitive firm. There is perfect symmetry of information across agents on the same island but asymmetries exist across islands which will be explained in detail in the subsequent sub-sections.

There is perfect consumption insurance within the island, permitting the existence of a representative household on each island. From here on, I shall refer to the representative household on island i as household i and the representative firm on island i as firm i. Household i owns and operates firm i. Thus, the household owns the claims to the firm's profit. There are incomplete markets across islands, i.e. households do not have access to a full set of Arrow securities. Households on each island accumulate capital, which they rent to firms on other islands through a centralized capital market. Labor is immobile across across islands. Thus, the only way islands can insure against island-specific risk is by investing in capital.

The monopolistically competitive firm on island i produces variety Y_i using capital and labor. It hires labor only from the island i labor market at wage $\omega_{i,t}$. However, it rents capital from an economy-wide capital market at the rental rate r. The final good is given by a Dixit-Stiglitz aggregator over all varieties $i \in [0, 1]$:

$$Y = \left[\int_0^1 Y_i^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

Consumption and investment is in terms of the final good which is also the numeraire.[4]

2.2.1 Exogenous Shocks to the Economy

Agents on island i in the economy are hit by both aggregate and idiosyncratic shocks. As in the standard business cycle literature, aggregate productivity in the economy varies over time. Productivity on each island is denoted by $z_{i,t}$ which is comprised of two components, z_t and $e_{i,t}$:

$$\ln z_{i,t} = \ln z_t + \ln e_{i,t}$$

where z_t is the aggregate component of productivity, which follows a mean reverting process:

$$\ln z_t = \rho_z \ln z_{t-1} + v_t \ , \ v_t \sim N(0, \sigma_v^2) \tag{2.1}$$

and $e_{i,t}$ is island specific idiosyncratic productivity. I assume $\ln e_{i,t} \sim N(0, \sigma_e^2)$ is i.i.d across time and satisfies an adding up constraint:

$$\int_0^1 \ln e_{i,t} di = 0, \forall t \tag{2.2}$$

Agents on island i can observe $z_{i,t}$ but not its individual components.

[4] The aggregate price level in this economy is defined by the standard price index $P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di\right]^{\frac{1}{\theta-1}}$. In the absence of a nominal anchor, this price P_t is undetermined. Hence, without loss of generality, I set $P_t = 1$, $\forall t$.

At the beginning of period t, the total capital stock available for production is given by $K_t + \zeta_t$, where $\zeta_t \sim N(0, \sigma_\zeta^2)$ is an aggregate shock to the availability of capital and is i.i.d across time. These shocks are not directly observed by agents. Agents can infer the realization of the shocks by observing the rental rate of capital. Shocks to the supply of capital may be seen as agents from outside the economy investing or supplying capital at the beginning of the period. These agents must be compensated for the use of their capital at the end of their period. Such shocks are common in the market micro-structure literature such as Grossman and Stiglitz (1980) and also in macro-finance literature such as Singleton (1987). This shock implies that prices such as the rental rate do not fully reveal the true state of aggregate productivity to agents. On observing a higher rental rate of capital, agents cannot conclude that this is due to a higher aggregate productivity as it could be due to a negative shock to the supply of capital available to firms. This shock is introduced as more of a modeling tool to ensure that prices do not reveal the true state of the economy, and the volatility of this shock can be set to a very low number to remove other direct effects it has on the economy.

Define $\varepsilon_{i,t} = \{v_t, \zeta_t, e_{i,t}\}$ and $\varepsilon_t = \{v_t, \zeta_t, 0\}$. Note that $\varepsilon_t = \int_0^1 \varepsilon_{i,t} di$. In addition, denote $\varepsilon_i^t = \{\varepsilon_{i,t-s}\}_{s=0}^{\infty}$ as the sequence of all realizations of ε_i through period t. Similarly, $\varepsilon^t = \{\varepsilon_{t-s}\}_{s=0}^{\infty}$ denotes the complete history of ε through period t. From this point forth, I shall refer to ε^t as *the true aggregate fundamentals* of the economy and to ε_i^t as the *true fundamentals for island i*.[5] Agents on island i do not

[5]Knowing $\varepsilon^{i,t}$ is sufficient for agents to infer the realizations of the aggregate component of productivity, the island specific component of productivity, the shock to the supply of capital.

observe $\varepsilon^{i,t}$ or ε^{t} and try to infer them using both endogenously and exogenously generated information. As is standard in the business cycle literature, I assume that the innovations are mutually independent. The unconditional distribution of $\varepsilon_{i,t}$ follows a normal distribution, $\varepsilon_{i,t} \sim N(0, \Sigma)$, where Σ is a diagonal matrix.[6]

The timing of events is as follows. For convenience, I divide each period into two sub-periods, which are labeled the "pre-production" phase and the "post-production" phase. In the pre-production stage of period t, firms and households on island i observe island-specific productivity $z_{i,t}$ and an exogenous signal of aggregate productivity φ_t, which is defined in the next sub-section. A shock to the supply of capital ζ_t is realized but not observed. This shock can temporarily increase or decrease the supply of capital to firms. Agents on island i also observe the rental rate for capital, r_t. Agents on an island use information from various sources to try and infer the true fundamentals of the economy. Given its inferences of the fundamentals of the economy, household i decides how much labor to supply, as well as how much to consume and invest. Actual Investment is a an ex-post residual and ensures that the budget constraint of the household is satisfied. Firm i decides how much labor to hire from the local labor market and the how much capital to rent from the economy-wide capital market. As firm i does not know the true aggregate state of the economy at the time it chooses its inputs, this implies that aggregate output/demand Y_t and actual demand for a firm's output Y_{it} are unknown.[7]. Once the firms and households finish making their decisions, the "pre-production" phase

[6]Even though Σ is assumed to be a diagonal matrix, the setup is flexible enough to allow for any structure of contemporaneous correlation.

[7]See Section 2.2.4 for more details.

ends. In the next phase, all firms undertake production, and relative prices of each variety, $\{p_{i,t}\}_{i\in[0,1]}$, adjust to clear markets. Firms observe the realization of the relative price $p_{i,t}$ of their own variety and pay out dividends/issue equity. This ends the "post-production" phase. The timeline is summarized in Figure 2.1.

2.2.2 Information Structure

A key deviation of this model from the standard business cycle literature is that agents cannot observe the shocks hitting the economy, i.e. agents cannot observe $\varepsilon_{i,t}$ directly. The economy is divided into a continuum of informationally isolated partitions or "islands" (Lucas Jr., 1972). There is perfect symmetry of information across agents on the same island but there is dispersion of information across islands. Agents on each island possess some private information about the true fundamentals of the economy. Since agents do not observe the true fundamentals of the economy, they must form inferences about $\varepsilon_{i,t}$ by looking at information available to them, which originates from both endogenous and exogenous sources.

Exogenous sources of information are not affected by market interactions. In the current model, this can be further divided into two sub-categories: private and public. Private exogenous sources refer to the signals that agents on island i receive about island specific productivity $z_{i,t}$. In addition to these private exogenous signals, each agent observes a noisy public signal of aggregate productivity.

$$\varphi_t = z_t + u_t \tag{2.3}$$

where $u_t \sim N(0, \sigma_u^2)$ is an i.i.d. measurement error which is common across agents. This signal is supposed to proxy for information that agents could get from non-market signals. Define $x_{i,t}^{exo} = \{z_{i,t}, \varphi_t\}$. Following Rondina and Walker (2012), information originating from exogenous sources is labeled $\mathbb{U}_t(x_i^{exo})$ which denotes the smallest sub-space spanned by the current and past realizations of $x_{i,t}^{exo}$.

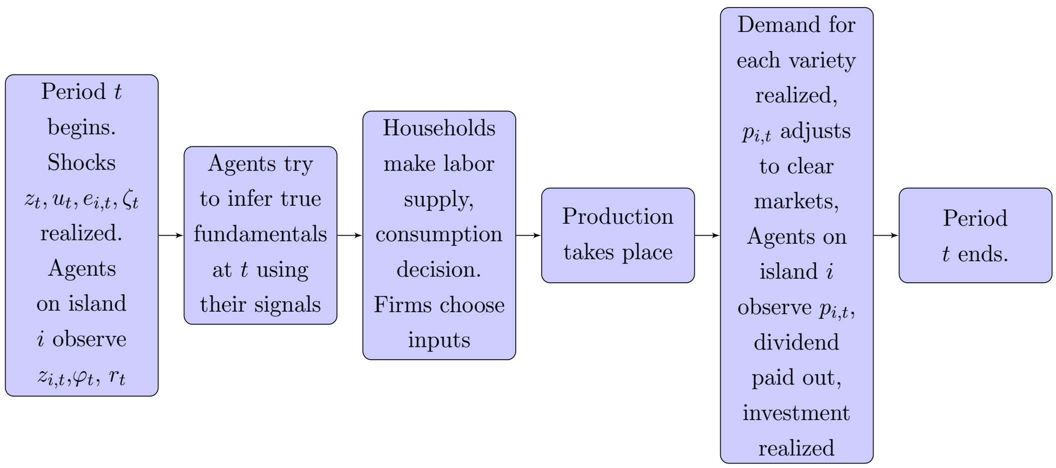

Figure 2.1: Timeline

Endogenous sources of information are those which are influenced by market interactions. In a setting with dispersed information, prices convey some of the private exogenous information of other agents. This is because prices aggregate information from decisions of other agents which are partly contingent on their private information. In the current setting, at time t agent i can observe market-consistent information, i.e. the history of all rental rates, r_t, and the relative price for variety i up to period $t-1$, $p_{i,t-1}$. Define $x_{i,t}^{end} = \{p_{i,t-1}, r_t\}$. Information from endogenous sources can also be split into two categories: $\mathbb{V}_t(x)$ and $\mathbb{M}_t$. $\mathbb{V}_t(x_i^{end})$ denotes the smallest closed sub-space spanned by current and past realizations of

$x_{i,t}^{end}$. $\mathbb{M}_t$ captures the notion of rational expectations and the assumption that agents know the equilibrium processes which generate x_t^{end} and x_t^{exo}. This component is referred to as the cross-equation restrictions associated with rational expectations equilibria.

The time t information of agents on island i is denoted by $\mathcal{I}_{i,t}$:[8]

$$\mathcal{I}_{i,t} = \mathbb{U}_t(x_i^{exo}) \vee \mathbb{V}_t(x_i^{end}) \vee \mathbb{M}_t \tag{2.4}$$

Given the information set of agent i at time t, $E_t[\cdot \mid \mathcal{I}_{i,t}] \equiv E_{i,t}[\cdot]$ denotes the projection of a variable onto the information set $\mathcal{I}_{i,t}$.[9]

Since agents do not observe shocks hitting the economy in an economy with dispersed information, aggregate shocks like z_t, u_t and ζ_t are not present in agent $i's$ information set. However, given their information sets, agents can forecast these aggregate fundamentals. Let $E_{i,t}z_t$ represent an agent on island i's forecast of contemporaneous aggregate productivity. I label the scenario where $E_{i,t}z_t > (<) z_t$ as agents on island i being optimistic (pessimistic) about aggregate productivity. Consistent with this definition, I label the scenario where $\overline{E}_t z_t > (<) z_t$ as the economy as a whole being characterized by optimism (pessimism) about aggregate productivity. $\overline{E}_t z_t$ refers to the average belief about aggregate productivity in the economy. The operator $\overline{E}$ is defined below. The definitions of pessimism and optimism are

[8]The operator $\vee$ is defined as follows: $A \vee B$ denotes the smallest closed sub-space which spans the sub-spaces A and B.

[9]The projection will be defined more concretely in the linear economy defined in Section 2.3. More precisely, the projection in the linear model is the linear projection operator which is equivalent to the conditional expectations operator under the assumption that $\varepsilon_{i,t}$ is a normal random variable.

summarized as follows:

Definition 1. *The economy is optimistic about X if*

$$\overline{E}_t X > X$$

and is pessimistic about X if

$$\overline{E}_t X < X$$

where $\overline{E}_t$ refers to the the average expectations operator and is defined below in definition 2.

This definition is different from the definition of optimism and pessimism in the news shocks literature. The news shock literature defines an exogenous positive signal about tomorrow's fundamentals as optimism. However, optimism or pessimism here refers to a phenomenon which is generated endogenously within the model.

Definition 2. *The average expectation of a random variable X across agents in the economy at time t is given by:*

$$\overline{E}_t X = \int_0^1 E_{i,t} X di \tag{2.5}$$

$\overline{E}_t X \equiv \overline{E}_t^{(1)} X$ is also referred to as the First-Order Expectation of X and represents the first moment of the distribution of forecasts of X.

At this stage, it is useful to point out the differences in assumptions in the papers which are closely related to the current setup. Unlike this paper, Angeletos and La'O (2008) assume that agents do not learn from endogenous signals, which in terms of notation implies that $\mathbb{V}_t(x_i^{end}) = \{0\}$ in their model. Graham and Wright (2010) assume that agents only have access to market-consistent information, i.e. endogenous public signals in the form of prices, which in terms of the current notation implies that $\mathbb{U}_t(x_i^{exo}) = \{0\}$ in their model. Some other recent papers like Eusepi and Preston (2011) have also tried to explain business cycle fluctuations on the basis of learning and expectations formation. The basic environment in these papers is distinct from the current setup as they deviate from rational expectations by assuming that agents do not have the knowledge contained in $\mathbb{M}_t$. Agents in these models try to learn about $\mathbb{M}$ over time. In the current setup, I stay within the framework of rational expectations. Agents know the equilibrium pricing functions. In fact, without the knowledge of these functions, they would not be able to filter as much information out of the prices. Agents utilize the knowledge of these equilibrium pricing functions to infer the true fundamentals, which play a part in determining the equilibrium prices.

Notice that in the economy with dispersed information, each agent has an independent private signal of the aggregate productivity:

$$\ln z_{i,t} = \ln z_t + \ln e_{i,t} \tag{2.6}$$

If agents were able to share information, averaging the sequence of signals $\{z_{i,t}\}_{i\in[0,1]}$ across the agents would yield the true value of aggregate productivity, z_t in that period and the model would become identical to the full information model.[10] I assume that there is no market for information where agents can explicitly trade their private information. This is a reasonable assumption, as information markets do not exist widely and the ones that do exist are not very deep. This is because there is reason to believe that agents may strategically choose to withhold information.[11] Despite the absence of an explicit market where agents can trade information, I allow agents in my model to learn about the private information of other agents from their market interactions. In particular, prices in this setup provide information about the private information of others, and hence the true fundamentals. Thus, prices can potentially reveal the true state of the economy. However, as shown in Section 2.3.2, prices fail to be fully informative because of the infinite regress in expectations induced by the presence of dispersed information. Market incompleteness is very important for this result to hold. If agents had access to a full set of Arrow securities, the prices of those securities would provide information about the true state of the economy and might reveal it to agents. If agents could recover the true fundamentals of the economy by observing these prices, the economy would converge to the symmetric full information case.

[10]This follows from the adding up constraint (2.2).

[11]There is a large literature studying the strategic motives that may cause agents to withhold their private information. For example, Okuno-Fujiwara et al. (1990) show how agents holding private information may strategically provide information which is less precise than their private information. Another example is the "cheap talk" literature such as Crawford and Sobel (1982). This literature studies how strategic motives may not allow an agent to communicate his information to others.

2.2.3 Households

Household i chooses consumption, $C_{i,t}$, and labor supply, $H_{i,t}$, at the beginning of the period and investment in capital $K_{i,t+1}$, is the residual. The timing described above implies that at the beginning of period t, households observe the wage rate, $\omega_{i,t}$, and the rental rate, r_t. However, dividends are not paid to the household until after production takes place and relative prices clear the markets. The household is constrained in its choices of consumption and labor supply such that tomorrow's capital stock $K_{i,t+1}$ is non-negative for any realizations of $\Pi_{i,t}$. The problem of the representative household on island i can be written as:[12]

$$\max_{\{C_{i,t},H_{i,t},K_{i,t+1}\}_{t=0}^{\infty}} E_{i,0}\sum_{t=0}^{\infty}\beta^t \ln\left\{C_{i,t}-\Psi\frac{H_{i,t}^{1+\nu}}{1+\nu}\right\}$$

subject to

$$\begin{aligned} C_{i,t}+K_{i,t+1} &= \omega_{i,t}H_{i,t}+[1+r_t-\delta]K_{i,t}+\Pi_{i,t} \\ K_{i,t+1} &\geq 0 \end{aligned}$$

where $\Pi_{i,t}$ refers to the profit of firm i. $E_{i,0}[\cdot]$ refers to the expectations conditional on the information set of agent i at time 0. The optimality conditions for household i can be expressed as follows:

$$H_{i,t}^{\nu} = \frac{\omega_{i,t}}{\Psi} \tag{2.7}$$

[12]Note that solving the model is simplified by the absence of an income effect on labor supply.

$$1 \leq \beta E_{i,t} \left\{ \left[\frac{(1+\nu)C_{i,t} - \Psi H_{i,t}^{1+\nu}}{(1+\nu)C_{i,t+1} - \Psi H_{i,t+1}^{1+\nu}} \right] [1 + r_{t+1} - \delta] \right\} \ , \forall \ \Pi_{i,t} \tag{2.8}$$

The assumption of dispersed information can be seen in the fact that the expectations of agent i are indexed by i since the information sets across islands are heterogeneous. This implies that the agent uses his own beliefs to calculate expected benefits in the future from investing in capital. These beliefs differ across islands.

2.2.4 Firms

There is a representative monopolistically competitive firm on island i which is owned by the representative household on the island. Prices are flexible. The firm's objective is static and is to maximize expected dividend payments to its owner. Firm i can only employ labor, $L_{i,t}$, from the same island. Since the firm is the only employer of labor on island i, it internalizes the fact that hiring more labor is going to raise the wage rate.[13] Firm i also rents capital, $J_{i,t}$, from an economy-wide rental market at a rental rate r_t, which it takes as given.

As per the standard Dixit-Stiglitz demand system, demand for a particular

[13] I assume that the firm internalizes the price effect in the labor market to introduce increasing marginal costs in the firm's production decision. It is well known that with a constant return to scale production function and linear costs of hiring inputs, the cost minimization problem of a firm yields a constant marginal cost function [see Mas-Colell et al. (1995, chapter 5) for details]. With a Dixit-Stiglitz demand system, the iso-elastic demand curves with constant marginal cost do not generate strategic complementarity in a firm's output decision. However, if the firm internalizes the price effect of hiring additional labor, its cost of hiring labor is no longer linear in labor and is associated with increasing marginal costs [See equation 2.17]. This is sufficient to generate strategic complementarity in the firm's output decision. If the firm produces too little relative to aggregate output, its realized relative price is too high and it could have increased profits by producing slightly more; if it produces too much relative to aggregate output, increasing marginal costs erode its profits.

firm's good is a function of both its relative price and the level of aggregate demand/output:

$$Y_{i,t} = P_{i,t}^{-\theta} Y_t$$

At the time of making its input decisions, a firm only observes its own productivity and can neither observe nor infer the level of aggregate productivity. Consequently, firms cannot infer the output of all other firms, and hence Y_t is unknown. As such, a firm must forecast the level of aggregate demand in choosing its level of output. Thus, its input decisions depend on the perceived levels of aggregate demand. This feature of the model arises because dispersed information prevents agents from inferring the true fundamentals.[14] Conversely, if firms have full information, they would be able to compute Y_t with certainty, and thus perceptions about output would not matter. With dispersed information, uncertainty about productivity manifests itself in the form of demand uncertainty even in a neoclassical model.

Once all firms have produced output and demands are realized, prices clear markets. Thus, prices adjust to make the demand relationship hold ex-post. The objective for the firm can be written as:

$$\max_{L_{i,t}, J_{i,t}} E_{i,t} \left[P_{i,t} Y_{i,t} - \omega_{i,t} L_{i,t} - r_t J_{i,t} \right] \tag{2.9}$$

[14] See Section 2.3.2.

subject to

$$
\begin{aligned}
F_{i,t}(J, L) &= z_{i,t} J_{i,t}^{\alpha} L_{i,t}^{1-\alpha} && (2.10)\\
Y_{i,t} &= P_{i,t}^{-\theta} Y_t && (2.11)\\
L_{i,t}^{\nu} &= \frac{\omega_{i,t}}{\Psi} && (2.12)
\end{aligned}
$$

Equations (2.10) and (2.11) are the production technology of firm i and the ex-post demand curve facing firm i, respectively. Equation (2.12) is the labor supply on island i and is derived from equation (2.7) from the maximization problem of household i. The solution to the firms' problem can be characterized by demands for labor, $L_{i,t}$, and capital, $J_{i,t}$:

$$
L_{i,t} = \left\{ \left[\frac{\theta-1}{\theta}\right]^{\theta} z_{i,t}^{\theta-1} \left[\frac{\alpha}{r_t}\right]^{\alpha(\theta-1)} \left[\frac{1-\alpha}{\Psi(1+\nu)}\right]^{1+(1-\alpha)(\theta-1)} \left[E_{i,t} Y_t^{\frac{1}{\theta}}\right]^{\theta} \right\}^{\frac{1}{1+\nu[\alpha+\theta(1-\alpha)]}} \tag{2.13}
$$

$$
\begin{aligned}
J_{i,t} = \Bigg\{ \left[\frac{\theta-1}{\theta}\right]^{\theta(1+\nu)} z_{i,t}^{(\theta-1)(1+\nu)} \left[\frac{\alpha}{r_t}\right]^{1-\alpha+\theta(\alpha+\nu)} \left[\frac{1-\alpha}{\Psi(1+\nu)}\right]^{(1-\alpha)(\theta-1)} \cdots \\
\times \left[E_{i,t} Y_t^{\frac{1}{\theta}}\right]^{\theta(1+\nu)} \Bigg\}^{\frac{1}{1+\nu[\alpha+\theta(1-\alpha)]}} \qquad (2.14)
\end{aligned}
$$

Output of firm i is given by

$$
\begin{aligned}
Y_{i,t}^{\frac{1}{\theta}} = z_{i,t}^{\frac{1+\nu}{1+\nu[\alpha+\theta(1-\alpha)]}} \left\{\frac{\theta-1}{\theta} E_{i,t} Y_t^{\frac{1}{\theta}}\right\}^{\frac{1+\alpha\nu}{1+\nu[\alpha+\theta(1-\alpha)]}} \left[\frac{\alpha}{r_t}\right]^{\frac{\alpha(1+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}} \cdots \\
\times \left[\frac{1-\alpha}{\Psi(1+\nu)}\right]^{\frac{1-\alpha}{1+\nu[\alpha+\theta(1-\alpha)]}} \qquad (2.15)
\end{aligned}
$$

Equations (2.13) and (2.14) show that firm i's decisions of how much labor and capital to hire depend on its perception of aggregate output. Equation (2.15) shows that a firm's output responds not only to fundamentals but also to the firm's perception of aggregate output/demand in the economy. A pessimistic view of aggregate demand causes a firm to contract its output by scaling down its inputs. Thus, this strictly neoclassical economy with no nominal rigidities displays Keynesian features where firms respond to perceptions about the level of aggregate demand. This is not the case in the model without dispersed information as $E_{i,t}Y_t^{\frac{1}{\theta}} = Y_t^{\frac{1}{\theta}}$ under full information. Hence, dispersed information coupled with strategic complementarity in the firm's output decision implies that perception of aggregate demand becomes a state variable in the firm's problem.

The market clearing price for variety i can be written as:

$$P_{i,t} = \left[\frac{\theta}{\theta - 1} \frac{Y_t^{\frac{1}{\theta}}}{E_{i,t}Y_t^{\frac{1}{\theta}}} \right] \overline{mc}_{i,t}(Y_{i,t}) \tag{2.16}$$

where $\overline{mc}_{i,t}(Q)$ is the marginal cost faced by firm i in period t to produce quantity Q and is given by:

$$\overline{mc}_{i,t}(Q) = \left[\frac{1}{z_{i,t}} \right]^{\frac{1+\nu}{1+\alpha\nu}} \left[\frac{\Psi(1+\nu)}{1-\alpha} \right]^{\frac{1-\alpha}{1+\alpha\nu}} \left[\frac{r_t}{\alpha} \right]^{\frac{\alpha(1+\nu)}{1+\alpha\nu}} Q^{\frac{\nu(1-\alpha)}{1+\alpha\nu}} \tag{2.17}$$

Equation (2.16) shows that dispersed information leads to time-varying markups unlike in the standard symmetric full information setting. In the symmetric full information setting, $E_{i,t}Y_t^{\frac{1}{\theta}} = Y_t^{\frac{1}{\theta}}$ and the price is a constant markup $\frac{\theta}{\theta-1} > 1$ over marginal costs. This markup is associated with the dead-weight loss associated with monopolistic competition. Imperfect information introduces a time-varying component to the markup which depends on the ratio of the actual and perceived level of aggregate demand.

2.2.5 Equilibrium

2.2.5.1 Labor Market Equilibrium

The firm on each island internalizes the labor supply curve when choosing the amount of labor to hire. In equilibrium, $H_{i,t} = L_{i,t} \equiv N_{i,t}$. The equilibrium level of employment on island i is given by:

$$N_{i,t} = \left\{ \left[\frac{\theta - 1}{\theta}\right]^{\theta} z_{i,t}^{\theta-1} \left[\frac{\alpha}{r_t}\right]^{\alpha(\theta-1)} \left[\frac{1-\alpha}{\Psi(1+\nu)}\right]^{1+(1-\alpha)(\theta-1)} \left[E_{i,t} Y_t^{\frac{1}{\theta}}\right]^{\theta} \right\}^{\frac{1}{1+\nu[\alpha+\theta(1-\alpha)]}} \tag{2.18}$$

The equilibrium wage rate on island i is given by

$$\omega_{i,t} = \Psi N_{i,t}^{\nu} \tag{2.19}$$

2.2.5.2 Capital Market Equilibrium

At time t, the rental rate r_t adjusts to equate the total supply of capital $K_t + \zeta_t = \int_0^1 K_{i,t} di + \zeta_t$ to the total demand for capital $J_t = \int_0^1 J_{i,t} di$

$$\begin{aligned} & \left\{ \left[\frac{\theta-1}{\theta}\right]^{\theta(1+\nu)} \left[\frac{1-\alpha}{\Psi(1+\nu)}\right]^{(1-\alpha)(\theta-1)} \left[\frac{\alpha}{r_t}\right]^{1-\alpha+\theta(\alpha+\nu)} \right\}^{\frac{1}{\Lambda}} \int_0^1 \left\{ z_{i,t}^{\theta-1} \left[E_{i,t} Y_t^{\frac{1}{\theta}}\right]^{\theta} \right\}^{\frac{1+\nu}{\Lambda}} di \\ = \; & K_t + \zeta_t \end{aligned} \tag{2.20}$$

where $\Lambda = 1 + \nu[\alpha + \theta(1-\alpha)]$.

2.2.5.3 Goods Market Equilibrium

The relative price of each variety adjusts to clear the market. Aggregate market clearing implies

$$Y_t = C_t + I_t + (1 + r_t - \delta)\zeta_t \tag{2.21}$$

where C is the aggregate consumption given by

$$C_t = \int_0^1 C_{i,t} di$$

and I_t is the aggregate investment

$$I_t = K_{t+1} di - (1-\delta) K_t di$$

and $(1 + r_t - \delta)\zeta_t$ represents the resources that are owed to the foreigners who lent capital to the economy.

Definition 3 (**Equilibrium**). *Equilibrium in the economy is defined as a set of sequences of allocations* $\{\{C_{i,t}, H_{i,t}, K_{i,t+1}, J_{i,t}, L_{i,t}\}_{i\in[0,1]}\}_{t=0}^{\infty}$*, prices* $\{r_t, \{\omega_{i,t}, p_{i,t}\}_{i\in[0,1]}\}_{t=0}^{\infty}$ *and information sets* $\{\{\mathcal{I}_{i,t}\}_{i\in[0,1]}\}_{t=0}^{\infty}$ *along with the sequence of exogenous processes* $\{z_t, \zeta_t, u_t\{e_{i,t}\}_{i\in[0,1]}\}$ *such that the following conditions are true:*

- *The information sets* $\{\{\mathcal{I}_{i,t}\}_{i\in[0,1]}\}_{t=0}^{\infty}$ *are defined by equation (2.4).*
- $\{\{C_{i,t}, K_{i,t+1}, H_{i,t}\}_{t=0}^{\infty}\}_{i\in[0,1]}$ *satisfy equations (2.7) and (2.8) for each household* $i \in [0,1]$ *and thus, solve the problem of the household on island* $i \in [0,1]$ *given the sequence of information sets* $\{\{\mathcal{I}_{i,t}\}_{i\in[0,1]}\}_{t=0}^{\infty}$.
- $\{\{L_{i,t}, J_{i,t}\}_{t=0}^{\infty}\}_{i\in[0,1]}$ *satisfy equations (2.13) and (2.14) for each* $i \in [0,1]$ *and solve firm* $i \in [0,1]$*'s problem in each period given the sequence of information sets* $\{\{\mathcal{I}_{i,t}\}_{i\in[0,1]}\}_{t=0}^{\infty}$.
- *The sequence of equilibrium wage rates* $\{\{\omega_{i,t}\}_{t=0}^{\infty}\}_{i\in[0,1]}$ *is determined by the firm's employment decision taking the labor supply curve as given.* $(L_{i,t} = H_{i,t})$ *each period on island* $i \in [0,1]$ *and must satisfy equation (2.19)*
- *The sequence of rental rates* $\{r_t\}_{t=0}^{\infty}$ *equates the total demand for capital by firms* J_t *to the total supply of capital* $K_t + \zeta_t$ *every period, i.e. equation (2.20) is satisfied* $\forall t$.
- *The sequences of relative prices for each variety* i, $\{\{p_{i,t}\}_{t=0}^{\infty}\}_{i\in[0,1]}$ *equate demand and supply for each variety in every period, i.e.* $p_{i,t}$ *satisfies equation*

(2.11) $\forall t$.

- *The resource constraint for the economy is satisfied $\forall t$, i.e. equation (2.21) is satisfied at all times.*

- *Consistency requires the following*

$$\begin{aligned}\int_0^1 C_{i,t} di &= C_t \\ \int_0^1 K_{i,t+1} di &= K_{t+1} \\ \int_0^1 J_{i,t} di &= J_t\end{aligned}$$

2.3 The Linear Economy

From this point forth, I will focus exclusively on the log-linearized version of the economy. This is to facilitate a tractable solution to the filtering problem.[15] I compute a log-linear approximation of the model around the non-stochastic steady state with a degenerate distribution of capital. A detailed description of the steady state is available in Appendix B.1. The equations describing the log-linearized economy are presented in Appendix B.2. In the log-linearized economy, all hatted variables are in terms of log deviations from their steady state values and variables that are not hatted are deviations in levels from their steady state values.

[15] Agents in this model are permitted to learn from endogenous signals such as prices. In a linearized model, prices are linear functions of the past and current innovations to the shocks in the economy. Agents try to infer the innovations to these fundamentals of the economy by using prices as signals. The filtering problem is made tractable by the fact that these signals are linear in the innovations. A higher order approximation would result in prices depending not just linearly on the innovations which makes the filtering problem much harder to solve.

The structure of this section is as follows. First, in sub-section 2.3.1, I present the equilibrium under the assumption of Full Information. Next, in sub-section 2.3.2, I present the equilibrium of the model with dispersed information. This allows me to contrast the predictions of the model under the two different assumptions on information sets.

2.3.1 Full Information

It is standard in the business cycle literature to assume that agents observe all contemporaneous shocks hitting the economy. Using the terminology of section 2.2.2, this implies that the exogenous sources of information at time t include the realizations of $\{\varepsilon_{i,t}\}_{i\in[0,1]}$. Notice that in the baseline case, the public signal of productivity given by equation (2.3) is redundant as all agents observe aggregate productivity without any noise.

Definition 4 (Information Set under Full Information). *The information set of any agent on island i at time t under the full information benchmark is given by:*

$$\mathcal{I}_{i,t} = \mathcal{I}_t^{FI} = \mathbb{U}_t(\{\varepsilon_j\}_{j\in[0,1]}) \vee \mathbb{M}_t \ , \forall i \in [0,1]$$

Following Whiteman (1983), I compute an equilibrium where the decisions of agents on island i at time t are driven solely by the Gaussian process ε_i^t. Linearity of the model along with the adding up constraint (2.2) ensure that the aggregate allocations and prices depend only on the Gaussian process, ε^t. More precisely, this means that the decision of agents on island i at time t lie in $\mathcal{H}_t^\beta(\varepsilon_i)$ and aggregate quantities and economy-wide prices lie in $\mathcal{H}_t^\beta(\varepsilon)$, where $\mathcal{H}_t^\beta(\varepsilon_i)$ and $\mathcal{H}_t^\beta(\varepsilon)$ denote the space of time independent beta-summable linear combinations of current and past realizations of ε_i and ε respectively. This restriction rules out the existence of sunspots and ensures the existence of a unique solution. Thus, I ignore the noisy public signal for the rest of this subsection.

In the benchmark economy, all agents in the economy are assumed to have symmetric information, implying that they form the same expectation of future fundamentals.

$$E_t[X] \equiv E[X \mid \mathcal{I}_{i,t}] = E[X \mid \mathcal{I}_t^{FI}] \ , \forall i \in [0,1]$$

where X is any random variable. The forecast by any agent in the economy of a random variable X is given by $E[X \mid \mathcal{I}_t]$, which refers to the linear projection of the random variable X onto $\mathcal{I}_t$. Since innovations $\varepsilon_{i,t}$ are assumed to be jointly normal, the linear projections correspond to conditional expectations.

The problem of the household and the firm under the benchmark case is identical to the one described in section 2.2.3 along with the additional assumption that all agents have the same information set. The full information equilibrium can be summarized by the following propositions. Details can be found in Appendix B.3.

Proposition 6 (**Decisions made by agents on island** i)**.**

- *The labor supply and capital accumulation decisions of the household on island* *i are given by:*

$$
\begin{aligned}
\widehat{H}_{i,t} &= \frac{\omega_{i,t}}{\nu} && (2.22)\\
\widehat{K}_{i,t+1} &= K_{i,t} - \frac{\alpha\beta\Psi\nu(\theta-1)H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\widehat{r}_t \\
&+ \left[\frac{\beta r}{\kappa K} + \frac{\alpha\nu(1-\beta)(\theta-1)\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right] E_t \sum_{j=0}^{\infty} \beta^{j+1}\widehat{r}_{t+j+1} \\
&- \left[\frac{Y}{\theta K} + \frac{\nu\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right]\left[(1-\beta)E_t \sum_{j=0}^{\infty} \beta^{j+1}\widehat{Y}_{t+j+1} - \beta\widehat{Y}_t\right] \\
&- \left[\frac{(\theta-1)Y}{\theta K} + \frac{\nu(\theta-1)\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right]\left[\frac{(1-\beta)\beta\rho_z}{1-\beta\rho_z}\right]\widehat{z}_t \\
&+ \left[\frac{(\theta-1)Y}{\theta K} + \frac{\nu(\theta-1)\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right]\beta\widehat{z}_{i,t} && (2.23)
\end{aligned}
$$

- *The representative firm on island i's demands for labor and capital and its output are given by:*

$$
\begin{aligned}
\widehat{L}_{i,t} &= \frac{(\theta-1)\widehat{z}_{i,t}}{1+\nu[\alpha+\theta(1-\alpha)]} - \frac{\alpha(\theta-1)\widehat{r}_t}{1+\nu[\alpha+\theta(1-\alpha)]} \\
&\quad + \frac{\widehat{Y}_t}{1+\nu[\alpha+\theta(1-\alpha)]} && (2.24)
\end{aligned}
$$

$$
\begin{aligned}
\widehat{J}_{i,t} &= \frac{(\theta-1)(1+\nu)\widehat{z}_{i,t}}{1+\nu[\alpha+\theta(1-\alpha)]} - \frac{[1-\alpha+\theta(\alpha+\nu)]\widehat{r}_t}{1+\nu[\alpha+\theta(1-\alpha)]} \\
&\quad + \frac{(1+\nu)\widehat{Y}_t}{1+\nu[\alpha+\theta(1-\alpha)]} && (2.25)
\end{aligned}
$$

$$
\begin{aligned}
\widehat{Y}_{i,t} &= \frac{\theta(1+\nu)\widehat{z}_{i,t}}{1+\nu[\alpha+\theta(1-\alpha)]} - \frac{\alpha\theta(1+\nu)\widehat{r}_t}{1+\nu[\alpha+\theta(1-\alpha)]} \\
&\quad + \frac{(1+\alpha\nu)\widehat{Y}_t}{1+\nu[\alpha+\theta(1-\alpha)]} && (2.26)
\end{aligned}
$$

Proof. See Appendix B.3 □

Proposition 7 (**Aggregate Quantities and Prices**).

- *Aggregate Capital stock evolves according to the following law of motion:*

$$\widehat{K}_{t+1} = \lambda_k \widehat{K}_t + \lambda_z \widehat{z}_t + \lambda_\zeta \zeta_t \tag{2.27}$$

where λ_k, λ_z and λ_ζ are constants and are defined in Appendix B.3.

- *The rental rate is given by:*

$$\widehat{r}_t = \frac{1+\nu}{\alpha+\nu}\widehat{z}_t - \frac{\nu(1-\alpha)}{\alpha+\nu}\widehat{K}_t - \frac{\nu(1-\alpha)}{\alpha+\nu}\zeta_t \tag{2.28}$$

- *Aggregate output is given by*

$$\widehat{Y}_t = \frac{1+\nu}{\alpha+\nu}\widehat{z}_t + \frac{\alpha(1+\nu)}{\alpha+\nu}\widehat{K}_t + \frac{\alpha(1+\nu)}{\alpha+\nu}\zeta_t \tag{2.29}$$

Proof. See Appendix B.3 □

Equations (2.24)-(2.26) demonstrate that the decisions of firm i are driven solely by the realized level of productivity on island i, the rental rate $\widehat{r}_t$ and actual aggregate output $\widehat{Y}_t$. Since all agents have the same 'correct' belief, all decisions of agents in the full information economy are driven by changes in fundamentals and there is no room for fluctuations caused by sentiments. It is straightforward to see that this economy is characterized by constrained efficiency.

2.3.2 Dispersed Information

As was explained earlier, agents on island i have access to various exogenous and endogenous sources of information. The endogenous sources of information are prices in the markets in which agents from island i participate, i.e. the rental rate $\widehat{r}_t$ and the relative price of variety i from last period, $\widehat{p}_{i,t-1}$.

Proposition 8 (Information contained in prices). .

1. Upon observing the realized relative price, $\widehat{p}_{i,t-1}$ at the end of $t-1$, a firm can infer aggregate output $\widehat{Y}_{t-1}$ without any error.

2. Wages $\omega_{i,t}$ on island i provide the agents on the island with no additional information since labor markets are local.

Firms at the time of making their output decisions do not know the actual level of aggregate demand. As a result, they do not know the market clearing relative price for their product with certainty. However, the market clearing relative price is revealed after all firms have produced. At this juncture, each firm is able to observe only its' own product's relative price. The ex-post demand curve facing each firm i at the end of period $t-1$ is given by

$$\widehat{Y}_{i,t-1} = -\theta \widehat{p}_{i,t-1} + \widehat{Y}_{t-1}$$

Thus, given $\widehat{Y}_{i,t-1}$, observing the relative price $\widehat{p}_{i,t-1}$ is the same as observing $\widehat{Y}_{t-1}$. Thus, from now on I treat $\widehat{Y}_{t-1}$ as the signal instead of $\widehat{p}_{i,t-1}$. From now on I shall refer to the vector of signals at time t as $x_{i,t} = (\widehat{z}_{i,t}, \varphi_t, \widehat{r}_t, \widehat{Y}_{t-1})$.

Definition 5. *In the economy with dispersed information, the information set of agents on island i at time t is given by:*

$$\mathcal{I}_{i,t} = \mathbb{U}_{i,t}(\widehat{z}_{i,t}, \varphi_t) \vee \mathbb{V}_t(\widehat{r}_t, \widehat{Y}_{t-1}) \vee \mathbb{M}_t$$

Given an information set, the expectation of any random variable X by agent i, denoted by $E_{i,t}X$, refers to the linear projection of X on to $\mathcal{I}_{i,t}$. Given that the shock process ϵ_t is Gaussian in this setup, these linear projections correspond to the conditional expectations. Since $\mathcal{I}_{i,t} \neq \mathcal{I}_{j,t}$, in general $E_{i,t}X \neq E_{j,t}X$.

Solving a dynamic model in which agents learn from endogenous signals is not trivial. In models with only exogenous noisy sources of information, the agent's problem of filtering this information can be solved independently of their economic decisions. This is because economic decisions do not affect the structure of information available to agents. This is the case in models such as Angeletos and La'O (2008) where agents only learn from exogenous sources of information.

However, if agents are learning from prices, this is no longer the case. As was discussed in section 2.2.2, prices aggregate the decisions of other agents, which are partly contingent on their private information. Hence, prices provide provide a summary of the private information of other agents. Prices affect the economic decisions agents make and are in turn affected by these economic decisions. Thus, agents' inference problem cannot be solved independently of their economic decisions. Moreover, the quality of information available from prices depends on which signals agents put more weight on when making their decisions. In turn, weight agents put on prices as a source of information depends on the quality of information they provide. This implies that the quality of information available from prices is determined endogenously within the model by the decisions of all agents.

I show that in such a setting, price will not provide perfectly precise signals of the true fundamentals, and that agents are therefore unable to infer the true fundamentals $\varepsilon_{i,t}$ from the sequence of all realized pubic and private information. Solving the model requires me to fully determine the behavior of agents and the informational content of prices. I follow a two step procedure. First, I solve the problem of the firm and household on each island $i \in [0,1]$, taking as given the informational content of prices. This induces a particular information set for agents on island i, $\mathcal{I}_{i,t}$. Agents make decisions measurable with respect to this information set. Prices then aggregate these decisions and the informational content of these prices can be determined. Consistency requires that the realized informational content of prices be the same as the initial assumption. This approach is operationalized in the following subsections. The full solution is presented in Appendix B.2.

For the ease of exposition define the two vectors $\xi_{i,t} = \{v_t, u_t, \zeta_t, e_{i,t}\}$ and $\xi_t = \{v_t, u_t, \zeta_t, 0\}$. Notice that $\xi_t = \int_0^1 \xi_{i,t} di$. Similar to the full information section, I compute the unique equilibrium under which agents decisions lie in $\mathcal{H}_t^{\beta}(\xi_i)$ and aggregates and prices lie in $\mathcal{H}_t^{\beta}(\xi)$, following the adding up constraint. Agents cannot observe the true fundamentals and infer the true fundamentals from their various sources of information, which include the noisy signal about aggregate productivity. Thus, I allow for agents decisions to not just depend on ε_i^t but also on $\{u_{t-j}\}_{j=0}^{\infty}$.

2.3.2.1 Step 1: The Decision of Agents given the informational content of prices

In this subsection, I fix the informational content of prices and solve for the decisions of these agents. The decisions of households i under information set $\mathcal{I}_{i,t}$ are summarized by the following proposition.

Proposition 9. *The labor supply and capital accumulation decisions of the household on island i are summarized below:*

$$\nu\widehat{H}_{i,t} = \widehat{\omega}_{i,t} \tag{2.30}$$

$$\begin{aligned}\Delta\widehat{K}_{i,t+1} = & -\frac{\alpha\beta\nu(\theta-1)\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\widehat{r}_t + \frac{\beta Y}{\theta K}\widehat{Y}_t - \left(\frac{\beta Y}{\theta K} - \frac{\nu\Psi H^{1+\nu}}{1+\nu(\alpha+\theta(1-\alpha)}\right)E_{i,t}\widehat{Y}_t \\ & + \left[\frac{\beta r}{\kappa K} + \frac{\alpha\nu(1-\beta)(\theta-1)\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right]E_{i,t}\sum_{j=0}^{\infty}\beta^{j+1}\widehat{r}_{t+j+1} \\ & - \left[\frac{Y}{\theta K} + \frac{\nu\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right]\left(\frac{1-\beta}{\beta}\right)E_{i,t}\sum_{j=0}^{\infty}\beta^{j+1}\widehat{Y}_{t+j} \\ & - \left[\frac{(\theta-1)Y}{\theta K} + \frac{\nu(\theta-1)\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right]\left[\frac{(1-\beta)\beta\rho_z}{1-\beta\rho_z}E_{i,t}\widehat{z}_t - \beta\widehat{z}_{i,t}\right]\end{aligned} \tag{2.31}$$

Proof. See Appendix B.2. □

The labor supply decision is identical to the one under full information. However, there is a subtle difference in the savings decision under dispersed information. Notice that the savings decision in part depends on the expected benefit to holding a unit of capital in the future. Under dispersed information, this expected benefit is with respect to the beliefs of household i. These beliefs in general are different from those of household j and different from the objective full-information belief. This opens the door for fluctuations in investment depending on perceptions. If household i is pessimistic about the benefit of holding a unit of capital in the future, it cuts back on investment. In the same vein, if all agents are pessimistic about the benefit of holding capital in the future, they all cut back on their investment, and this leads to a reduction in capital stock tomorrow. Thus pessimism can lead to swings in investment behavior.

Dispersed information also alters the firm's decisions compared to the full information case. The decisions of firm i are summarized in the following proposition.

Proposition 10. *The labor and capital demand of the firm on island i are summarized below:*

$$\widehat{L}_{i,t} = \frac{(\theta-1)\widehat{z}_{i,t}}{1+\nu[\alpha+\theta(1-\alpha)]} - \frac{\alpha(\theta-1)\widehat{r}_t}{1+\nu[\alpha+\theta(1-\alpha)]} + \frac{E_{i,t}\widehat{Y}_t}{1+\nu[\alpha+\theta(1-\alpha)]} \quad (2.32)$$

$$\widehat{J}_{i,t} = \frac{(\theta-1)(1+\nu)\widehat{z}_{i,t}}{1+\nu[\alpha+\theta(1-\alpha)]} - \frac{[1-\alpha+\theta(\alpha+\nu)]\widehat{r}_t}{1+\nu[\alpha+\theta(1-\alpha)]} + \frac{(1+\nu)E_{i,t}\widehat{Y}_t}{1+\nu[\alpha+\theta(1-\alpha)]} \quad (2.33)$$

Consequently, the output decision of firm i can be expressed as:

$$\widehat{Y}_{i,t} = \frac{\theta(1+\nu)\widehat{z}_{i,t}}{1+\nu[\alpha+\theta(1-\alpha)]} - \frac{\alpha\theta(1+\nu)\widehat{r}_t}{1+\nu[\alpha+\theta(1-\alpha)]} + \frac{(1+\alpha\nu)E_{i,t}\widehat{Y}_t}{1+\nu[\alpha+\theta(1-\alpha)]} \quad (2.34)$$

Proof. See Appendix B.2 □

Equations (2.32)-(2.34) show that the output decision of firm i is characterized by strategic complementarity. The input demand and output decisions of the firm depend on its perception of aggregate demand, $E_{i,t}\widehat{Y}_t$. This result is presented formally in the following Lemma.

Lemma 4. *Firm i's demand for inputs is increasing in its perception of aggregate demand/output. As a result, a firm's output is increasing in its perception of aggregate demand.*

$$\frac{\partial \widehat{Y}_{i,t}}{\partial E_{i,t}\widehat{Y}_t} = \frac{1+\alpha\nu}{1+\nu[\alpha+\theta(1-\alpha)]} > 0 \tag{2.35}$$

i.e., a firm's output decision is characterized by ***strategic complementarity****.*

Proof. The claim follows from (2.34). □

This strategic complementarity in output implies that the firm wants to hire more labor and capital when it perceives the level of aggregate demand to be high. This can be seen from equations (2.32) and (2.33). At the time of making production decisions, firm i does not know the true fundamentals, and thus it cannot predict with certainty the decisions of the other firms. As a result, it cannot ascertain the exact level of aggregate demand $\widehat{Y}_t$. Since the profits of the firm depend on its relative output, uncertainty about fundamentals manifests itself in the form of demand uncertainty. Each firm must decide how much to produce given its perception of aggregate demand rather than actual aggregate demand.

In an economy with dispersed information where agents' individual decisions are affected by their perceptions of the fundamentals, aggregate quantities depend on the distribution of beliefs across the economy. In particular, average perceptions about the fundamentals affect the aggregate quantities.

Proposition 11. *Aggregate capital accumulation by agents in the economy depends on average perceptions of the state of the economy:*

$$\begin{aligned}
\Delta\widehat{K}_{t+1} \quad = \quad & -\frac{\alpha\beta\nu(\theta-1)\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\widehat{r}_t + \frac{\beta Y}{\theta K}\widehat{Y}_t - \left(\frac{\beta Y}{\theta K} - \frac{\nu\Psi H^{1+\nu}}{1+\nu(\alpha+\theta(1-\alpha)}\right)\overline{E}_t\widehat{Y}_t \\
& + \left[\frac{\beta r}{\kappa K} + \frac{\alpha\nu(1-\beta)(\theta-1)\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right]\overline{E}_t\sum_{j=0}^{\infty}\beta^{j+1}\widehat{r}_{t+j+1} \\
& - \left[\frac{Y}{\theta K} + \frac{\nu\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right]\left(\frac{1-\beta}{\beta}\right)\overline{E}_t\sum_{j=0}^{\infty}\beta^{j+1}\widehat{Y}_{t+j} \\
& - \left[\frac{(\theta-1)Y}{\theta K} + \frac{\nu(\theta-1)\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right]\left[\frac{(1-\beta)\beta\rho_z}{1-\beta\rho_z}\overline{E}_t\widehat{z}_t - \beta\widehat{z}_t\right] \qquad (2.36)
\end{aligned}$$

and so do the total demands for labor and capital:

$$\widehat{L}_t = \frac{(\theta-1)\widehat{z}_t}{1+\nu[\alpha+\theta(1-\alpha)]} - \frac{\alpha(\theta-1)\widehat{r}_t}{1+\nu[\alpha+\theta(1-\alpha)]} + \frac{\overline{E}_t\widehat{Y}_t}{1+\nu[\alpha+\theta(1-\alpha)]} \qquad (2.37)$$

$$\widehat{J}_t = \frac{(\theta-1)(1+\nu)\widehat{z}_t}{1+\nu[\alpha+\theta(1-\alpha)]} - \frac{[1-\alpha+\theta(\alpha+\nu)]\widehat{r}_t}{1+\nu[\alpha+\theta(1-\alpha)]} + \frac{(1+\nu)\overline{E}_t\widehat{Y}_t}{1+\nu[\alpha+\theta(1-\alpha)]} \qquad (2.38)$$

Consequently, so do aggregate output and prices:

$$\widehat{Y}_t = \frac{(1-\alpha)\overline{E}_t\widehat{Y}_t}{1-\alpha+\theta(\alpha+\nu)} + \frac{\theta(1+\nu)\widehat{z}_t}{1-\alpha+\theta(\alpha+\nu)} + \frac{\alpha\theta(1+\nu)\,(K_t+\zeta_t)}{1-\alpha+\theta(\alpha+\nu)} \qquad (2.39)$$

$$
\begin{aligned}
\widehat{r}_t &= \frac{(1+\nu)\overline{E}_t\widehat{Y}_t}{1-\alpha+\theta(\alpha+\nu)} + \frac{(\theta-1)(1+\nu)\widehat{z}_t}{1-\alpha+\theta(\alpha+\nu)} \\
&\quad - \frac{\{1+\nu[\alpha+\theta(1-\alpha)]\}(K_t+\zeta_t)}{1-\alpha+\theta(\alpha+\nu)} \qquad (2.40)
\end{aligned}
$$

Proof. Aggregating equation (2.31) over all households yields equation (2.36), while aggregating equation (2.34) over all firms yields equation (2.39). Equations (2.37)-(2.38) can be derived by aggregating equations (2.32)-(2.33). Equation (2.40) can be derived by aggregating equation (2.33) and imposing capital market clearing $\widehat{J}_t = \widehat{K}_t + \zeta_t$. □

Proposition 11 is in strong contrast to Proposition 7. In the full information case, aggregate quantities and prices do not depend on average perceptions about the state of the economy and are driven purely by fundamentals. Equations (2.39)-(2.40) highlight that the distribution of beliefs is also a force in determining the behavior of aggregate quantities and prices in an economy with dispersed information. In particular, equation (2.39) shows that aggregate output is affected by average beliefs about aggregate demand. A higher average expectation about the level of aggregate demand results in an increase in actual output. If all firms are optimistic (pessimistic) about the level of aggregate demand, they will choose to produce more (less). This reliance of actual output on the perceived level of output acts like an amplification mechanism. In the previous section, firm i was able to observe the productivity of each of the firms in the economy, and as a result, it could ascertain with certainty the output decisions of these firms. Therefore, its decision of how much to produce only depended on actual aggregate demand and not on perceived aggregate demand.

Lemma 5. *Aggregate output responds positively to perceptions about aggregate demand*

$$\frac{\partial \widehat{Y}_t}{\partial \overline{E}_t \widehat{Y}_t} = \frac{1-\alpha}{1-\alpha+\theta(\alpha+\nu)} > 0$$

Proof. The claim follows from equation (2.39). □

Similarly, equation (2.36) shows how pessimism and optimism can affect output in the future. Investment at time t depends on perceptions about current and future aggregate demand as well as average perceptions about the return from holding capital tomorrow. Average perception of a low return to capital induces agents to invest less, which implies that the capital stock tomorrow is smaller and hence the production possibility frontier for tomorrow moves inwards. Pessimistic average beliefs about the return from holding capital can arise from pessimistic beliefs about aggregate demand today. Pessimistic beliefs about aggregate demand today cause firms to cut back on their demand for capital, which reduces the rental rate today. A low rental rate today is indicative of a low return to capital in the near future as productivity is persistent. This results in a cut back on investment spending. Fluctuations to perceptions not driven by shocks to fundamentals can thus cause current and future aggregate output to contract or expand depending on the average beliefs today. This is the dynamic mechanism through which dispersed information acts to make downturns longer and deeper. Other macro models of dispersed information such as Angeletos and La'O (2008) do not focus on this dynamic mechanism as they abstract from capital or any other persistent endogenous state variable.

Lorenzoni (2009) shows how nominal rigidities combined with non-fundamental fluctuations in beliefs can have dynamic effects on output. This paper highlights that non-fundamental fluctuations in beliefs can have dynamic persistent effects even in a neoclassical rational expectations economy with no nominal rigidities or other explicitly Keynesian features. Dispersed information results in a neoclassical economy displaying a *Keynesian* flavor, in that both perceptions about aggregate demand determine and the actual output in the economy. The average expectations are analogous to the Keynesian idea of *animal spirits*. Section 2.4 fleshes out how such optimism or pessimism may arise.

2.3.2.2 Step 2: Filtering and prediction problem

The previous sub-section provided a solution to an agent's problem taking as given the informational content of prices. This section pins down the informational content of prices as an equilibrium object. As is clear from the propositions in the previous section, agents' decisions depend on their perception of how prices and aggregate demand evolve in the future. Equations (2.30)-(2.34) imply that the household on island i needs to predict $\sum_{j=0}^{\infty}\beta^{j+1}\widehat{r}_{t+j+1}, \widehat{z}_t, \sum_{j=0}^{\infty}\beta^{j+1}\widehat{Y}_{t+j}$ and $\widehat{Y}_t$ in order to make an investment decision. These predictions are the linear projections of these objects onto $\mathcal{I}_{i,t}$. More generally, these projections are linear combinations of past and current realizations of the signals $\{x_{i,t-s}\}_{s=0}^{\infty}$, which themselves are functions of the past and current innovations to the shocks $\{\xi_{i,t-s}\}_{s=0}^{\infty}$. Thus, without loss of generality, we can assume that the predictions are linear combinations of past and current ξ_i's.

3.2.2.1 Technical discussion of the difficulty in solving dispersed information models

.

Solving the model requires one to solve the prediction problem of an agent with information set $\mathcal{I}_{i,t}$. The problem of infinite regress in expectations is well known since Townsend (1983). Prices and future aggregate demand depend on the decision rules of all agents in the economy, who make output and investment decisions in the presence of private information. As a result, the distribution of beliefs plays an important role in determining the aggregates of the economy. To be able to predict prices, capital stock and output in the future, agents must forecast the private information available to the other agents. This leads to the problem of infinite regress and makes the state space explosive. For example, consider firm i's output decision in period t. In order to decide on the level of output to produce, firm i must predict the level of aggregate demand $\widehat{Y}_t$. Applying the expectation operator to equation (2.39), we have:

$$E_{i,t}\widehat{Y}_t = \frac{\theta(1+\nu)E_{i,t}\widehat{z}_t}{1+\nu[\alpha+\theta(1-\alpha)]} - \frac{\alpha\theta(1+\nu)\widehat{r}_t}{1+\nu[\alpha+\theta(1-\alpha)]} + \frac{(1+\alpha\nu)E_{i,t}\overline{E}_t\widehat{Y}_t}{1+\nu[\alpha+\theta(1-\alpha)]} \quad (2.41)$$

However, this depends on agent i's expectation of the average expectations of aggregate demand. The average expectation can be calculated by averaging the beliefs of all agents:

$$\begin{aligned}\overline{E}_t\widehat{Y}_t &\equiv \int_0^1 E_{i,t}\widehat{Y}_t = \frac{\theta(1+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\overline{E}_t\widehat{z}_t - \frac{\alpha\theta(1+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{r}_t \\ &+\frac{1+\alpha\nu}{1+\nu[\alpha+\theta(1-\alpha)]}\overline{E}_t^{(2)}\widehat{Y}_t\end{aligned}$$

where $\overline{E}_t^{(2)}\widehat{Y}_t = \int_0^1 E_{i,t}\overline{E}_t\widehat{Y}_t di$ is the average expectation of the average expectation of aggregate output. Plugging this equation into equation (2.41) shows that agent i's expectation of aggregate output depends his expectation of both the average expectations of aggregate productivity and also the average expectation of the average expectation of aggregate output. This implies that agent i's expectation of aggregate demand depends on the second order average expectations. In fact, if we repeat the above procedure to substitute higher order expectations, we will find that firm i's belief about aggregate output and hence the decision of how much to produce depend on the entire set of higher order beliefs about aggregate productivity:

$$\begin{aligned}E_{i,t}\widehat{Y}_t &= \frac{\theta(1+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\sum_{n=0}^{\infty}\left[\frac{1+\alpha\nu}{1+\nu[\alpha+\theta(1-\alpha)]}\right]^n E_{i,t}\overline{E}_t^{(n)}\widehat{z}_t \ldots \\ &-\frac{\alpha\theta(1+\nu)\widehat{r}_t}{1+\nu[\alpha+\theta(1-\alpha)]-\theta(1+\nu)}\end{aligned} \tag{2.42}$$

Since firms are symmetric, aggregate output depends on the entire set of higher order expectations about aggregate output:

$$\begin{aligned}\widehat{Y}_t &= \frac{\theta(1+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\sum_{n=0}^{\infty}\left[\frac{1+\alpha\nu}{1+\nu[\alpha+\theta(1-\alpha)]}\right]^n \overline{E}_t^{(n)}\widehat{z}_t \ldots \\ &-\frac{\alpha\theta(1+\nu)\widehat{r}_t}{1+\nu[\alpha+\theta(1-\alpha)]-\theta(1+\nu)}\end{aligned} \tag{2.43}$$

Note that the notation $\overline{E}_t^n$ implies the average expectations of order n. We can define $\overline{E}_t^{(n)}$ recursively as follows:

$$\overline{E}_t^{(n)}\widehat{z}_t = \int_0^1 E_{i,t}\overline{E}_t^{(n-1)}\widehat{z}_t di$$

and

$$\overline{E}_t^{(0)}\widehat{z}_t = \widehat{z}_t$$

The problem stated above is not tractable as it is easy to see that it gives rise to an infinite dimensional state space. Agents need to track an infinite number of objects $\{\overline{E}_t^{(n)}\widehat{z}_t\}_{n=0}^{\infty}$. Standard techniques of filtering, used in incomplete information models, such as the standard Kalman Filter, are ill-suited for such a problem as they require a well defined finite dimensional state space. Other recent papers have come across a similar problem. Woodford (2002) deals with this infinite regress problem by using a clever technique which allows him to predict only a finite number of objects. In particular, Woodford (2002) restricts agents to only learn from exogenously generated signals, which allows him to employ the above strategy and still use the Kalman Filter. However, this technique cannot be generalized to models in which agents learn from endogenous signals as the convergence properties of the standard Kalman filter are not straightforward. Graham and Wright (2010) extend the Kalman Filter to handle endogenous signals and use the technique suggested in Nimark (2010), which involves truncating the higher order beliefs at some large order $k < \infty$. Thus, rather than tracking $\{\overline{E}_t^{(n)}\widehat{z}_t\}_{n=0}^{\infty}$, the solution truncates the higher order beliefs up to k. As a result, $\{\overline{E}_t^{(n)}\widehat{z}_t\}_{n=0}^k$ are the state variables of the model. Since, k is finite, this allows the definition of a finite dimensional state space and use of the Kalman Filter. However, this is a computationally intensive procedure.

The problem of having to track higher order expectations stems from private information. The decisions of agents depend on their expectations of current and future aggregate quantities and prices. However, agents are informed differentially about the fundamentals today, implying that their forecasts of these objects are also heterogeneous. Since agents cannot infer the true fundamentals today without error, their perceptions about productivity, aggregate demand and so on depend on all the past signals they observe. As agents use all past and current signals to infer the fundamentals today, perceptions about fundamentals depend potentially on the entire history of shocks. These decisions are then aggregated to form the rental rate, which consequently also depends on the entire history of realizations of ξ. Previous papers have dealt with this problem by assuming that all information becomes common knowledge after a certain lag, as in Hellwig (2002) and Lorenzoni (2009). Nimark (2010) shows that these truncation techniques may inadvertently alter the properties of the equilibrium.

In this paper, I use techniques suggested by Kasa (2000) to deal with the infinite regress in expectations. Kasa (2000) shows how to translate the problem into the frequency domain, where the filtering problem is no longer infinite dimensional, allowing one to compute an exact solution. In addition, Wiener-Kolmogrov forecasting methods can be used to project prices and output in the future onto the information sets of agents in a tractable fashion, as this technique does not require a well-defined state space. The current paper adapts this method to solve for equilibrium. This is a methodological contribution; to my knowledge no existing work adapts frequency-domain methods to the solution of business cycle models with dispersed information.

3.2.2.2 Solution to the filtering problem .

Following Kasa (2000), I transform the problem into the frequency domain and use techniques adapted from Whiteman (1983) to compute a rational expectations equilibrium. A detailed explanation can be found in Appendix B.2.5. Solving the model requires one to calculate an agent's expectations of the fundamentals today and in the future. The procedure to compute the equilibrium is explained below in brief.

The decisions of households and firms depend on the perceived level of aggregate demand. The perceptions of aggregate demand on island i depend potentially on all past and current signals observed on the island. These signals are in turn functions of current and all past fundamentals. Thus, without loss of generality assume that agent i's perception of aggregate demand, $E_{i,t}\widehat{Y}_t$, is given by[16]

$$E_{i,t}\widehat{Y}_t = \mathcal{Y}_v(L)v_t + \mathcal{Y}_u(L)u_t + \mathcal{Y}_\zeta(L)\zeta_t + \mathcal{Y}_e(L)e_{i,t} \equiv \mathcal{Y}(L)\xi_{i,t} \quad (2.44)$$

which in turn implies that average perception in the economy about aggregate demand takes the following form:

$$\overline{E}_t\widehat{Y}_t = \mathcal{Y}_v(L)v_t + \mathcal{Y}_u(L)u_t + \mathcal{Y}_\zeta(L)\zeta_t \equiv \mathcal{Y}(L)\xi_t \quad (2.45)$$

where $\{\mathcal{Y}_j(L)\}_{j\in\{v,u,e,\zeta\}}$ are potentially infinite order beta-summable lag polynomials.

The capital accumulation decisions of households on each island depend on their own perception about the current and future state of the economy. As a result, the aggregate capital stock depends on average beliefs, which in turn depend on the realizations of all past and current realizations of ξ. In particular, assume that aggregate capital evolves according to the following process:

$$K_{t+1} = \mathcal{K}_v(L)v_t + \mathcal{K}_u(L)u_t + \mathcal{K}_\zeta(L)\zeta_t \quad (2.46)$$

where $\mathcal{K}_v$ and $\mathcal{K}_u$ are potentially infinite order beta-summable lag polynomials. The lag polynomials $\{\mathcal{K}_j(L), \mathcal{Y}_j(L)\}_{j\in\{v,u,e,\zeta\}}$ need to be determined in equilibrium.

[16]The structure imposed on the predicted values implicitly assumes that the equilibrium is symmetric. The functions defining the perceptions are identical across islands, i.e. the lag polynomials do not have an i subscript. Since the perceptions of agents on island i depend potentially on the entire history of shocks, recursive filtering techniques which require a well defined state space become harder to apply.

Given the guesses in equations (2.44)-(2.46), the perceived law of motion of rental rates and aggregate output can be defined using by plugging in equations (2.44)-(2.46) into equations (2.39) and (2.40).[17]

If rental rates were fully revealing, agents would be able to infer the current fundamentals ε_t by observing it. But as is shown in the next section, agents will not be able to infer the current fundamentals by observing their set of signals. However they are able to infer some convolution of current fundamentals with the past fundamentals. These combinations of past and current fundamentals are determined by deriving an observationally equivalent fundamental Wold representation of equation (2.47) below. The procedure follows Rozanov (1967) and is explained in Appendix B.2.5. This also allows one to pin down the information contained in prices.

The next step of the solution involves taking these perceived laws of motion as given and computing the decisions of households and firms. Aggregating these decisions yields actual laws of motion of the aggregate capital stock, output, rental rates and perceived aggregate demand. Rational expectations implies that the actual and perceived laws of motions should be consistent with each other. This implies that the perceived laws of motion have to be matched with the actual derived ones. This completes the procedure, which identifies the unique equilibrium in $\mathcal{H}_t^\beta(\xi)$.

2.4 Results

This section characterizes the exact linear equilibrium of the dispersed information economy and contrasts its properties to the full information case. The

[17] These are given by equations (B.2.18)-(B.2.19) in Appendix B.2.5.

numerical results are based the following calibration. The period in the model is set to one quarter. Parameters that govern preferences and production technology are calibrated to match long-run values of postwar US aggregates. I follow standard calibration procedures as explained in Prescott (1986) and Cooley and Prescott (1995). The discount factor β is set to 0.99 to match an annual interest rate of 4%. The share of capital in total output, α, is set to 0.4 to match the labor share in national income. The parameter Ψ, which scales disutility of labor, is set to set match labor supply to 31% of the time in steady state. Following Fernandez-Villaverde et al. (2009), I set ν to 1.35 to match a Frisch elasticity of labor supply of 0.74. The persistence of the aggregate component of productivity ρ_z is set to 0.95, and the standard deviation of the innovation to aggregate productivity σ_v, is set to 0.007. I set the standard deviation of idiosyncratic productivity shocks σ_e to 0.1. This is consistent with Castro et al. (2011), who find that mean standard deviation of firm level productivity is within the range of 8-10 percent. I set the elasticity of substitution across different varieties θ to 10 to match a steady state markup of 10 percent. δ is set to 0.025 to match a quarterly depreciation rate 2.5%. I set the volatility of the noise shock σ_u to 2.1% which is consistent with the findings of L'Huillier et al. (2009).

2.4.1 Informationally Inefficient Prices

This section shows that in the economy with dispersed information, households and firms on island $i \in [0, 1]$ are unable to infer the true fundamentals $\varepsilon_{i,t}$ at time t

from their signals $x_{i,t}$.

Writing the signals available to agents on island i in their innovations representations yields equation (2.47):

$$\begin{bmatrix} \widehat{z}_{i,t} \\ \varphi_t \\ \widehat{r}_t \\ \widehat{Y}_{t-1} \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\rho_z L} & 0 & 0 & 1 \\ \frac{1}{1-\rho_z L} & 1 & 0 & 0 \\ \mathcal{R}_v(L) & \mathcal{R}_u(L) & \mathcal{R}_\zeta(L) & 0 \\ L\mathbb{Y}_v(L) & L\mathbb{Y}_u(L) & L\mathbb{Y}_\zeta(L) & 0 \end{bmatrix} \times \begin{bmatrix} v_t \\ u_t \\ \zeta_t \\ e_{i,t} \end{bmatrix} \tag{2.47}$$

or

$$x_{i,t} = \mathcal{M}(L)\xi_{i,t}$$

where $\{\mathcal{R}_j, \mathbb{Y}_j\}$ are defined in equations (B.2.18)-(B.2.19) in Appendix B.2.5. Since $Det[\mathcal{M}(z)] = 0$ at $z = 0$, $\varepsilon_{i,t}$ is not fundamental for $x_{i,t}$ and $\mathcal{M}(L)$ does not possess an inverse only in positive powers of L. Since in equation (2.47) is not a fundamental Wold representation, agent i cannot infer the contemporaneous shocks $\xi_{i,t}$ to the economy from observing all the past and current realizations of the signals $\{x_{i,t-j}\}_{j=0}^{\infty}$.

Notice that the above equation is a mapping of the signals in terms of all the past and current realization of fundamentals along with noise for agents on island i. If price were fully revealing, agents would have been able to infer $\xi_{i,t}$ by observing the realizations of the signals $x_{i,t}$ up to period t. However, in this case, agents are no longer able to infer $\xi_{i,t}$ from the signals they observe. The agent has as many signals as objects that he needs to infer, so this inability of agents to infer the true innovations to the economy is not because of not having enough signals.[18] Rather, prices are informationally inefficient because of the infinite regress in expectations. Agents cannot observe the true fundamentals at the beginning of each period. As a result they cannot infer aggregate demand/output. The rental rate r_t provides information about the current state of aggregate productivity but is convoluted by the shock to the supply of capital, which is not observed by agents directly. Since aggregate capital available today depends in part on investment decisions made last period, it depends on the average expectations about the fundamentals in $t-1$. Thus, to infer aggregate productivity from the rental rate an agent must infer the average beliefs about fundamentals in $t-1$. However, this is not a trivial exercise. Agents on all islands learn from rental rates, and hence the average beliefs about current fundamentals depend on the rental rate today. Note that average beliefs today depend on the average beliefs yesterday through the rental rate. Hence average belief in period t depends on all average beliefs in the past. Thus, market-consistent information is governed by fundamentals not just from today but also from the infinite past. This can be seen as arising from the infinite regress in expectations. If information was not dispersed, agents could infer the true fundamentals about aggregate productivity from observing the current rental rate, as they would know what the average beliefs about fundamentals yesterday were.

Thus, in the economy with dispersed information, agents are unable to isolate

[18] Equation (2.47) is an infinite order VAR in innovations representation. The informational inefficiency of prices is similar to the problems in interpreting VAR's which are discussed in Fernandez-Villaverde et al. (2007). They provide conditions under which an econometrician can infer the *true* structural shocks from a VAR. The non-invertability of $\mathcal{M}$ is synonymous with the failure of these conditions. As a result, agents (who are doing the jobs of econometricians !) cannot infer the *true* contemporaneous fundamentals $\varepsilon_{i,t}$.

the effects of changes in fundamentals from noise. In fact, agents' forecast errors in predicting aggregate productivity are serially correlated:

$$\begin{aligned} \widehat{z}_t - E_{i,t-1}\widehat{z}_t &= \frac{v_t}{1-\rho_z L} + \rho_z(-\gamma_{v1}\frac{v_{t-1}}{1-\rho L} - \gamma_{v2}\frac{v_{t-2}}{1-\rho L} - \gamma_u u_{t-1} + \gamma_\zeta \zeta_{t-1} \\ &\quad -\gamma_e e_{i,t-1}) \end{aligned} \tag{2.48}$$

Consequently, the average forecast error is serially correlated

$$\widehat{z}_t - \overline{E}_{t-1}\widehat{z}_t = \frac{v_t}{1-\rho L} + \rho_z(-\gamma_{v1}\frac{v_{t-1}}{1-\rho L} - \gamma_{v2}\frac{v_{t-2}}{1-\rho L} - \gamma_u u_{t-1} + \gamma_\zeta \zeta_{t-1}) \tag{2.49}$$

where $\{\gamma_j\}_{j\in\{v1,v2,u,\zeta,e\}}$ are positive constants which depend on model parameters and steady state values. Since forecast errors are serially correlated, agents' forecasts are characterized by waves of pessimism and optimism.

2.4.2 Response of the Economy to a Aggregate Productivity Shock

This section discusses the reaction of the economy to a negative innovation to aggregate productivity.

On Impact Following a decrease in the aggregate productivity in period t, firm i observes a lower realization of his island specific productivity $z_{i,t}$ and also of the noisy public signal φ_t. The firm is unable to dis-entangle whether the lower observations are driven by an actual fall in aggregate productivity or by some combination of decreases in idiosyncratic productivity and noise. As a result, it tempers its beliefs of a fall in aggregate productivity. In addition to these exogenous signals, the firm also observes the rental rate for capital, from which it tries to infer the perceptions of others in the economy regarding aggregate productivity. A lower rental rate reinforces the belief that productivity is low, and that average perception in the economy about aggregate productivity is low, but since prices are not fully revealing, firm i still cannot be fully certain that aggregate productivity has gone down. It follows from equation (2.49) that the average perception of aggregate productivity is higher than actual productivity.

Since the decline in perceived aggregate productivity is less than the actual decline, demand for capital falls by less on impact compared to the full information case. As a result, the rental rate falls by less. Nevertheless, a lower rental rate reaffirms that average perceptions about productivity are low. Each firm believes that firms on other islands respond to lower average perceptions of aggregate productivity by decreasing production. Given the anticipated decline in production by firms on other islands, the firm perceives that aggregate demand will be low. Strategic complementarity in output decisions implies that firms will respond to lower perceptions of aggregate demand by reducing their own output. Compared to the full information case, firms in a dispersed information economy decrease output when (1) they perceive aggregate productivity to be low and (2) when they perceive that average beliefs of aggregate demand are also low. As such, Figure 2.2 highlights that perceived aggregate demand falls on impact of a negative productivity shock. Notably, the first panel of Figure 2.2 demonstrates the effect of strategic complementarity, since firms perceive aggregate demand to have declined, each firm lowers its actual production. This causes actual aggregate output to fall by more than the

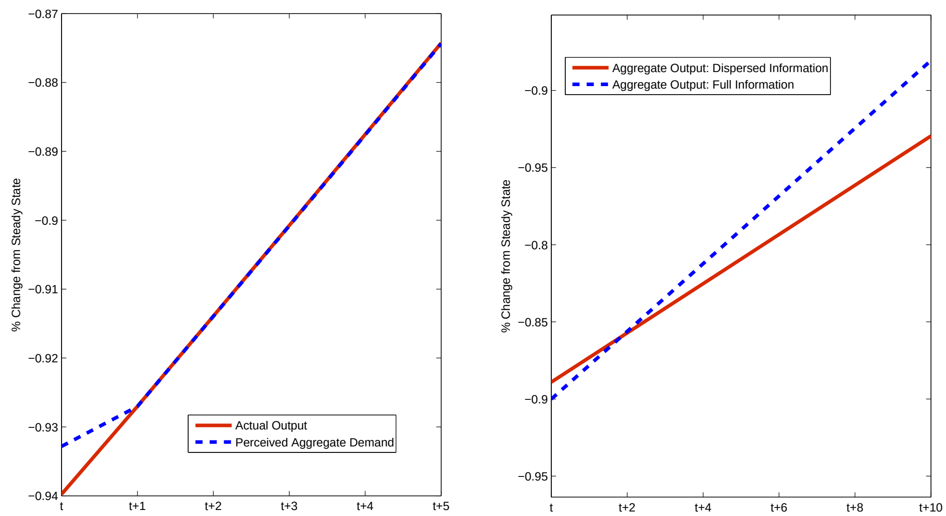

Figure 2.2: Evolution of Perceived Aggregate Demand and Actual Aggregate Output after a negative productivity shock

decline in perceived aggregate demand. However, the decline in output is marginally less severe initially compared to the fall in output in the full information economy as demonstrated by the second panel in Figure 2.2. Following equation (2.39), agents could not disentangle the true decline in aggregate productivity, and consequently tempered expectations about the fall in aggregate demand causes output to decline by less.

Since firms and households on an island are symmetrically informed, the households' perception about aggregate productivity is also higher than the actual level of productivity on impact. As productivity is persistent, households anticipate that the demand for capital in $t+1$ will continue to be low and consequently they expect the benefit from investing in capital to be low. As such, households cut back on investment, which lowers the capital stock in the future.

Dynamic Effects As shown above, a negative productivity shock has less effect on economic activity on impact than in the full information case. However, the negative impulse to productivity is amplified in subsequent quarters under dispersed information, as can be seen in Figure 2.2.

Agents on each island are unsure about the extent of the fall in aggregate productivity. Initially they attribute only a portion of the weak signals to a decrease in aggregate productivity. At the end of period t, after production takes place everyone observes aggregate output which is much lower than their expectations which causes each agent to revise their expectations about aggregate productivity downwards.

This is where pessimism has an impact. In the period following the initial shock, the full information economy is already on its way to a recovery, as aggregate productivity is mean reverting and starts to increase again. Each agent observes this increase in aggregate productivity and this causes output, consumption and investment to rebound. However, this is not the case in the economy with dispersed information. In the dispersed information economy, each firm observes that its island specific productivity has increased, and also a higher realization of the noisy public signal than the previous period. However, on observing this information, each firm still cannot infer whether the observed increase in productivity is particular to them or whether others also observe similar signals. Each firm is cautious in increasing their capital demand and hence output in response to this new information. Each firm fears that other firms do not share this information and hence will not increase their output, which will result in losses to the firm. As a result, each firm reacts cautiously to this new information, increasing their capital demand and hence their output slightly.

Since each firm tempers its increased capital demand, the rental rate does not increase enough to clearly reflect the improvement in productivity. Even though each firm observes signals that suggest improvement in productivity, the relatively small increase in the rental rate signals to each firm that the capital demand by other firms is low. Consequently, each firm infers that all other firms are pessimistic. Since each firm anticipates low aggregate demand, it lowers its own capital demand and hence output. A lower realization of aggregate output then reaffirms the belief of each firm that the others are pessimistic about aggregate productivity.

In other words, even though each firm observes signals of improving conditions, they infer from the price signals that others do not share their good news about improving conditions. As a result, each firm believes that the average perceptions about aggregate productivity and hence aggregate demand/output are low. This implies that the forecasts of all agents are negatively biased. The negatively biased forecasts are what corresponds to *pessimism* in the economy. Thus, compared to the full information economy where output started to recover immediately, output in the dispersed information economy does not rebound as strongly. In fact, Figure 2.2 demonstrates that output in the dispersed information economy is below the level of output in the full information economy in periods following the initial drop.

The presence of dispersed information and strategic complementarity implies that each firm is cautious and hence slow in responding to new information that others might not share. Prices which aggregate these decisions, are therefore also slow to reflect new information. Since prices are public sources of information, they reinforce the pessimism/optimism, which causes agents to temper their decisions further and hence further delays the incorporation of private information about improving conditions into decisions and hence prices. Thus, a key component of the mechanism is the reinforcing role played by the endogenous price signals. The mechanism is similar to that in Woodford (2002) where dispersed information interacts with strategic complementarity in pricing to create delayed responses to a monetary policy shock that is not common knowledge. However, a key difference is that since Woodford (2002) does not feature learning from endogenous price signals, there is no mechanism to reinforce the pessimism or optimism.

Dispersed information and strategic complementarity have dynamic as well as static effects. Since households share the same information as firms, each households also believes that there is a widespread sense of pessimism about aggregate productivity in the economy, which results in lower aggregate demand/output. Since aggregate productivity is persistent, they anticipate that this pessimism will also persist over time, and hence predict that the demand for capital will be lower in the future too. Anticipating a lower benefit from holding capital tomorrow, households lower their investment expenditure today. This results in a lower capital stock in the future, which causes the production possibility frontier to contract. This slows down the recovery of output. A smaller capital stock and thus a higher rental rate following a higher marginal productivity of capital tomorrow should encourage investment. However, the pessimistic beliefs about aggregate demand today result in a negative wealth effect as households predict smaller dividends from each firm.

Thus, this pessimism about the level of aggregate productivity and hence lower aggregate demand/output causes income of households to decline giving rise to a fall in consumption and in investment spending. Declines in investment and consumption spending feed pessimistic beliefs about the state of aggregate demand which through endogenous price signals reinforce more pessimism. The fall in consumption in the dispersed information economy is smaller than the full information economy because the incentive of households to invest falls drastically. This causes the recession to be deeper and more protracted. Figure 2.3 summarizes the effect of a downward revision in forecasts on investment and consumption and highlights that investment falls by a much larger proportion in the dispersed information economy compared to the full information economy.

The dispersed information economy features more volatile and persistent fluctuations than its full information counterpart. In particular, output fluctuations can result from both changes in fundamentals and changes in average perceptions about fundamentals. Standard business cycle models predict a high correlation between productivity shocks and changes in output. This prediction is not supported empirically. This correlation is attenuated in the dispersed information economy. Output falls by less on impact of the technology shock and the recovery slower than the subsequent improvement in technology due to lagging perceptions about fundamentals. In addition, the model is consistent with the strong empirical correlation between consumer sentiment indices and short run fluctuations.

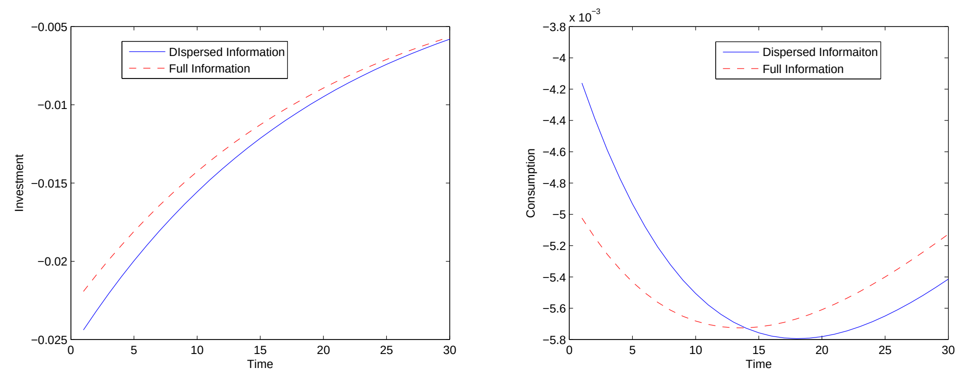

Figure 2.3: Declines in Capital, Employment, Consumption after a negative productivity shock

2.4.3 Response of the Economy to a Noise Shock

Angeletos and La'O (2008) and Lorenzoni (2009) discuss how noise that is completely unrelated to the fundamentals of the economy can result in fluctuations in economic activity in an economy with dispersed information with strategic complementarity. I interpret the noise shock as the error u_t in the public signal of aggregate productivity φ_t. u_t is assumed to be i.i.d across time. Noise shocks in this setting have transient effects on economic activity. Figure 2.4 shows the responses of output, rental rate, consumption and capital stock to a positive noise shock.

On impact of a positive noise shock, agents are unable to distinguish the shock from an actual increase in productivity and hence attribute a part of signal to an increase in aggregate productivity. Since the noise shock does not affect the fundamentals of the economy,it does not raise any firm's island specific productivity is unchanged. Thus, a typical firm infers that its idiosyncratic island specific productivity shock has fallen along with a rise in aggregate productivity, leaving $\widehat{z}_{i,t}$

unchanged. Since firm i does not observe the island specific productivities of other firms, it infers based on the higher noisy public signal that other firms are likely to have observed higher island specific productivity and will increase production. Strategic complementarity implies that firm i wants to align its production decisions with others, and hence firm i increases capital and labor demand to increase output.

Following equation (2.34), firm i tempers its increase in output as its own island specific productivity hasn't risen, even though it perceives aggregate demand/output to have increased. If there had been an actual increase in aggregate productivity, firm i would have observed both an increase in the public signals and its own island specific productivity and would have responded more strongly in terms of the increase in output.

Since firms are symmetric, the average firm observes no change in island specific productivity but infers that productivity has gone up on other islands. This causes each firm to increase its capital demand and hence production, but to a lesser extent than in the case of increasing productivity. Since each firm tempers its increase in capital demand, rental rates do not increase as sharply as in the case where there has been a true increase in aggregate productivity. Observing that the rental rate did not respond strongly to the increase in the noisy public signal, each firm cuts its forecast of aggregate productivity. This reinforces the incentive of a firm to temper its increase in output. Since each firm increases its output only a little, realized aggregate output does not rise substantially. At the end of the period, each firm observes this small increase in aggregate output, and this again reinforces their

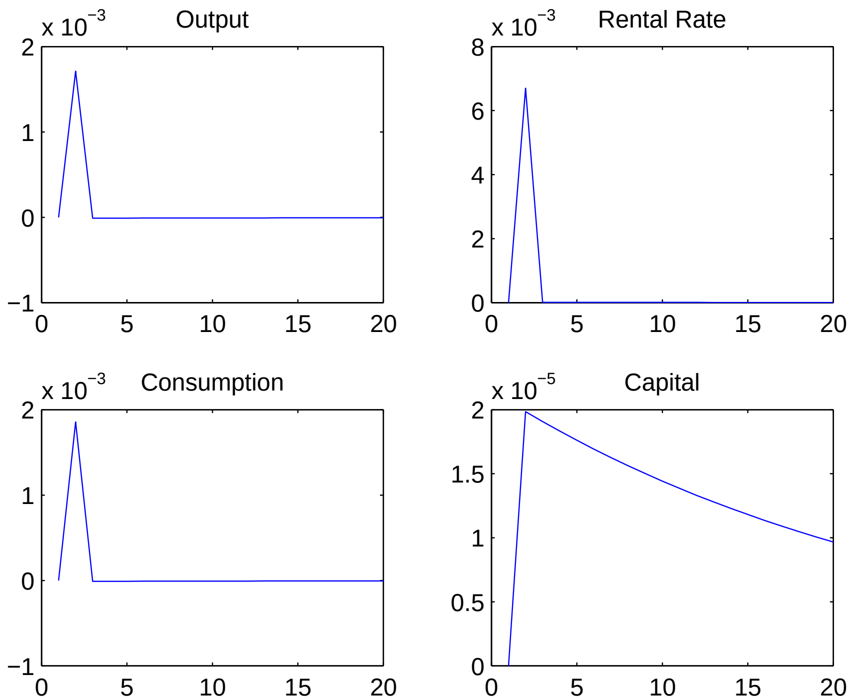

Figure 2.4: Response to a positive noise shock

belief that aggregate productivity did not increase much in the first place.

In the next period, the average firm observes that its island specific productivity remains unchanged, while the noisy public signal reverts back to the steady state level. This further suggests to the firm that there was in fact no change in aggregate productivity in the previous period. Given that the firm recognizes that other firms likely share the same beliefs, the effect of an initial noise shock dies out very quickly, as can be seen in Figure 2.4. Thus, a noise shock leads only to a lukewarm initial response in aggregate output which is not persistent.

The key reason why there is no persistent effect of noise in this setup is that agents learn from prices, which are endogenous signals that inform agents about the beliefs of others. The findings of this section suggest that prices are fairly good at eliminating fluctuations in an economy arising due to noise shocks. However, as was clear from the previous sections, this was not the case when the economy was hit by a productivity shock. In that case, the price signals actually reinforced the pessimistic beliefs in the economy and resulted in more volatile fluctuations compared to the full information case. Thus, this paper demonstrates that the efficiency with which the pricing system aggregates information about the shocks affecting an economy depends on the type of shock.

Earlier work such as Angeletos and La'O (2008) and Lorenzoni (2009) show that noise shocks can cause expectation driven business cycle fluctuations. This paper shows that pure exogenous noise shocks have only a transitory effect on the economy once agents can learn from endogenous signals such as prices. Rather it is the noisy informational content of endogenous price signals and the bias embedded in them from correlated forecast errors that generate fluctuations based on sentiments.

2.5 Normative Analysis

To explore the normative implications of dispersed information, I first describe the benchmark economy to which the decentralized economy is compared. Earlier literature such as Angeletos and La'O (2010), Angeletos and Pavan (2009) and others argue that comparing an economy with dispersed information to a full information economy is not a sensible comparison. They argue that the appropriate benchmark for comparison is one where agents still have to rely on their private information to make their decisions but internalize the collective welfare of all agents in their decision making.

With the benchmark economy defined as above, it is still not trivial to define a planner's problem in this setup. Recent papers that deal with the normative implications of informational frictions, such as Adam (2007), Angeletos and La'O (2010), Lorenzoni (2010) and Paciello and Wiederholt (2011), all assume the existence of perfect consumption insurance across households and that all households have identical beliefs. This is not so in the current setup as there is imperfect consumption insurance across islands, and in addition households on each island have different beliefs. Thus, the planner's problem which corresponds to the benchmark economy as defined above takes the following form.

The planner maximizes the sum of expected utility of the households on each island. The expected utility of each household is calculated with respect to their own beliefs. In addition, the planner must make use of the capital market to engineer redistribution. Thus, the planner's problem is not the traditional one which is independent of prices. The problem of the planner can be written as follows:

$$\max \int_0^1 E_{i,0} \sum_{t=0}^{\infty} \beta^t \ln \left\{ C_{i,t} - \Psi \frac{H_{i,t}^{1+\nu}}{1+\nu} \right\} di$$

subject to

$$\begin{aligned} C_{i,t} + K_{i,t+1} - (1-\delta) K_{i,t} &= Y_t^{\frac{1}{\theta}} \left[z_{i,t} J_{i,t}^{\alpha} H_{i,t}^{1-\alpha} \right]^{\frac{\theta-1}{\theta}} \\ &\quad + r_t (K_{i,t} - J_{i,t}) \ , \ \forall i \in [0,1] \end{aligned} \tag{2.50}$$

$$\int_0^1 J_{i,t} di = \int_0^1 K_{i,t} di + \zeta_t \tag{2.51}$$

$$Y_t^{\frac{\theta-1}{\theta}} = \int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di \tag{2.52}$$

$$K_{i,t+1} = K_{i,t+1}(\mathcal{I}_{i,t}) \ , \ H_{i,t} = H_{i,t}(\mathcal{I}_{i,t}) \ , \ J_{i,t} = J_{i,t}(\mathcal{I}_{i,t}) \tag{2.53}$$

Equation (2.50) represents the budget constraint of each household i and equation (2.52) defines aggregate output. Equation (2.51) is the capital market clearing condition and is the non-standard component in the planner's problem, since the planner is now restricted by a market in choosing allocations. The solution to the planner's problem provides the optimal allocations. As was the case in the earlier sections, I solve the log-linear approximation around the steady state to the planner's problem. The steady state of solution to the planner's problem is different from the decentralized steady state defined in Appendix B.1. From now on, I shall refer to the steady state of the planner's problem as the undistorted steady state and the steady state defined in Appendix B.1 as the distorted steady state. Appendix B.1.2 defines the undistorted steady state. In the undistorted steady state, the monopolistic wedge is absent. In addition, there is an additional wedge in the distorted steady state between the marginal rate of substitution and the marginal product of labor arising from the assumption that the firm internalizes the upward sloping labor supply. This wedge is absent from the undistorted steady state as well.

Proposition 12. *The planner chooses the output produced on each island according* *to:*

$$\begin{aligned}\widehat{Y}_{i,t} &= \frac{\kappa(1+\alpha\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\left[C(\widehat{C}_{i,t}-\widehat{C}_t)-\Psi H^{1+\nu}(\widehat{H}_{i,t}-\widehat{H}_t)\right]\ldots\\ &+\frac{(\theta-1)(1+\alpha\nu)}{\theta(1+\nu[\alpha+\theta(1-\alpha)])}\left[\widehat{Y}_t+E_{i,t}\widehat{Y}_t\right]+\frac{(2-\theta)(1+\alpha\nu)}{\theta(1+\nu[\alpha+\theta(1-\alpha)])}\overline{E}_t\widehat{Y}_t\ldots\\ &+\frac{\theta(1+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{z}_{i,t}+\frac{\alpha\theta(1+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{r}_t \qquad (2.54)\end{aligned}$$

where C and H represent the undistorted steady state consumption and labor. Aggregate output is given by:

$$\widehat{Y}_t=\frac{(1-\alpha)\overline{E}_t\widehat{Y}_t}{1-\alpha+\theta^2(\alpha+\nu)}+\frac{\theta^2(1+\nu)\widehat{z}_t}{1-\alpha+\theta^2(\alpha+\nu)}+\frac{\alpha\theta^2(1+\nu)\left(K_t+\zeta_t K^{-1}\right)}{1-\alpha+\theta^2(\alpha+\nu)} \qquad (2.55)$$

A major difference between the planner's allocations and the decentralized economy is that the planner internalizes the effect of each firm's output on total output. As a result, a planner chooses to produce more and to reduce the monopolistic wedge. In addition, the planner reduces the effective strength of the strategic complementarity by tempering the response of a firm's output to changes in perceptions about aggregate demand.

Proposition 13. *The response of aggregate output to changes in perceptions of aggregate demand is smaller in the the planner's solution than in the decentralized economy.*

Proof. It can be seen from equations (2.39) and (2.55) that in the planner's solution, the response of output to changes in average perceptions about aggregate demand is smaller than the decentralized case:

$$\left.\frac{\partial \widehat{Y}_t}{\partial \overline{E}_t \widehat{Y}_t}\right|_{planner} = \frac{1-\alpha}{1-\alpha+\theta^2(\alpha+\nu)} < \frac{1-\alpha}{1-\alpha+\theta(\alpha+\nu)} = \left.\frac{\partial \widehat{Y}_t}{\partial \overline{E}_t \widehat{Y}_t}\right|_{decentralized}$$

as $\theta > 1$. □

In fact, given the calibration of the model, the planner would reduce the responsiveness of aggregate output to changes in average beliefs by one order of magnitude. The planner increases the responsiveness of output to aggregate productivity and capital but these elasticities are of the same order of magnitude as in the decentralized case.

To understand the planner's motive, it is important to understand why the planner seeks to reduce the degree of strategic complementarity in a firm's output decision. As was discussed earlier, the primary reason why recessions are more persistent and why recoveries are inhibited is because of the interaction of dispersed information and strategic complementarities in firms' output decisions. In such a setting, the firms do not incorporate new information in their decisions if they fear that other firms do not share the same information. Since prices aggregate the information from the decisions of agents, prices fail to reflect new information about changing economic conditions. The above mechanism can also be seen in terms of an information externality. The coordination motive amongst firms causes each firm to place an inefficiently high weight on public information (prices) in making its decisions and consequently to put too little weight on its private information. This results in inefficiently low "production of information" as each agent puts too little weight on its private information. Agents put too much weight on prices, which are reinforcing the pessimism/optimism in the first place. This is the main driver for inefficiently (unconditionally)volatile and persistent fluctuations in the decentralized economy.

To correct this information externality, the planner wants to impose a Pigouvian tax on the use of public information so as to align private incentives in information use with the social incentives. By increasing the weight on private signals, the planner is indirectly increasing the informational content of prices. This can be seen as an increase in the signal to noise ratio in the price signal as not a lower weight is assigned to past innovations which implies a lower part of total variance ascribed to these past innovations. Since prices now weight information about current conditions more, they reinforce pessimism/optimism less. As a result, aggregate output responds less to changes in perceptions in the planner's solution. Notice that the planner does not provide any new information to the agents in the economy but merely changes the weight that agents put on their private information in their decisions and as a result reduces the information externatlity. In other words, the planner does not directly influence the filtering problem of agents but can still construct an improvement in welfare.

Notice, however, that the planner does not entirely eliminate the response of output to changes in perception about aggregate demand. There are two reasons for this. First, fluctuations caused by actual changes in productivity are efficient, and these also affect the perceptions about aggregate demand. Thus, not all fluctuations caused by a change in perceptions are inefficient. Second, the planner faces a trade-off in reducing the strategic complementarity. By reducing the strategic complementarity, output on each island responds more to idiosyncratic productivity, as shown in the first term of equation (2.54). This increases the cross-sectional dispersion of output, which reduces welfare. Because of the incomplete markets, a higher cross-sectional dispersion of output across islands also implies higher dispersion in consumption which reduces the average welfare in the economy. The planner's objective is concave, and thus, he prefers to equate marginal utilities across islands.

Angeletos and La'O (2008) conduct a normative analysis of their setup under dispersed information and conclude that the decentralized economy is constrained efficient. This is not the case here. The primary reason for this difference is that in the current setup, agents are allowed to learn from endogenous sources of information such as prices. Note that the forecast errors embedded in prices arise due to the agents assigning relatively less importance to their private signals. Since the planner can influence the weights agents assign to their various sources of information, the planner can actually control the informational content of prices and reduce the size of the forecast error.

The other difference between the current setup and Angeletos and La'O (2008) is that the fluctuations in their model are driven by noise in exogenous signals. As was discussed in the previous sections, the importance of these noise shocks is greatly diminished when agents can learn from endogenous price signals such as rental rates. In fact, the "non-fundamental" volatility in the current model is driven by endogenous price signals which are informationally noisy. This "noise" in prices is caused by the endogenously generated forecast errors. Since "noise" in this setup is an outcome of the model rather than a primitive, this implies policy can be directed to influence the extent of noise.[19]

2.5.1 Implementing the Efficient Allocations using Fiscal Policy

This subsection shows how the efficient allocation can be implemented by a policy-maker by using taxes and subsidies contingent only on aggregates that are observed by everyone in the economy. The key ingredient in this implementation is that agents in the economy are forward-looking and the planner can use state-contingent subsidies and taxes to affect the weights that agents place on public and private information in their decisions. In particular, a policy-maker can implement the Pigouvian tax on the use of public information by instituting a countercyclical sales subsidy contingent on the realization of aggregate output (which is publicly observed in this economy). Thus, the policy-maker need not possess an informational advantage over the private sector to implement a tax/subsidy to achieve the efficient allocation.

To implement the efficient allocation in the decentralized economy, the planner has access to a payroll tax/subsidy, τ_t^h, and a proportional sales subsidy, τ_t^s at his disposal. In addition I assume that the authority has access to a uniform lump-sum tax to balance the budget. Lump sum taxes are not of consequence in this setup as policy only works through the anticipatory effects of the distortionary taxes as will be seen below. Allowing for lump sum taxes is common in the normative literature

[19] This stands in contrast to a scenario where the noise is a primitive of the model. In the latter, policy obviously cannot affect the level of noise.

on informational frictions (See Angeletos and La'O (2010)). Taxes are collected at the end of the post-production stage and the government is restricted to run a balanced budget. Since the tax authority has at most the same information as the private sector, these tax rates depend only on quantities which are publicly observed. I assume that these taxes are functions of the realization of aggregate output, which is observed by all agents at the end of the post-production stage. More precisely, I assume that the (deviation from steady state values of) tax/ subsidy rates take on the following form:

$$\tilde{\tau}_t^s = \phi^s \widehat{Y} t \tag{2.56}$$

$$\tilde{\tau}_t^h = \phi^h \widehat{Y} t \tag{2.57}$$

where ϕ^s and ϕ^h are constants. $\phi^s < 0$ implies that the sales subsidy co-varies negatively with output. $\phi^h > 0$ implies that the payroll tax co-varies positively with output. The problem of household i and firm i in the presence of taxes is presented in Appendix B.4.

By restricting taxes/subsidies to depend only on publicly available information, I am implicitly assuming that taxes are uniform across islands. Given these uniform taxes, the tax authority cannot hope to address all the concerns of the planner in implementing the efficient allocation. To implement a set of efficient allocations, the tax authority must would need to have access to island-specific taxes and subsidies which depend on information available on that island. In this paper, I focus on the former case and leave the latter for future research.

Given the tax system, the optimal decisions of firm i can be summarized by:

$$\widehat{H}_{i,t} = \frac{[1+(\theta-1)\phi^s - \alpha\phi^h - \theta(1-\alpha)\phi^h]E_{i,t}\widehat{Y}_t}{1+\nu(\alpha+\theta(1-\alpha)} + \frac{(\theta-1)\widehat{z}_{i,t}}{1+\nu(\alpha+\theta(1-\alpha)} \cdots - \frac{\alpha(\theta-1)\widehat{r}_t}{1+\nu(\alpha+\theta(1-\alpha)} \tag{2.58}$$

$$\widehat{J}_{i,t} = \frac{\{(1+\nu)[1+(\theta-1)\phi^s] - (1-\alpha)(\theta-1)\phi^h\}E_{i,t}\widehat{Y}_t}{1+\nu(\alpha+\theta(1-\alpha)} + \frac{(\theta-1)(1+\nu)\widehat{z}_{i,t}}{1+\nu(\alpha+\theta(1-\alpha)} \cdots - \frac{[1-\alpha+\theta(\alpha+\nu)]\widehat{r}_t}{1+\nu(\alpha+\theta(1-\alpha)} \tag{2.59}$$

The output of firm i is given by:

$$\widehat{Y}_{i,t} = \frac{(1+\alpha\nu)[1-\phi^s(\theta-1)] - \theta(1-\alpha)\phi^h}{1+\nu(\alpha+\theta(1-\alpha)} E_{i,t}\widehat{Y}_t + \frac{\theta(1+\nu)}{1+\nu(\alpha+\theta(1-\alpha)}\widehat{z}_{i,t} \cdots - \frac{\alpha\theta(1+\nu))}{1+\nu(\alpha+\theta(1-\alpha)}\widehat{r}_t \tag{2.60}$$

Notice that ϕ^s and ϕ^h affect the degree of strategic complementary by affecting how firms respond to a change in perception about aggregate demand. Aggregate output in this economy is given by:

$$\widehat{Y}_t = \frac{\theta(1+\nu)}{1-\alpha+\theta(\alpha+\nu)} z_t + \frac{\alpha\theta(1+\nu)}{1-\alpha+\theta(\alpha+\nu)}\left(K_t + \frac{\zeta_t}{K}\right) + \frac{(1-\alpha)(1+\theta(\phi^s-\phi^h)-\phi^s)}{1-\alpha+\theta(\alpha+\nu)}\overline{E}_t\widehat{Y}_t \tag{2.61}$$

Notice that by appropriately choosing ϕ^s and ϕ^h, the tax authority can reduce the response of output to changes in perceptions. By setting ϕ^s and ϕ^h appropriately, the planner can equate the elasticity of output with respect to changes in perception about aggregate demand in equations (2.55) and (2.61).

Given that the tax authority is constrained to only make taxes contingent on a linear function of output, it cannot affect the responsiveness of output to actual productivity and capital to match those from the planner's problem. However, as was noted earlier, the planner increases the sensitivity of output to actual changes in productivity, but these changes aren't very large. Many different configurations of (ϕ^s, ϕ^h) can be used to reduce the responsiveness of output to changes in average perceptions. The two most obvious candidates being setting one of the two to zero. Setting $\phi^h = 0$, given the calibration of the model, $\phi^s = -0.09$ which implies that the sales subsidy is countercyclical and that the payroll tax does not vary with the cycle. The other preferred alternative is to set $\phi^s = 0$ and to use the payroll tax. This yields $\phi^h = 0.08$ which implies a procyclical payroll tax and an acyclical sales subsidy. This second option is more in line with the old Keynesian prescription of a counter-cyclical fiscal policy through a pro-cyclical payroll tax. Simulations of the model confirm that this indeed increases welfare and reduces the volatility of output fluctuations.

To see why a countercyclical subsidy can implement the Pigouvian tax consider the following. The reason recessions were protracted was because each firm was unwilling to act on positive private information as it was worried that the others did not share the same positive news. Strategic complementarity in a firm's output decision implies that it loses profits by producing a quantity very different from other firms. If a firm expects others to produce low output, it too reduces its output.

Further, as was explained earlier, this causes prices to not reflect information about improving conditions and thus extends and deepens recessions. The countercyclical sales subsidy protects a firm against this very contingency. If a firm acts on its positive private information and produces more while others do not respond, the sales subsidy makes up for the firm's lost profits from not coordinating. As a result, the sales subsidy reduces the reliance of a firm's payoffs to the decisions of other firms. In particular, it reduces the effective degree of strategic complementarity in a firm's output decision.

With a lower effective degree of strategic complementarity, each firm has a smaller incentive to coordinate and hence has an incentive to react more to its private information. In other words, each firm reduces the weight it puts on public information and increases the weight it puts on private information in their decision making process. Since decisions now reflect newer information, so do prices and hence there is more "production of information" which reduces the effect of pessimism and optimism on output fluctuations. Thus, even without directly providing new information to the agents, the policy maker has the ability to reduce the inefficiently large volatility and persistence. Note that the policy of providing a counter-cyclical sales subsidy to firms operates only because agents are forward looking and change their behavior in anticipation of events in the future. This is reminiscent of the results in King (1982) and Weiss (1980) who demonstrated that policy can be used to affect the informational content of prices in models of monetary policy where agents learn from commonly observed prices.

Notice that the policy analysis assumed uniform taxation across islands. More generally, taxes/ subsidies could be tailored to local conditions. Allowing taxes and subsidies to be based on island specific information could reduce the tension in the planner's decision which forces him to accommodate some sentiment-driven fluctuations, i.e. the planner is unable to entirely eliminate the presence of sentiment-driven fluctuations. By conditioning on local characteristics, the planner has another tool to control the cross-sectional dispersion of output which can allow him to further reduce aggregate fluctuations arising from sentiments without imposing further welfare losses that are due to increased cross-sectional variance across islands. This is left for future research.

2.6 Concluding Remarks

This paper argues that in economies with incomplete markets, dispersed information in the presence of strategic complementarities results in agents having to predict the information that others possess. I highlight that the information inefficiency of the pricing system may result in excessively volatile and persistent fluctuations. In particular, if agents infer the perceptions of others from an informationally inefficient pricing system, they form forecast errors which are correlated. These correlated forecast errors play the role of pessimism and optimism and interact with production and investment decisions of firms and households to generate unconditionally more volatile and persistent swings in output.

Allowing agents to learn from prices reduces the impact of pure noise shocks in driving fluctuations in the economy. However, dispersed information causes more pronounced booms and recessions in response to productivity shocks. These fluctuations are inefficient and can be thought of as being amplified by non-fundamental forces such as pessimism and optimism.

In addition to the positive implications of dispersed information, I also conduct a normative analysis which shows that the decentralized economy is characterized by inefficient volatility and persistence of output in response to shocks. Countercyclical fiscal policy can be used to remedy these inefficient fluctuations. However, this analysis assumed that the planner can credibly commit to policies. As is well known from work by Svensson and Woodford (2004) and others, that time consistency is an issue in the choice of policy with forward looking agents in environments of incomplete information. Finding a time-consistent implementation is much harder in the current setup and is left for future work.

The current paper presents a tractable method of solving business cycle models with heterogeneous beliefs. The basic structure of this model shows that even the simplest business cycle models can become very complicated when agents are allowed to have different beliefs. Future work in this area involves exploring the implications of dispersed beliefs in more sophisticated models with asset markets and how this affects the business cycle through the financing decisions of firms. Another avenue of future research is to explore non-linear solutions to the current problem as the linear methods used in this paper impose symmetry on the model. Applying non-linear methods will allow an exploration of whether information dispersion itself changes over the business cycle.

Chapter A: Appendix for Chapter 1

A.1 Proofs

A.1.1 Existence of a constant equilibrium nominal interest rate

Let $R(t) = R$, then, by definition

$$Q(t) = e^{Rdt} E_t\{Q(t+dt)\}$$

From equation (1.6):

$$E_t\left(\frac{Q(t+dt)}{Q(t)}\right) = e^{-\rho dt} E_t\left(\frac{R(t)M^D(t)}{R(t+dt)M^D(t+dt)}\right)$$

Impose $R(t) = R(t+dt) = R$:

$$E_t\left(\frac{Q(t+dt)}{Q(t)}\right) = e^{-\rho dt} E_t\left(\frac{M^D(t)}{M^D(t+dt)}\right) = e^{\left(-\rho-\mu+\frac{\sigma_m^2}{2}\right)dt}$$

Thus,

$$R = \rho + \mu - \frac{\sigma_m^2}{2}$$

A.1.2 Proof of Lemma 1

The Full-Information case is when $F_m = F_z = 0$. All firms adjust prices in response to all shocks at every instant. Plugging in equations (4) - (9) and imposing market clearing into the firm's objective can be rewritten as

$$E_0^i \quad \left\{ \frac{1}{\lambda} \int_0^\infty e^{-\rho t} \left[\left(\frac{RM(t)}{P(t)} \right)^{\frac{1}{\gamma}-1} \left(\frac{P_i(t)}{P(t)} \right)^{1-\epsilon} - \frac{\alpha}{[AZ_i(t)]^{\frac{1}{\theta}}} \left(\frac{RM(t)}{P(t)} \right)^{\frac{1}{\gamma\theta}} \left(\frac{P_i(t)}{P(t)} \right)^{-\frac{\epsilon}{\theta}} \right] dt \right.$$
$$\left. -\alpha \left[F_m \int_0^\infty e^{-\rho t} dD_i^m(t) dt + F_z \int_0^\infty e^{-\rho t} dD_i^z(t) dt \right] \right\} \tag{A.1.1}$$

Each firm sets $P_i(t)$ so as to maximize (A.1.1):

$$P_i^f(t) = \left[\frac{\epsilon\alpha}{\theta(\epsilon-1)} \frac{R^{\frac{1+\theta\gamma-\theta}{\gamma\theta}}}{A^{\frac{1}{\theta}}} \right]^{\frac{\theta}{\theta(1-\epsilon)+\epsilon}} Z_i(t)^{-\frac{1}{\theta(1-\epsilon)+\epsilon}} M(t)^{\frac{1+\gamma\theta-\theta}{\gamma(\theta(1-\epsilon)+\epsilon)}} P(t)^{1-\frac{1+\gamma\theta-\theta}{\gamma(\theta(1-\epsilon)+\epsilon)}}$$

Define $A = \left[\frac{\epsilon\alpha}{\theta(\epsilon-1)} R^{\frac{1+\gamma\theta-\theta}{\gamma\theta}} \right]^\theta$ so that the initial constant term goes to 1. Thus,

$$P_i^f(t) = Z_i(t)^{-\frac{1}{\theta(1-\epsilon)+\epsilon}} M(t)^{\frac{1+\gamma\theta-\theta}{\gamma(\theta(1-\epsilon)+\epsilon)}} P(t)^{1-\frac{1+\gamma\theta-\theta}{\gamma(\theta(1-\epsilon)+\epsilon)}}$$

Taking logs on both sides

$$\ln P_i^f(t) = \zeta \ln Z_i(t) + r \ln P^f(t) + (1-r) \ln M(t) \tag{A.1.2}$$

where $\zeta = \frac{-1}{\theta(1-\epsilon)+\epsilon}$ and $r = 1 - \frac{1+\gamma\theta-\theta}{\gamma(\theta(1-\epsilon)+\epsilon)}$.

A.1.3 Proof of Proposition 1

Subtract μt from both sides of equation (A.1.2) to get

$$\ln P_i^f(t) - \mu t = \zeta[\ln Z_i(t) - 0] + r[\ln P^f(t) - \mu t] + (1-r)[\ln M(t) - \mu t]$$

which implies

$$\hat{P}_i^f(t) = \zeta \hat{Z}_i(t) + r\hat{P}^f(t) + (1-r)\hat{M}(t) \tag{A.1.3}$$

where the hats imply deviations from the symmetric steady state equilibrium.

The price index $P = \left[\int_0^1 P_i(t)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$ can be approximated by

$$p(t) = \int_0^1 p_i(t)di$$

around any symmetric equilibrium. Thus, integrating equation (A.1.3) over $i \in [0,1]$ yields:

$$\begin{aligned}\hat{P}^f(t) &= \int_0^1 \hat{P}_i^f(t)di = \zeta \int_0^1 \hat{Z}_i(t)di + r\hat{P}^f(t) + (1-r)\hat{M}(t) \\ &= r\hat{P}^f(t) + (1-r)\hat{M}(t)\end{aligned}$$

which implies

$$\hat{P}^f(t) = \hat{M}(t)$$

A.1.4 Proof of Lemma 2

This proof mirrors the steps in Hellwig and Veldkamp (2009). Guess that $p(t)$ follows the path

$$p(t) = \sigma_m \int_{-\infty}^{t} g_t(\tau) dW(\tau) + h_t(\tau)t$$

Plugging this guess into the expression for $p^*(t)$ yields

$$p^*(t) = \sigma_m \int_{-\infty}^{t} [1 - r + rg_t(\tau)] dW(\tau) + (1 - r)\mu t + rh_t(\tau)t$$

Note that

$$E\{p^*(t) \mid \mathcal{I}_{\hat{\tau}_m}\} = \sigma_m \int_{-\infty}^{\hat{\tau}_m} [1 - r + rg_t(\tau)] dW(\tau) + (1 - r)\mu t + rh_t(\tau)t$$

and

$$E\{z_i(t) \mid \mathcal{I}^i_{\hat{\tau}_z}\} = z_i(\hat{\tau}_z) e^{-\eta(t-\hat{\tau}_z)}$$

Thus, from equation (1.18), firm i with the information set $\mathcal{I}^i_t = \mathcal{I}_{\hat{\tau}_m} \times \mathcal{I}^i_{\hat{\tau}_z}$ is

$$p_i(t) = \sigma_m \int_{-\infty}^{\hat{\tau}_m} [1 - r + rg_t(\tau)] dW(\tau) + (1 - r)\mu t + rh_t(\tau)t + \zeta z_i(\hat{\tau}_z) e^{-\eta(t-\hat{\tau}_z)}$$

The aggregate (log) price can be derived by integrating over the two distributions Γ_t^m and Γ_t^z.

$$p(t) = \sigma_m \int_{-\infty}^{t} [1 - \Gamma_t^m(\tau)][1 - r + rg_t(\tau)]dW(\tau) + (1-r)\mu t + rh_t(\tau)t$$

Using the method of undetermined coefficients yields

$$g_t(\tau) = \frac{[1 - \Gamma_t^a(\tau)](1-r)}{1 - r + r\Gamma_t^a(\tau)} \text{ and } h_t(\tau) = \mu$$

Thus,

$$p(t) = \int_{-\infty}^{t} \frac{(1-r)(1 - \Gamma_t^m(\tau))}{1 - r + r\Gamma_t^m(\tau)} dW(\tau) + \mu t$$

Plugging this into the expression for

$$\begin{aligned} p^*(t) &= r\sigma_m \int_{-\infty}^{t} \frac{([1 - \Gamma_t^a(\tau)]1 - r)}{1 - r + r\Gamma_t^a(\tau)} dW(\tau) + r\mu t + (1-r)\sigma_m \int_{-\infty}^{t} dW(\tau) + (1-r)\mu t \\ &= \sigma_m \int_{-\infty}^{t} \frac{1-r}{1 - r + r\Gamma_t^a(\tau)} dW(\tau) + \mu t \end{aligned}$$

The next claim follows from the fact that $W(t)$ is a standard Brownian motion.

A.1.5 Proof of Proposition 2

The problem of finding the optimal planning horizon about the aggregate state can be reformulated with the time since when last information was acquired. Define $\delta_m = t - \hat{\tau}_m$ and $\delta_z = t - \hat{\tau}_z$ as the time since firm i last acquired information about the aggregate state and about the idiosyncratic state respectively. Thus, the two

Bellman equations above can be rewritten as

$$\mathbb{L}_1(\delta_m) = \min_{\delta'_m \geq \delta_m} \int_0^{\delta'_m - \delta_m} e^{-\rho s} L_1(s) ds + e^{-\rho(\delta'_m - \delta_m)} [\mathcal{C}_m + \mathbb{L}_1(0)] \quad \text{(A.1.4)}$$

$$\mathbb{L}_2(\delta_z) = \min_{\delta'_z \geq \delta_z} \int_0^{\delta'_z - \delta_z} e^{-\rho s} L_2(s) ds + e^{-\rho(\delta'_z - \delta_z)} [\mathcal{C}_z + \mathbb{L}_2(0)] \quad \text{(A.1.5)}$$

where

$$L_1(\delta_m) = \begin{cases} \sigma_m^2 \int_0^{\delta_m} \frac{(1-r)^2}{(1-r\frac{s}{T_m})^2} ds & \text{if } \delta_m \leq T_m \\ \sigma_m^2 \int_0^{T} \frac{(1-r)^2}{(1-r\frac{s}{T_m})^2} ds + \sigma_m^2(\delta_m - T_m) & \text{if } \delta_m > T_m \end{cases}$$

and

$$L_2(\delta_z) = \zeta^2 \sigma_z^2 \int_0^{\delta_z} ds = \zeta^2 \sigma_z^2 \delta_z$$

The solution to the first Bellman equation is characterized by the optimal planning horizon (for aggregate money shocks) T_m^*, iff $t > s + T_m^*$. The first order conditions with respect to $\hat{\delta}_m$ for the problem described in equation (A.1.4), using the fact that the optimal horizon is T_m^*, we can write

$$L_1(T_m^*) = \rho[\mathcal{C}_q + \mathbb{L}_1(0)] \quad \text{(A.1.6)}$$

where

$$\mathbb{L}_1(0) = \frac{\int_0^{T_m^*} e^{-\rho s} L_1(s) ds + e^{-\rho T_m^*} \mathcal{C}_m}{1 - e^{-\rho T_m^*}} \quad \text{(A.1.7)}$$

Following equations (A.1.6) and (A.1.7), in equilibrium since $T_m = T_m^*$, it must be the case that

$$L_1(T_m^*) = \frac{\rho}{1-e^{-\rho T_m^*}} \left(\int_0^{T_m^*} e^{-\rho s} L_1(s) ds + \mathcal{C}_m \right)$$

or

$$\mathcal{C}_m = \int_0^{T_m^*} e^{-\rho \delta} [L_1(T_m^*) - L_1(\delta)] d\delta$$

Recall that

$$L_1(\delta) = \sigma_m^2 \int_0^{\delta} \frac{(1-r)^2}{\left(1 - r\frac{s}{T_m}\right)^2} ds$$

if $\delta \leq T_m$. Define $\Theta = \frac{s}{T_m^*}$, $L_1(\delta)$ can be written as

$$L_1(\delta) = \sigma_m^2 T_m^* \int_0^{\delta/T_m^*} \left(\frac{1-r}{1-r\Theta} \right)^2 d\Theta = \sigma_m^2 (1-r)^2 T_m^* \frac{\delta}{T_m^* - r\delta}$$

which can be used to write

$$L_1(T_m^*) - L_1(\delta) = (1-r) T_m^* \frac{T_m^* - \delta}{T_m^* - r\delta}$$

Therefore, T_m^* is implicitly defined by

$$F_m(\sigma_m, r, \mathcal{C}_m, T_m^*) = 0$$

where

$$F_m(\sigma_m, r, \mathcal{C}_m, T_m) = \sigma_m^2 T_m \int_0^{T_m} (1-r) e^{-\rho s} \frac{T_m - s}{T_m - rs} ds - \mathcal{C}_m$$

Note that

$$\frac{\partial F_m}{\partial T_m^*} = \sigma_m^2(1-r)\int_0^{T_m^*} \frac{T_m^* - s}{T_m^* - rs} ds + \sigma_m^2(1-r)T_m^* \int_0^{T_m^*} \frac{s(1-r)}{(T_m^* - rs)^2} ds > 0 \text{ for } 0 < r < 1$$

Since, $F_m(\sigma_m, r, \mathcal{C}_m, 0) = -\mathcal{C}_m < 0$ and $\frac{\partial F_m}{\partial T_m^*} > 0$, F_m crosses zero only once, $T_m^* > 0$ is unique.

A.1.6 Derivation of the Phillips Curve

To estimate the Phillips curve, I use a discrete time approximation to the problem. In the staggered equilibrium, a particular firm i which last updated information at date τ, at time t wants to set the (log) price

$$p_i(t; \tau) = E_\tau \left[p(t) + (1-r)y(t) \right]$$

where F_τ denotes the forecast of the (log) target price at time t based on information as of time τ. $E_\tau p(t)$ is the expected log price at t as of τ and $E_\tau y(t)$ is the expected output gap (in percentages) as at t as of τ.

Since in the staggered symmetric equilibrium, a constant fraction $\frac{1}{T}$ of firms update their information every period, at time t, no firm has information older than $t-T-1$, where T is the optimal planning horizon(that was determined as an equilibrium object). Thus, a mass $\frac{1}{T}$ of firms have information which is current as of date $s \in [t-T+1, t]$. Thus, the aggregate (log) price at time t is the average of the price set by each set of firms with different vintages:

$$\begin{aligned} p(t) &= \sum_{s=0}^{T-1} \frac{E_{t-s}\left[p(t)+(1-r)y(t)\right]}{T} \\ &= -E_{t-T}\left[\frac{p(t)+(1-r)y(t)}{T-1}\right] + \sum_{s=0}^{T-1} \frac{E_{t-1-s}\{p(t)+(1-r)y(t)\}}{T-1} \\ &\quad + \frac{(1-r)y(t)}{T-1} \end{aligned} \tag{A.1.8}$$

Similarly, the aggregate (log) price at $t-1$ can be written as

$$p(t-1) = \sum_{s=0}^{T-1} \frac{E_{t-1-s}\{p(t-1)+(1-r)y(t-1)\}}{T} \tag{A.1.9}$$

Subtract equation (A.1.8) from (A.1.9) to get equation (1.32) minus the measurement error:

$$\begin{aligned} \pi(t) &= \frac{p(t)+(1-r)y(t)}{T} + \sum_{s=0}^{T-1} \frac{E_{t-s-1}\{\pi(t)+(1-r)\Delta y(t)\}}{T} \\ &\quad - \frac{E_{t-T}\{p(t)+(1-r)y(t)\}}{T} \end{aligned} \tag{A.1.10}$$

A.2 Data Appendix

A.2.1 Estimation of the SIPC

The standard SIPC (Mankiw and Reis, 2002) can be written as

$$\pi_t = \frac{(1-\lambda)(1-r)}{\lambda} y_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j}[\pi_t + (1-r)\Delta y_t]$$

where $1 - \lambda$ is the exogenously specified rate of arrival of new information and also corresponds to the fraction of firms receiving new information every period. It is necessary to truncate the number of lagged variables as estimating an equation with infinite regressors is not feasible. However, the determination of the number of lags to be included is not supported by any economic theory or restriction from within the model. Additionally, truncation introduces another error term which is usually correlated with the regressors, causing estimates to be generally inconsistent. [See Coibion (2010) for a more detailed discussion on this inconsistency]

The setup in the present paper presents a slightly modified approach which lets one circumvent these problems. Given that each firm updates its information every T quarters, in a staggered setting, a fraction $\frac{1}{T}$ of firms update their information each period. This implies that, at any date t, there is no firm that has information older than vintage $t - T$. This makes the optimal choice of lags equal to T. This forms a long run identifying restriction ensuring that monetary policy is neutral in the long run. Since all firms know about any changes to monetary policy prior to $t-T+1$, the aggregate price has responded proportionally to those changes in policy and hence there is no effect on output anymore. The standard SIPC imposes a different long run identifying restriction to ensure long run neutrality: $\lim_{t\to\infty} \lambda(1-\lambda)^t = 0$. This restriction in the standard SIPC implies that all firms receive new information at least asymptotically. However, this long run restriction does not provide a criterion for picking the number of lags to be used in the estimation.

A.2.1.1 Maximum Likelihood Estimation

The equation to estimate is

$$\begin{aligned}\pi(t) &= \frac{p(t)+ry(t)}{T}+\sum_{s=0}^{T-1}\frac{E_{t-s-1}\{\pi(t)+r\Delta y(t)\}}{T} \\ &\quad -\frac{E_{t-T}\{p(t)+ry(t)\}}{T}+\epsilon_t \qquad \text{(A.2.1)}\end{aligned}$$

where $\epsilon_t \sim N(0,\sigma^2)$ is a measurement error i.i.d across time. The parameter space is defined as follows

$$\Theta = \{(T,r,\sigma) \mid T \in \{2,3,4\}, r \in \mathbb{R}, \sigma \in \mathbb{R}_+\}$$

I estimate the parameters using Maximum Likelihood estimation. The Likelihood function given data X can be written as

$$\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^{t_N-t_1} exp\left\{-\frac{1}{2\sigma T}\sum_{t=t_1}^{t_N}\Bigg(T\pi(t)-[p(t)+ry(t)]+E_{t-T}[p(t)+ry(t)]\cdots \right. \\ \left. -\sum_{s=0}^{T-1}E_{t-s-1}[\pi(t)+r\Delta y(t)]\Bigg)^2\right\}$$

A.2.1.2 Hypothesis Testing

To test Proposition 5, I estimate the model over two subsamples. Thus, the model to be estimated is

$$
\begin{aligned}
\pi(t) \quad = \quad & \mathcal{I}(t < break)\left[\frac{p(t) + r_{pre}y(t)}{T_{pre}} - \frac{E_{t-T_{pre}}\{p(t) + r_{pre}y(t)\}}{T_{pre}} \dots \right. \\
& \left. + \sum_{s=0}^{T_{pre}-1} \frac{E_{t-s-1}\{\pi(t) + r_{pre}\Delta y(t)\}}{T_{pre}}\right] \dots \\
& + \mathcal{I}(t \geq break)\left[\frac{p(t) + r_{post}y(t)}{T_{post}} - \frac{E_{t-T_{post}}\{p(t) + r_{post}y(t)\}}{T_{post}} \dots \right. \\
& \left. + \sum_{s=0}^{T_{post}-1} \frac{E_{t-s-1}\{\pi(t) + r_{post}\Delta y(t)\}}{T_{post}}\right] + \varepsilon_t
\end{aligned}
$$

To implement the Likelihood ratio test I calculate the maximized log likelihood of the restricted model where I restrict $T_{pre} \leq T_{post}$ and the unrestricted model where T_{pre} and T_{post} are left unrestricted. The test statistic takes the following form

$$LR = -2\left[\ln \mathcal{L}(restricted) - \ln \mathcal{L}(unrestricted)\right]$$

which is distributed as a χ^2 with 1 degree of freedom. The critical values for 1% is 6.6349, 5% is 3.8415 and for 10% is 2.7055.

For the first breakpoint, with the median forecasts, the test statistic is 10.386 and the null can be rejected with 99% confidence.; with mean forecasts, the test statistic is 4.6132 and the null can be rejected with 95% confidence.

For the second breakpoint, with the median forecasts, the test statistic is 10.386 and the null can be rejected with 99% confidence.; with mean forecasts, the test statistic is 4.6132 and the null can be rejected with 95% confidence.

Chapter B: Appendix for Chapter 2

B.1 Steady State

This section defines the undistorted and the distorted steady states:

B.1.1 Distorted Steady State of the Economy

I focus on the non-stochastic steady state with a degenerate distribution capital. The non negativity constraint on capital is assumed to be slack in steady. There is no uncertainty about fundamentals in steady state and thus all islands are symmetrically informed about the true state of the economy. Thus, the steady state of the dispersed information economy corresponds to the one for the symmetric information case. The steady state can be characterized as follows:

$$z_i = 1 \ , \forall i \tag{B.1.1}$$

$$e_i = 0 \ , \forall i \tag{B.1.2}$$

$$\zeta_i = 0 \ , \forall i \tag{B.1.3}$$

$$r = \frac{1}{\beta} - 1 + \delta \tag{B.1.4}$$

$$K_i = K = \left[\frac{\alpha\beta}{1-\beta(1-\delta)}\right]^{\frac{\alpha+\nu}{\nu(1-\alpha)}} \left[\frac{1-\alpha}{\Psi(1+\nu)}\right]^{\frac{1}{\nu}} \left[\frac{\theta-1}{\theta}\right]^{\frac{1+\nu}{\nu(1-\alpha)}} , \forall i \tag{B.1.5}$$

$$N_i = N = \left[\frac{\alpha\beta}{1-\beta(1-\delta)}\right]^{\frac{\alpha}{\nu(1-\alpha)}} \left[\frac{1-\alpha}{\Psi(1+\nu)}\right]^{\frac{1}{\nu}} \left[\frac{\theta-1}{\theta}\right]^{\frac{1}{\nu(1-\alpha)}} , \forall i \tag{B.1.6}$$

$$Y_i = Y = \left[\frac{\alpha\beta}{1-\beta(1-\delta)}\right]^{\frac{\alpha(1+\nu)}{\nu(1-\alpha)}} \left[\frac{1-\alpha}{\Psi(1+\nu)}\right]^{\frac{1}{\nu}} \left[\frac{\theta-1}{\theta}\right]^{\frac{1+\alpha\nu}{\nu(1-\alpha)}} , \forall i \tag{B.1.7}$$

$$C_i = C = \left[\frac{\alpha\beta}{1-\beta(1-\delta)}\right]^{\frac{\alpha(1+\nu)}{\nu(1-\alpha)}} \left[\frac{1-\alpha}{\Psi(1+\nu)}\right]^{\frac{1}{\nu}} \left[\frac{\theta-1}{\theta}\right]^{\frac{1+\alpha\nu}{\nu(1-\alpha)}} \dots \times \left[1 - \frac{\alpha\beta\delta(\theta-1)}{\theta[1-\beta(1-\delta)]}\right] , \forall i \tag{B.1.8}$$

$$\omega_i = \omega = \left[\frac{1-\alpha}{1+\nu}\right] \left[\frac{\alpha\beta}{1-\beta(1-\delta)}\right]^{\frac{\alpha\nu}{\nu(1-\alpha)}} \left[\frac{\theta-1}{\theta}\right]^{\frac{\nu}{\nu(1-\alpha)}} , \forall i \tag{B.1.9}$$

B.1.2 Undistorted Steady State of the Economy

$$z_i = 1 \ , \forall i \tag{B.1.10}$$

$$e_i = 0 \ , \forall i \tag{B.1.11}$$

$$\zeta_i = 0 \ , \forall i \tag{B.1.12}$$

$$r = \frac{1}{\beta} - 1 + \delta \tag{B.1.13}$$

$$\alpha Y = rK \tag{B.1.14}$$

$$(1-\alpha)Y = \Psi H^{1+\nu} \tag{B.1.15}$$

$$Y = K^{\alpha} N^{1-\alpha} \tag{B.1.16}$$

$$C_i = C = Y - \delta K \tag{B.1.17}$$

B.2 Log-Linearized Economy

This section provides the log-linearized version of the economy defined in the main body of the paper. The log-linearization is arou+nd the non-stochastic steady state defined in Appendix B.1.

B.2.1 Shocks

The log-linearized shock processes are as follows:

Productivity   The productivity on island i can be expressed as

$$\widehat{z}_{i,t} = \widehat{z}_t + e_{i,t} \tag{B.2.1}$$

where

$$\widehat{z}_t = \rho_z \widehat{z}_{t-1} + v_t \tag{B.2.2}$$

Shock to the supply of Capital   The supply shock $\zeta_t \sim N(0, \sigma_\zeta^2)$ is i.i.d across time.

B.2.2 Households

The log-linearized form of household i's problem can be written as

$$\nu \widehat{H}_{i,t} \;=\; \widehat{\omega}_{i,t} \tag{B.2.3}$$

$$\begin{aligned} rE_{i,t}\widehat{r}_{t+1} &= \frac{\kappa}{\beta}\left\{CE_{i,t}\Delta\widehat{C}_{i,t+1} - \Psi H^{1+\nu}E_{i,t}\Delta\widehat{H}_{i,t+1}\right\} && \text{(B.2.4)} \\ C\widehat{C}_{i,t} - \Psi H^{1+\nu}\widehat{H}_{i,t} &= -K\left[\widehat{K}_{i,t+1} - \frac{\widehat{K}_{i,t}}{\beta}\right] + \frac{Y}{\theta}\left[\widehat{Y}_t + (\theta-1)\widehat{z}_{i,t}\right] \\ &\quad + \nu\Psi H^{1+\nu}\widehat{H}_t && \text{(B.2.5)} \end{aligned}$$

where $\kappa = \left[\frac{1+\nu}{(1+\nu)C - \Psi H^{1+\nu}}\right]$. Equation (B.2.3) is the labor supply, while equation (B.2.4) is the bond and capital Euler equations, and equation (B.2.5) is the budget constraint. Equations (B.2.4),(B.2.5) and (B.2.10) can be solved to yield the following capital accumulation decisions:

$$\begin{aligned}\Delta\widehat{K}_{i,t+1} &= -\frac{\alpha\beta\nu(\theta-1)\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\widehat{r}_t + \frac{\beta Y}{\theta K}\widehat{Y}_t - \left(\frac{\beta Y}{\theta K} - \frac{\nu\Psi H^{1+\nu}}{1+\nu(\alpha+\theta(1-\alpha)}\right)E_{i,t}\widehat{Y}_t \\ &+ \left[\frac{\beta r}{\kappa K} + \frac{\alpha\nu(1-\beta)(\theta-1)\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right]E_{i,t}\sum_{j=0}^{\infty}\beta^{j+1}\widehat{r}_{t+j+1} \\ &- \left[\frac{Y}{\theta K} + \frac{\nu\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right]\left(\frac{1-\beta}{\beta}\right)E_{i,t}\sum_{j=0}^{\infty}\beta^{j+1}\widehat{Y}_{t+j} \\ &- \left[\frac{(\theta-1)Y}{\theta K} + \frac{\nu(\theta-1)\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right]\left[\frac{(1-\beta)\beta\rho_z}{1-\beta\rho_z}E_{i,t}\widehat{z}_t - \beta\widehat{z}_{i,t}\right] \quad \text{(B.2.6)}\end{aligned}$$

B.2.3 Firms

Labor and capital demand for firm i is given by

$$\begin{aligned}\widehat{L}_{i,t} &= \frac{\theta-1}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{z}_{i,t} - \frac{\alpha(\theta-1)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{r}_t \\ &+ \frac{1}{1+\nu[\alpha+\theta(1-\alpha)]}E_{i,t}\widehat{Y}_t \qquad \text{(B.2.7)} \\ \widehat{J}_{i,t} &= \frac{(\theta-1)(1+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{z}_{i,t} - \frac{1-\alpha+\theta(\alpha+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{r}_t \\ &+ \frac{1+\nu}{1+\nu[\alpha+\theta(1-\alpha)]}E_{i,t}\widehat{Y}_t \qquad \text{(B.2.8)}\end{aligned}$$

The output of firm i is given by:

$$\begin{aligned}\widehat{Y}_{i,t} &= \frac{\theta(1+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{z}_{i,t} - \frac{\alpha\theta(1+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{r}_t \\ &+ \frac{1+\alpha\nu}{1+\nu[\alpha+\theta(1-\alpha)]}E_{i,t}\widehat{Y}_t \qquad \text{(B.2.9)}\end{aligned}$$

B.2.4 Market Clearing

B.2.4.1 Labor Market Equilibrium

Since labor is immobile across islands, the labor market on each island has to clear in equilibrium. Firm i internalizes that the household i has an upward sloping labor supply curve when choosing how much labor to hire. Equilibrium employment $\widehat{N}_{i,t}$ is given by:

$$\begin{aligned}\widehat{N}_{i,t} &= \frac{\theta-1}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{z}_{i,t} - \frac{\alpha(\theta-1)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{r}_t \\ &+ \frac{1}{1+\nu[\alpha+\theta(1-\alpha)]}E_{i,t}\widehat{Y}_t \end{aligned} \tag{B.2.10}$$

and the wage on island i is given by:

$$\begin{aligned}\widehat{\omega}_{i,t} &= \frac{\nu(\theta-1)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{z}_{i,t} - \frac{\alpha\nu(\theta-1)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{r}_t \\ &+ \frac{\nu}{1+\nu[\alpha+\theta(1-\alpha)]}E_{i,t}\widehat{Y}_t \end{aligned} \tag{B.2.11}$$

Aggregate employment is given by:

$$\begin{aligned}\widehat{N}_{t} &= \frac{\theta-1}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{z}_{t} - \frac{\alpha(\theta-1)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{r}_t \\ &+ \frac{1}{1+\nu[\alpha+\theta(1-\alpha)]}\overline{E}_{t}\widehat{Y}_t \end{aligned} \tag{B.2.12}$$

B.2.4.2 Capital Market Equilibrium

Total demand for capital is given by

$$\begin{aligned}\widehat{J}_t &= \frac{(\theta-1)(1+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{z}_t - \frac{1-\alpha+\theta(\alpha+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{r}_t \\ &+ \frac{1+\nu}{1+\nu[\alpha+\theta(1-\alpha)]}\overline{E}_t\widehat{Y}_t \end{aligned} \tag{B.2.13}$$

Aggregating equation (B.2.6)yields the law of motion of aggregate capital stock:

$$\begin{aligned}\widehat{K}_{t+1} &= \widehat{K}_t + \frac{\beta r}{\kappa K}\overline{E}_t\sum_{j=0}^{\infty}\beta^{j+1}\widehat{r}_{t+j+1} - \frac{Y}{\theta K}\left[\left(\frac{1-\beta}{\beta}\right)\overline{E}_t\sum_{j=0}^{\infty}\beta^{j+1}\widehat{Y}_{t+j} - \overline{E}_t\widehat{Y}_t\right] \\ &- \frac{(\theta-1)Y}{\theta K}\left[\frac{(1-\beta)\beta\rho_z}{1-\beta\rho_z}\overline{E}_t\widehat{z}_t - \beta\widehat{z}_t\right] \end{aligned} \tag{B.2.14}$$

Market clearing at time t requires that the rental rate $\widehat{r}_t$ equate $\widehat{K}_t$ and $\widehat{J}_t$:

$$\begin{aligned}\widehat{r}_t &= \frac{(\theta-1)(1+\nu)}{1-\alpha+\theta(\alpha+\nu)}\widehat{z}_t - \frac{1+\nu[\alpha+\theta(1-\alpha)]}{1-\alpha+\theta(\alpha+\nu)}(\widehat{K}_t+\zeta_t) \\ &+ \frac{1+\nu}{1-\alpha+\theta(\alpha+\nu)}\overline{E}_t\widehat{Y}_t \end{aligned} \tag{B.2.15}$$

B.2.4.3 Goods Market Equilibrium

Total ouptut is calculated by aggregating the output of all the firms.

$$\begin{aligned}\widehat{Y}_t &= \frac{\theta(1+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{z}_t - \frac{\alpha\theta(1+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{r}_t \\ &+ \frac{1+\alpha\nu}{1+\nu[\alpha+\theta(1-\alpha)]}\overline{E}_t\widehat{Y}_t \end{aligned} \tag{B.2.16}$$

B.2.5 Solution

Assume that aggregate capital evolves according to the following law of motion:

$$\widehat{K}_{t+1} = \mathcal{K}_v(L)v_t + \mathcal{K}_u(L)u_t + \mathcal{K}_\zeta(L)\zeta_t \tag{B.2.17}$$

where $\mathcal{K}_v(L), \mathcal{K}_u(L)$ and $\mathcal{K}_\zeta(L)$ are analytic over the complex disk of radius $\sqrt{\beta}$. Given the guesses above, the rental rate and aggregate output can be expressed as:

$$\begin{aligned}\widehat{r}_t = & \left[\frac{(\theta-1)(1+\nu)}{1-\alpha+\theta(\alpha+\nu)}\frac{1}{1-\rho_z L} - \frac{1+\nu[\alpha+\theta(1-\alpha)]}{1-\alpha+\theta(\alpha+\nu)}L\mathcal{K}_v(L)\ldots\right.\\ & \left.+\frac{1+\nu}{1-\alpha+\theta(\alpha+\nu)}\mathcal{Y}_v(L)\right]v_t + \left[-\frac{1+\nu[\alpha+\theta(1-\alpha)]}{1-\alpha+\theta(\alpha+\nu)}L\mathcal{K}_u(L)\ldots\right.\\ & \left.+\frac{1+\nu}{1-\alpha+\theta(\alpha+\nu)}\mathcal{Y}_u(L)\right]u_t + \left[-\frac{1+\nu[\alpha+\theta(1-\alpha)]}{1-\alpha+\theta(\alpha+\nu)}(L\mathcal{K}_\zeta(L)+1)\ldots\right.\\ & \left.+\frac{1+\nu}{1-\alpha+\theta(\alpha+\nu)}\mathcal{Y}_\zeta(L)\right]\zeta_t\end{aligned} \tag{B.2.18}$$

$$\begin{aligned}\widehat{Y}_t = & \left[\frac{1}{1-\rho_z L} - \frac{1-\alpha}{1+\nu}\mathcal{R}_v(L) + \frac{1+\alpha\nu}{1+\nu}L\mathcal{K}_v(L)\right]v_t - \left[-\frac{1-\alpha}{1+\nu}\mathcal{R}_u(L)\right.\\ & \left.-\frac{1+\alpha\nu}{1+\nu}L\mathcal{K}_u(L)\right]u_t - \left[-\frac{1-\alpha}{1+\nu}\mathcal{R}_\zeta(L) - \frac{1+\alpha\nu}{1+\nu}L\mathcal{K}_\zeta(L)\right]\zeta_t\end{aligned} \tag{B.2.19}$$

These equations are then collected into equation (2.47). As was discussed in the main text, equation (2.47) does not constitute a fundamental Wold representation, and must be transformed into one so that it can be used to forecast the future values of the fundamentals. This is carried out using Blaschke matrices. Using Blaschke matrices, one can construct an observationally equivalent Wold representation of equation (2.47). Following the methodology in Rozanov (1967), the observationally equivalent fundamental Wold representation can be written as:

$$x_{i,t} = \mathcal{M}^*(L)\xi^*_{i,t} \tag{B.2.20}$$

where $\mathcal{M}^*(L) = \mathcal{M}(L)W\mathcal{B}(L)$, $\varepsilon^*_{i,t} = \mathcal{B}(L^{-1})'W'\varepsilon_{i,t}$, $WW' = I$ and $\mathcal{B}(L)$ is a Blaschke matrix with a pole at infinity.

By construction, $Det[\mathcal{M}^*(z)] \neq 0$, $\forall |z| < 1$. Hence, $\varepsilon^*_{i,t}$ is fundamental for $x_{i,t}$. $\{\xi^*_{i,t-j}\}_{j=0}^{\infty}$ and $\{x_{i,t}\}$ span the same space, i.e. agent i can recover $\xi^*_{i,t}$ by observing all past and current realizations of signals $x_{i,t}$. Note that $\xi^*_{i,t}$ is comprised of linear combinations of past and current realizations of the *true* shocks to the economy. Agent i cannot back out the *true* current state of the economy, however, he can infer a linear combination of todays shocks and shocks from previous periods. Using this fundamental representation, I then use the Wiener Kolmogrov forecasting formulas and Hansen Sargent formula to compute the following expectations:

$$E_{i,t}\widehat{z}_t = \mathbf{e}_1'\frac{\mathcal{M}^*(L) - \mathcal{M}^*(0)}{\rho_z L}\varepsilon^*_{i,t} \tag{B.2.21}$$

$$E_{i,t}\widehat{Y}_t = \mathbf{e}_4'\frac{\mathcal{M}^*(L) - \mathcal{M}^*(0)}{L}\varepsilon^*_{i,t} \tag{B.2.22}$$

$$\begin{aligned} E_{i,t}\sum_{j=0}^{\infty}\beta^{j+1}\widehat{r}_{t+j+1} &= \mathbf{e}_3'\frac{L\mathcal{M}^*(L) - \beta\mathcal{M}^*(\beta)}{L-\beta}\varepsilon^*_{i,t} - \widehat{r}_t \\ &= \mathrm{e}_4'\left[\beta\frac{\mathcal{M}^*(L) - \mathcal{M}^*(\beta)}{L-\beta}\right]\varepsilon^*_{i,t} \end{aligned} \tag{B.2.23}$$

$$\begin{aligned} E_{i,t}\sum_{j=0}^{\infty}\beta^{j+1}\widehat{Y}_{t+j} &= \mathbf{e}_4'\frac{L\mathcal{M}^*(L) - \beta\mathcal{M}^*(\beta)}{L-\beta}\varepsilon^*_{i,t} - \widehat{Y}_{t-1} \\ &= \mathrm{e}_4'\left[\beta\frac{\mathcal{M}^*(L) - \mathcal{M}^*(\beta)}{L-\beta}\right]\varepsilon^*_{i,t} \end{aligned} \tag{B.2.24}$$

where $\mathbf{e}_i$ is a 4×1 selection vector with 1 in the i-th position and 0 in all others.

Next, I plug these into equations (B.2.14) and (2.44) and match the z-transforms of the lag polynomials $\{\mathcal{K}_j(L), \mathcal{Y}_j(L)\}$. Finally, the lag polynomials $\{\mathcal{K}_j(L), \mathcal{Y}_j(L)\}$ are not analytic inside the disc at one point. I set $\{\mathcal{K}_j(\beta), \mathcal{Y}_j(\beta)\}$ to remove these singularites. This completes the solution.

B.3 Symmetric Information Economy

B.3.1 Housheholds

Households and firms on each island have symmetric information both within and across islands. Agents can observe all the shocks. Thus, in this section, all agents in the economy have the same information set and also do not need to filter the true fundamentals from market signals which implies the following:

$$E_t[\cdot] \equiv E[\cdot \mid \mathcal{I}_{i,t}] = E[\cdot \mid \mathcal{I}_t] \ , \forall i \in [0,1]$$
$$\overline{E}_t[\cdot] \equiv \int_0^1 E[\cdot \mid \mathcal{I}_{i,t}]di = E[\cdot \mid \mathcal{I}_t] = E_t[\cdot]$$

Thus, the equations describing the symmetric information model are the same equations as in Appendix B.2, setting $E_{i,t} = E_t$, $\forall i \in [0,1]$. Thus, the key equations which describe the decisions of agents are as follows.

The decisions of the household on island i can be expressed as:

$$\nu \widehat{H}_{i,t} = \widehat{\omega}_{i,t} \tag{B.3.1}$$

$$\begin{aligned}
\widehat{K}_{i,t+1} &= \widehat{K}_{i,t} - \frac{\alpha\beta\Psi\nu(\theta-1)H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\widehat{r}_t \\
&+ \left[\frac{\beta r}{\kappa K} + \frac{\alpha\nu(1-\beta)(\theta-1)\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right] E_t \sum_{j=0}^{\infty} \beta^{j+1}\widehat{r}_{t+j+1} \\
&- \left[\frac{Y}{\theta K} + \frac{\nu\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right]\left[\left(\frac{1-\beta}{\beta}\right) E_t \sum_{j=0}^{\infty} \beta^{j+1}\widehat{Y}_{t+j} - \widehat{Y}_t\right] \\
&- \left[\frac{(\theta-1)Y}{\theta K} + \frac{\nu(\theta-1)\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right]\left[\frac{(1-\beta)\beta\rho_z}{1-\beta\rho_z}\widehat{z}_t - \beta\widehat{z}_{i,t}\right]
\end{aligned} \tag{B.3.2}$$

$$\begin{aligned}
C\widehat{C}_{i,t} - \Psi N^{1+\nu}\widehat{H}_{i,t} &= -K\left[\widehat{K}_{i,t+1} - \frac{\widehat{K}_{i,t}}{\beta}\right] + \frac{Y}{\theta}\widehat{Y}_t + \frac{(\theta-1)Y}{\theta}\widehat{z}_{i,t} \\
&+ \nu\Psi H^{1+\nu}\widehat{H}_t
\end{aligned} \tag{B.3.3}$$

Equation (B.3.1) is the labor supply, equation (B.3.2) is the savings decision, and equation (B.3.3) is the budget constraint.

B.3.2 Firms

The labor and capital demand for firm i is given by

$$\begin{aligned}
\widehat{L}_{i,t} &= \frac{\theta-1}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{z}_{i,t} - \frac{\alpha(\theta-1)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{r}_t \\
&+ \frac{1}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{Y}_t
\end{aligned} \tag{B.3.4}$$

$$\begin{aligned}
\widehat{J}_{i,t} &= \frac{(\theta-1)(1+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{z}_{i,t} - \frac{1-\alpha+\theta(\alpha+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{r}_t \\
&+ \frac{1+\nu}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{Y}_t
\end{aligned} \tag{B.3.5}$$

The output of firm i is given by:

$$\begin{aligned}\widehat{Y}_{i,t} &= \frac{\theta(1+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{z}_{i,t} - \frac{\alpha\theta(1+\nu)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{r}_t \\ &\quad + \frac{1+\alpha\nu}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{Y}_t \end{aligned} \tag{B.3.6}$$

B.3.3 Market Clearing

B.3.3.1 Labor Market Equilibrium

Since labor is immobile across islands, the labor market on each island has to clear in equilibrium. Employment $\widehat{N}_{i,t}$ is chosen by firm i taking the labor supply function as given. Equilibrium employment $\widehat{N}_{i,t}$ is given by:

$$\begin{aligned}\widehat{N}_{i,t} &= \frac{\theta-1}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{z}_{i,t} - \frac{\alpha(\theta-1)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{r}_t \\ &\quad + \frac{1}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{Y}_t \end{aligned} \tag{B.3.7}$$

The wage on each island is given by:

$$\begin{aligned}\widehat{\omega}_{i,t} &= \frac{\nu(\theta-1)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{z}_{i,t} - \frac{\alpha\nu(\theta-1)}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{r}_t \\ &\quad + \frac{\nu}{1+\nu[\alpha+\theta(1-\alpha)]}\widehat{Y}_t \end{aligned} \tag{B.3.8}$$

Aggregate employment is given by:

$$\widehat{N}_t = \frac{\theta - 1}{1 + \nu[\alpha + \theta(1-\alpha)]}\widehat{z}_t - \frac{\alpha(\theta - 1)}{1 + \nu[\alpha + \theta(1-\alpha)]}\widehat{r}_t + \frac{1}{1 + \nu[\alpha + \theta(1-\alpha)]}\widehat{Y}_t \tag{B.3.9}$$

B.3.3.2 Capital Market Equilibrium

Total demand for capital is given by

$$\widehat{J}_t = \frac{1+\nu}{\nu(1-\alpha)}\widehat{z}_t - \frac{\alpha+\nu}{\nu(1-\alpha)}\widehat{r}_t \tag{B.3.10}$$

The law of motion of aggregate capital can be found by aggregating equation (B.3.2):

$$\begin{aligned}\widehat{K}_{t+1} = {} & \widehat{K}_t - \frac{\alpha\beta\Psi\nu(\theta-1)H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\widehat{r}_t \\ & + \left[\frac{\beta r}{\kappa K} + \frac{\alpha\nu(1-\beta)(\theta-1)\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right] E_t \sum_{j=0}^{\infty} \beta^{j+1}\widehat{r}_{t+j+1} \\ & - \left[\frac{Y}{\theta K} + \frac{\nu\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right]\left[\left(\frac{1-\beta}{\beta}\right) E_t \sum_{j=0}^{\infty} \beta^{j+1}\widehat{Y}_{t+j} - \widehat{Y}_t\right] \\ & - \left[\frac{(\theta-1)Y}{\theta K} + \frac{\nu(\theta-1)\Psi H^{1+\nu}}{(1+\nu[\alpha+\theta(1-\alpha)])\,K}\right]\left[\frac{(1-\beta)\beta\rho_z}{1-\beta\rho_z}\widehat{z}_t - \beta\widehat{z}_t\right]\end{aligned} \tag{B.3.11}$$

Market clearing at time t requires that the rental rate $\widehat{r}_t$ equate $\widehat{K}_t$ and $\widehat{J}_t$:

$$\widehat{r}_t = \frac{1+\nu}{\alpha+\nu}\widehat{z}_t - \frac{\nu(1-\alpha)}{\alpha+\nu}(\widehat{K}_t + \zeta_t) \tag{B.3.12}$$

B.3.3.3 Goods Market Equilibrium

Total output is exhausted by consumption, investment

$$\widehat{Y}_t = \frac{1+\nu}{\alpha+\nu}\widehat{z}_t + \frac{\alpha(1+\nu)}{\alpha+\nu}(\widehat{K}_t + \zeta_t) \tag{B.3.13}$$

B.3.4 Solution

As specified in the main text, I look for the unique equilibrium in which all island specific variables lie in $\mathcal{H}_t^\beta(\varepsilon_i)$ the aggregate quantities and prices lie in the space $\mathcal{H}_t^\beta(\varepsilon)$, i.e.all island i specific prices and quantities at time t are time-independent beta-summable linear functions of current and past realizations of shocks $\{v_{t-s}, e_{i,t-s}, \zeta_{t-s}\}_{s=0}^\infty$. As a consequence, all aggregate variables at time t are time independent beta-summable linear combinations of the current and past realizations of the sequence of shocks $\{v_{t-s}, \zeta_{t-s}\}_{s=0}^\infty$. I use the method of undetermined coefficients to compute the equilibrium.

Conjecture that the the law of motion of aggregate capital is given by:

$$\widehat{K}_{t+1} = \lambda_k \widehat{K}_t + \lambda_{k,z}\widehat{z}_t + \lambda_{k,\zeta}\zeta_t \tag{B.3.14}$$

where $\lambda_k < \sqrt{\beta}$ so as to restrict the solution to $\mathcal{H}_t^\beta(\varepsilon)$. Plugging in equations (B.3.12) - (B.3.14) into equation (B.3.11) and matching coefficients on K_t, z_t and ζ_t yields a system of equations in $\lambda_k, \lambda_{k,z}$ and $\lambda_{k,\zeta}$. The system is not presented in the text because it is intractably large to format. The system is quadratic in λ_k, and conditioning on λ_k, the system is linear in $\lambda_{k,z}$ and $\lambda_{k,\zeta}$. I choose the the solution of λ_k such that it lies in the disk of radius $\sqrt{\beta}$. Given the calibration, there is only one solution satisfying this requirement. Thus, the equilibrium is unique.

B.4 Policy

The planner chooses the sequences $\{\{C_{i,t}, H_{i,t}, K_{i,t+1}, J_{i,t}\}_{i\in[0,1]}\}_{t=0}^{\infty}$ to maximize the average welfare in the economy. It is not straight-forward to think of a welfare criteria in an environment with heterogeneous beliefs. I use the The policymaker is restricted to respect the beliefs of the household on each island and thus maximizes the sum of expected utilities for the household on each island where the expected utility on each island is calculated with respect to the beliefs on that island. The problem of the planner can be written as:

$$\max \int_0^1 E_{i,0} \sum_{t=0}^{\infty} \beta^t \ln \left\{ C_{i,t} - \Psi \frac{N_{i,t}^{1+\nu}}{1+\nu} \right\} di$$

subject to

$$C_{i,t} + K_{i,t+1} - (1-\delta)K_{i,t} = Y_t^{\frac{1}{\theta}} \left[z_{i,t} J_{i,t}^{\alpha} H_{i,t}^{1-\alpha} \right]^{\frac{\theta-1}{\theta}} + r_t(K_{i,t} - J_{i,t}) \quad \text{(B.4.1)}$$

$$\int_0^1 J_{i,t} di = \int_0^1 K_{i,t} di + \zeta_t \quad \text{(B.4.2)}$$

$$Y_t = \left[\int_0^1 \left(z_{i,t} J_{i,t}^{\alpha} H_{i,t}^{1-\alpha} \right)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \quad \text{(B.4.3)}$$

The optimal allocations which solve the planners problem is given by

$$Y_{i,t}^{\frac{\theta-1}{\theta}}\left[\frac{1}{\theta-1}\int_0^1 \frac{(1+\nu)C_{i,t}-\Psi H_{i,t}^{1+\nu}}{(1+\nu)C_{j,t}-\Psi H_{j,t}^{1+\nu}} Y_{j,t}^{\frac{\theta-1}{\theta}} E_{j,t} Y_t^{\frac{2-\theta}{\theta}} dj + E_{i,t} Y_t^{\frac{1}{\theta}}\right] = \frac{\theta \Psi H_{i,t}^{1+\nu}}{(\theta-1)(1-\alpha)}, \forall i \tag{B.4.4}$$

$$Y_{i,t}^{\frac{\theta-1}{\theta}}\left[\frac{1}{\theta-1}\int_0^1 \frac{(1+\nu)C_{i,t}-\Psi H_{i,t}^{1+\nu}}{(1+\nu)C_{j,t}-\Psi H_{j,t}^{1+\nu}} Y_{j,t}^{\frac{\theta-1}{\theta}} E_{j,t} Y_t^{\frac{2-\theta}{\theta}} dj + E_{i,t} Y_t^{\frac{1}{\theta}}\right] = \frac{\theta r_t J_{i,t}}{\alpha(\theta-1)}, \forall i \tag{B.4.5}$$

$$\beta E_{i,t}\left[\frac{(1+\nu)C_{i,t}-\Psi H_{i,t}^{1+\nu}}{(1+\nu)C_{i,t+1}-\Psi H_{i,t+1}^{1+\nu}}(1+r_{t+1}-\delta)\right] = 1, \forall i \tag{B.4.6}$$

To decentralize the efficient allocations, the planner can use taxes and subsides to affect the decisions of households and firms. Taxes and subsidies are only functions of aggregate output. The problem of household i with taxes can be written as:

$$\max_{\{C_{i,t},K_{i,t+1},H_{i,t}\}_{t=0}^{\infty}} E_{i,0}\sum_{t=0}^{\infty}\beta^t \ln\left\{C_{i,t}-\Psi\frac{H_{i,t}^{1+\nu}}{1+\nu}\right\}$$

subject to

$$C_{i,t}+K_{i,t+1} = \omega_{i,t}H_{i,t}+(1+r_t-\delta)K_{i,t}+(1-\tau_t^{\pi})\Pi_{i,t}$$

where $\Pi_{i,t}$ is the profit earned by firm i. This yields the familiar optimality conditions:

$$\begin{aligned} \omega_{i,t} &= \Psi H_{i,t}^{\nu} && \text{(B.4.7)} \\ 1 &= \beta E_{i,t}\left\{\frac{(1+\nu)C_{i,t}-\Psi H_{i,t}^{1+\nu}}{(1+\nu)C_{i,t+1}-\Psi H_{i,t+1}^{1+\nu}}(1+r_{t+1}-\delta)\right\} && \text{(B.4.8)} \end{aligned}$$

The problem of firm i can be written as:

$$\max_{\{H_{i,t},J_{i,t}\}} \mathrm{E}_{i,t}\left\{(1+\tau_t^s)Y_t^{\frac{1}{\theta}}\left(z_{i,t}J_{i,t}^{\alpha}H_{i,t}^{1-\alpha}\right)^{\frac{\theta-1}{\theta}}-(1+\tau_t^h)\Psi H_{i,t}^{1+\nu}-r_tJ_{i,t}\right\}$$

The tax authority is given the following instruments: a proportional sales subsidy τ_t^s, a proportional payroll tax τ_t^h, a tax on dividends τ_t^{π}. Note that these taxes/subsidies are uniform across islands. Respecting the informational constraints requires these taxes to be contingent on quantities which are common knowledge. Aggregate output is observed at the end of the period and I assume that taxes/subsidies only depend on the realization of aggregate output. More precisely,

$$\tilde{\tau}_t^s = \phi^s \ln\frac{Y_t}{Y} \tag{B.4.9}$$

where $\tilde{\tau}_t^s$ is the deviation of the sales subsidy from its steady state value of $\frac{1}{\theta-1}$. Similarly, define the deviation of the payroll subsidy from steady state by $\tilde{\tau}_t^h$ from its steady state value of $\frac{-\nu}{1+\nu}$ as follows:

$$\tilde{\tau}_t^h = \phi^h \ln\frac{Y_t}{Y}$$

The lump sum tax every period ensures that the budget is balanced.

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